PROCEEDINGS

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VOLUME XVII.

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VOL. XVII.

PLATES.

PROCEEDINGS

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The Effects of Hypertonic Solutions upon the Eggs of Echinus.

By J. GRAY, B.A., King's College. (Communicated by Mr F. A. Potts.)

[*Read* 28 October 1912.]

IN the *Proceedings* of this Society for last year, a preliminary account was given of the cytology of hybrid eggs obtained from the two species *Echinus esculentus* and *Echinus acutus.* The structure of such eggs was given as follows:

"In the cross esculentus ♀ × acutus ♂ the mitotic figures of the segmenting egg are perfectly normal, and do not differ recognisably from those of the pure species. In the converse cross acutus ♀ × esculentus ♂, however, a striking abnormality is constantly present in all the eggs examined. Until immediately after the dissolution of the nuclear membrane in the first segmentation division the behaviour is normal, and 38 normal chromosomes can be counted. As the spindle is formed, the chromosomes become scattered upon it irregularly, and gradually become collected in the equatorial plate. During this process it is seen that a considerable though variable number of them are either swollen up, or more commonly bear vesicles attached to their ends and sides. The staining of the vesicles is always less intense than that of the chromosomes, and is progressively fainter, the more the vesicle is developed, so giving the impression that the chromosome has swollen at one point, and that the chromatin is thus more thinly diffused in the wall of the vesicle than in the normal part of the chromosome. In the equatorial plate stage the vesicles may either remain attached to the chromosomes which produced them, or become separated from them; those which

become separated tend to take up positions round the edge of the equatorial plate, sometimes outside the spindle. The normal chromosomes, and those of which the normal shape has not become much altered by vesicle-production, then split longitudinally in the ordinary way, and begin to travel to the poles. It may sometimes be seen that a chromosome with a vesicle attached has split, and the vesicle, remaining attached to one half, is being carried with it towards the pole. It is possible that a few chromosomes, the greater part of which has become swollen into a vesicle, do not divide, but are carried entire to one or other pole. The vesicles which have become separated from their parent chromosomes appear to differ in their fate according to their position. If they lie among the chromosomes inside the spindle, they are carried with them to one or other pole and become included in the daughter nuclei. If, however, they are left on the edge of the spindle, as commonly happens with the larger vesicles, they remain outside the mitotic figure in the cytoplasm, and are not included in the nuclei of the daughter cells. In this case they usually contract and become small evenly stained spheres, not easily distinguishable from the larger yolk-granules, but usually recognisable after the cell-division is completed, lying in the cytoplasm near the boundary between the two cells.

In the second segmentation division, a similar process takes place, but is usually rather less pronounced; the vesicles are on the whole smaller, and we doubt whether complete chromosomes ever become vesicular*."

A similar phenemenon within the eggs of Echinoderms has recently been described by Konopacki. He treated the fertilised eggs of *Strongylocentrotus lividus* with hypertonic solutions of certain strengths for half an hour, and subsequently transferred to sea-water. Although no detailed description is given of the behaviour of the chromatin, a study of his figures leads to the conclusion that the effect of such solutions upon the fertilised eggs of *Echinus esculentus* and *E. acutus* would throw some light upon the cytology of hybrids derived from these species. The following are the chief results of such experiments carried out this year at Plymouth.

Eggs of *E. esculentus* were fertilised and allowed to remain in fresh sea-water for one hour: they were then transferred to a mixture of 50 c.c. sea-water + 6 c.c. $2\frac{1}{2}$ M. NaCl solution, and after half an hour were transferred to fresh sea-water until the first mitotic figure had been formed.

The eggs were then preserved and sectioned: the stain used being Heidenhain's Haematoxylin. Such preparations shewed that the most interesting effect of the hypertonic solution was to

* *Proc. Camb. Phil. Soc.* Vol. xvi. Pt. v. 1911, p. 415. Doncaster and Gray.

cause the formation of one vesicle in each nucleus exactly similar in appearance and behaviour to those seen in eggs of the hybrid *E. acutus* ♀ × *E. esculentus* ♂. There was, however, this important difference; whereas in the hybrid eggs the vesicles are formed directly from individual chromosomes and are never present inside the nuclear membrane, those in the hypertonic eggs of *E. esculentus* are always formed inside the nuclear membrane of the female pronucleus. It was found impossible to determine whether the number of chromosomes formed from such a nucleus was one less than the full somatic number for the species, but what evidence there is, is certainly not unfavourable to the view that this vesicle represents morphologically either the whole or part of one chromosome. It resembles the vesicles of the hybrid eggs in being either omitted from or included in the daughter nuclei of the first division.

The cytology of eggs of *E. acutus* which were treated in exactly the same way as is above described for *E. esculentus*, shews a marked difference between the two species. Within the nuclear membrane of the zygote nuclei, it is only very exceptional to find any trace of vesicles. As soon as the nuclear membrane disappears, however, numerous vesicles are found among the chromosomes. Their number varies from three to five, and there is absolutely no doubt that they are formed directly from individual chromosomes just as in the hybrid *E. acutus* ♀ × *E. esculentus* ♂, for not only is the number of normal chromosomes correspondingly reduced, but every stage between typical chromosomes and fully formed vesicles has been found. If it could be shewn that the vesicles are always derived from the same individual chromosomes in the complex, there would be an interesting proof of the physiological individuality of these bodies. Unfortunately so far this has been found to be impracticable.

The ultimate effect of these hypertonic solutions on the eggs of both species is to cause irregularity in segmentation of the cytoplasm. In no case did an egg develop beyond the blastula stage.

These experiments shew, therefore, that in the formation of vesicles and the consequent elimination of chromosomes, which normally takes place in the nuclei of the hybrid *E. acutus* ♀ × *E. esculentus* ♂, it is essentially the female chromatin that becomes pathological. It is extremely unfortunate that it is impossible to test this hypothesis by a study of the hybrid eggs themselves; yet the effects of hypertonic solutions on the eggs of the two species gives strong support for such an assumption.

It will be remembered, however, that the mitoses of the hybrid *E. esculentus* ♀ × *E. acutus* ♂ are quite normal, and therefore we cannot explain the pathological condition of the reverse cross by

the simple statement that the chromosomes of *E. acutus* are more susceptible to changed environment than those of *E. esculentus*. In order to explain these facts I would put forward the following suggestion in the most tentative way. It is based on the view that the relation of a cell to electrolytes in its surrounding media is of prime importance to the existence of that cell. The work of McClendon and R. S. Lillie has shewn that after fertilisation the surface membrane of the egg is more permeable to ions than before. It has also been demonstrated that the CO_2 production of a dividing cell changes just before each division, *i.e.*, there is a slight increase in permeability*. It is also highly probable that the effect of hypertonic solution upon a cell is to decrease its permeability to ions. (Sutherland; Lillie.)

The whole of the evidence in favour of the view that the permeability changes of the egg membranes (plasmic and nuclear) are of profound importance to the activities of the cell cannot be discussed in full, but the following extract from a recent paper by R.S. Lillie has a peculiar interest in connection with the phenomena discussed in this paper.

"The conclusion that many pathological conditions have their primary origin in abnormalities of the limiting membranes of cells is an obvious corollary of any view that regards such membranes—which are essentially insulating surface films of varying ionic permeability and electrical polarization—as largely controlling the rate and character of the cell processes. If stimulation depends primarily upon altered polarization of the plasma-membrane due to increased ionic permeability, it is clear that a normal response, in the case of any cell, implies a definite condition of the membrane. If this condition is permanently altered, the cell processes inevitably undergo derangement, and pathological changes follow. Such a deranged condition, if not too far advanced, may be rectified by restoring the membrane to its normal condition....The alteration caused by a toxic agent may consist primarily either in increasing or in decreasing the permeability normal to the membrane, or in altering in either direction the readiness with which the latter undergoes change....The plasma membrane cannot undergo marked and prolonged increase of permeability without alteration in the nature and proportion of the cell-constituents; this involves altered chemical organization and eventual derangement of the cell-processes."

It is, therefore, a possibility at least that the hypertonic solutions exert a toxic action upon the nucleus of a fertilised egg by upsetting the normal relationship of the cytoplasm to electrolytic ions.

* The susceptibility of a dividing cell to poisons has also been shewn to be rhythmical in the same way.

Now, the permeability of the egg is changed when a spermatozoon enters, and presumably the change is constant in degree for each species. When, however, the sperm of a foreign species is made to enter an egg, is it not possible that the change in permeability is not that which would have been caused by a sperm of the species to which the egg belongs? If this is so, and the degree of change in permeability of an egg when fertilised is therefore a function of the sperm, then the cytological behaviour of reciprocal crosses is explicable.

Let the change in permeability for

	E. acutus eggs (fertilised	by)	*E. acutus* sperm	be	P,
	E. esculentus	„ „	*E. esculentus*	„	P_1.
Then	*E. esculentus*	„ „	*E. acutus*	is	P,
and	*E. acutus*	„ „	*E. esculentus*	„	P_1.

Let the difference between P and P_1 be about equal to the change of permeability in normally fertilised eggs of *E. acutus* which is brought about by the action of hypertonic solutions of appropriate strength.

Now the chromatin of *E. esculentus* can withstand a change of permeability in the surrounding protoplasm equal to $P-P_1$, without becoming abnormal*, as is shewn by its reaction to a hypertonic solution capable of producing such a change. Hence when the egg of *E. esculentus* is fertilised by the sperm of *E. acutus*, and so attains the permeability peculiar to fertilised eggs of *E. acutus*, no abnormality occurs.

On the other hand, when the egg of *E. acutus* is fertilised by the sperm of *E. esculentus*, the *E. acutus* element becomes pathological, as it cannot endure a permeability in its surrounding protoplasm equal to that characteristic of an egg of *E. esculentus*.

This suggestion is, however, in entire opposition to most of the conclusions of other workers in hybridisation. The work of Kupelwieser, Baltzer and Tennent all tends to shew that it is the male chromatin which becomes abnormal in such cases. Tennent, however, has shewn that in the cross *Arbacia* × *Toxopneustes* not only are all the chromosomes derived from the male eliminated, but also some of those from the female parent. Again, G. Hertwig has shewn that fusion with an abnormal sperm can cause the female chromatin of a normal egg to be affected.

If the abnormalities are to be entirely confined to the male element in the egg, there is apparently no explanation for the cytology of such reciprocal crosses as *Sphaerechinus* × *Strongylocentrotus*, or *Echinus acutus* × *E. esculentus*; for if, as Baltzer

* Presumably the change $P-P_1$ is a little less than that caused in eggs of *E. esculentus* by 50 c.c. sea-water + 6 c.c. $2\frac{1}{2}$ M. NaCl, as in the latter case one vesicle is produced, while in the hybrid *E. acutus* × *E. esculentus* no such abnormality occurs.

suggests, the male chromatin goes wrong because it is "out of tune" with the female cytoplasm, why are not both reciprocal crosses abnormal? And in the case of *E. acutus* × *E. esculentus* we should expect the more sensitive *E. acutus* chromatin to go wrong when it enters the cytoplasm of the more resistent *E. esculentus*, and this is not the case.

The proof of the hypothesis here put forward would be in the demonstration of the fact that the permeability change of an egg when fertilised differs according to the species of sperm used, in other words, the degree of permeability change should be a function of the sperm*.

A more detailed account of this work will appear in the forthcoming number of the *Quarterly Journal of Microscopical Science* (Vol. LVIII.), where references will be found to all the works quoted in this paper.

* As in some cases of hybridisation there is no doubt whatever that it is the male chromatin that becomes abnormal it is obvious that changes of permeability of the cytoplasm, such as can be induced by the sperm, cannot be a sufficient explanation of all the abnormalities observed in cross-fertilised eggs.

Preliminary Note on the Inheritance of Self-sterility in Reseda odorata. By R. H. Compton, M.A., Gonville and Caius College.

[*Read* 28 October 1912.]

Among the numerous examples of self-sterility recorded for plants and animals the Mignonette is exceptional in that, as was discovered by Charles Darwin, certain individuals are completely self-sterile, others completely self-fertile. Clearly this affords a favourable opportunity for breeding experiments with the object of studying the inheritance of these obscure and puzzling phenomena. Each flower is normally self-pollinated, so that it is only necessary to exclude insects in order to ascertain whether a plant is self-fertile or self-sterile. Further the anthers are freely exposed before dehiscence, so that, although the flowers are small, emasculation is easy.

Seed was obtained from various tradesmen, and among the plants grown from it both classes of individuals were abundant. Experiments are in progress, and the results of the first generation may be described with the tentative hypothesis to which they point.

This hypothesis is that self-fertility is a simple Mendelian dominant character. In support of it may be mentioned the following facts:

(1) Self-sterile plants when bred *inter se* throw self-sterile offspring only. This is in accordance with the view that self-sterility is a Mendelian recessive.

(2) Certain self-fertile plants when self-fertilised yield self-fertile offspring only: when crossed with self-sterile plants the same result is obtained. These are regarded as homozygous for self-fertility.

(3) Other self-fertile plants when self-fertilised yield approximately three self-fertile to one self-sterile offspring: when crossed with self-sterile plants about half the progeny are self-fertile, half self-sterile. These are regarded as heterozygotes.

The publication of the data in full is deferred until the completion of the experiments, as is also discussion with reference to the literature.

Other characters are also being studied. As regards stature it appears that pure-breeding tall and dwarf races exist, and that the F_1 between them is intermediate in height.

An interesting pollen-character also seems to behave in a Mendelian fashion. Orange-red colour of pollen appears to be a simple dominant to bright yellow: self-fertilised heterozygotes throw about three reds to one yellow.

On the Anthropometric data collected by Professor J. Stanley Gardiner, F.R.S., in the Maldive Islands and Minikoi. By W. L. H. Duckworth, M.D., Sc.D., Jesus College.

[*Read* 28 October 1912.]

In the course of an expedition to the Maldive Islands and Minikoi, Professor J. Stanley Gardiner, F.R.S., collected a number of anthropometric data. These he handed to me, and they form the material upon which the following report is based.

1. The total number of individuals examined is 69. All are males. Of the 69, 20 are Minikoi men, while of the remainder 24 are from Addu Atoll. Other islets of the Maldive group supply a few representatives each. The complete list is as follows:

Island		*No. of individuals measured*
Minikoi		20 (caste is said to be of little moment).
Maldives:	Addu	24 (in 4 sets according to caste).
	Male	11 (in 3 sets according to caste).
	Hulule	9
	Kaharidu	1
	N. Mahlos	1
	Mulaku	1
	Nolewangfaro	2
	Total	69

2. Professor Gardiner records 13 measurements of each of these men. In addition to these data, five indices have been worked out. In preparing the list of the mean values and the range of variation I have been greatly aided by Dr Poole, of Sidney Sussex College. Dr Poole intended to prepare the whole of the report, but the claims of professional work made it necessary for him to hand the material back to me when he had nearly completed the determination (previously begun by me) of the means, the maximum and minimum values. As a result of all this, I am able to deal with the data re-arranged in the following list:

A.

I. Stature.
II. Height seated, *i.e.* height of torso.
III. Cephalic length.
IV. Cephalic breadth.
V. Face length.
VI. Upper face length.
VII. Face breadth.
VIII. Bigonial breadth.
IX. Nose length.
X. Nose breadth.
XI. Cephalic height.
XII. Cranial height.
XIII. Circumference of head.

B. Indices.

Cephalic.	Nasal.
Altitudinal.	Gonio-zygomatic.
Facial (Kollmann's).	

These data are arranged in three tables, of which No. I contains the original measurements made by Professor Gardiner, together with some of his comments on the same. Table II gives the complete list of indices, while in Table III, the means, the maxima and the minima are set forth in relation to the different groups of individuals described in paragraph 1 (*supra*). The absolute dimensions added to the indices give a total of 18 characters (cf. A and B above) for examination.

3. It is convenient to enquire first into the extent of variation and the manner in which it is exhibited by the several groups of men. For this purpose, the records of the maxima and minima as set forth in Table III may be employed. An examination of Table III leads to the following conclusions in this connection.

The maxima and minima are shared in the proportions given as follows:

Minimum values ... Minikoi 16 out of 18 characters.
Hulule 2 " " (sharing the lowest place once with Minikoi).
Addu 1 out of 18 characters.

Maximum values ... Minikoi 4 out of 18 characters.
Male 7 " "
Addu 7 " "

The tables shew clearly that the men of Minikoi are of smaller dimensions on the whole than the men from the Maldives. In addition to this, the Minikoi men are the most variable of the groups into which the data have been divided.

4. The standard deviation and the coefficient of variation will provide further evidence on the same point, viz. the relative variability of the different groups. Here we may begin the examination by a scrutiny of the seriations upon which the calculations of these data are based. Eight measurements and four indices are here available for study, but the analysis is not suitable for discussion in this place and it is presented in a tabulated form. Only a summary will be given here, and it is to the following effect.

(*a*) Stature: The Minikoi men are nearly at the bottom of the list in this respect. The Addu men (the most numerous group and therefore most fitly comparable) are the tallest and thus at the opposite end of the scale. The remaining groups are intermediate,

TABLE I. *Maldive Islands. No. 1. All*

Number	Height standing I	Height sitting II	Length of head III	Breadth of head IV	Face length V	Upper face length VI	Face breadth VII	Bigonial breadth VIII	Nose length IX	Breadth of nose X	Height of head XI	Height of cranium XII
1	148·9	75·8	19·1	14·0	10·5	6·1	13·2	11·5	5·1	3·8	20·5	12·4
2	156·8	73·3	17·8	13·6	11·1	6·3	12·6	10·3	5·3	3·5	19·9	12·0
3	160·4	81·1	20·4	14·6	10·8	6·1	12·5	11·1	4·5	3·4	21·8	12·[illegible]
4	163·2	84·2	19·5	15·0	11·1	6·2	12·2	10·0	5·0	3·7	22·3	13·0
5	161·4	80·7	18·9	15·4	11·2	6·3	13·5	11·1	5·1	4·2	21·2	11·[illegible]
6	150·4	76·0	19·0	14·0	10·4	5·9	12·6	10·1	4·7	2·8	21·6	12·[illegible]
7	155·9	79·5	19·6	15·3	11·2	6·4	13·3	10·8	4·9	3·5	22·4	12·[illegible]
8	153·2	83·1	17·8	14·4	10·8	6·5	12·8	11·6	5·5	3·3	23·6	12·[illegible]
9	147·8	79·2	18·2	14·4	10·8	5·8	12·7	10·2	4·8	3·3	20·5	12·[illegible]
	1398·0	*712·9*	*170·3*	*130·7*	*97·9*	*55·6*	*115·4*	*96·7*	*44·9*	*31·5*	*193·8*	*112·[illegible]*
10	156·7	83·7	20·0	15·7	10·7	6·5	13·6	11·6	4·2	3·8	21·1	13·[illegible]
11	172·9	89·2	18·5	14·3	11·2	6·7	13·9	13·1	5·0	4·2	24·2	13·[illegible]
12	158·1	80·9	19·4	16·0	11·7	6·5	14·5	13·0	4·8	4·5	22·8	13·[illegible]
13	173·6	88·5	20·5	16·0	11·4	7·0	14·2	12·0	5·1	3·7	25·0	14·[illegible]
	661·3	*342·3*	*78·4*	*62·0*	*45·0*	*26·7*	*56·2*	*49·7*	*19·1*	*16·2*	*93·1*	*55·[illegible]*
14	161·3	83·2	20·0	15·4	12·1	6·4	14·3	11·7	4·9	3·9	21·0	12·[illegible]
15	158·4	73·4	17·9	14·1	10·0	5·4	12·5	10·7	4·8	3·5	21·3	12·[illegible]
16	158·1	79·5	20·2	14·4	11·4	6·8	13·3	11·2	5·2	3·6	21·6	12·[illegible]
	477·8	*236·1*	*58·1*	*43·9*	*33·5*	*18·6*	*40·1*	*33·6*	*14·9*	*11·0*	*63·9*	*37·[illegible]*
17	155·6	76·8	18·8	14·3	11·3	6·8	13·0	11·7	5·3	4·0	22·2	13·[illegible]
18	147·4	79·9	18·6	14·2	10·9	6·4	12·0	9·7	5·2	3·3	22·4	13·[illegible]
19	156·7	78·2	18·6	14·6	12·2	7·5	12·8	11·0	5·7	3·7	21·7	11·[illegible]
20	161·5	79·4	19·0	14·8	11·6	6·9	12·7	10·4	4·9	3·7	21·9	12·[illegible]
	621·2	*314·3*	*75·0*	*57·9*	*46·0*	*27·6*	*50·5*	*42·8*	*21·1*	*14·7*	*88·2*	*50·[illegible]*
21	150·8	72·5	19·6	14·4	11·0	6·2	13·5	12·4	4·6	3·7	19·9	12·[illegible]
22	156·3	74·4	18·2	14·4	10·9	6·3	11·9	10·5	4·5	3·2	21·5	12·[illegible]
23	153·2	75·6	19·8	14·8	10·7	5·8	13·1	11·6	4·3	3·7	21·5	13·[illegible]
24	166·6	83·3	19·3	13·8	11·6	5·9	12·7	11·8	4·3	3·5	23·1	13·[illegible]
25	163·8	82·4	19·5	13·9	10·7	5·6	13·7	11·0	5·0	3·5	22·9	14·[illegible]
	790·7	*388·2*	*96·4*	*71·3*	*54·9*	*29·8*	*64·9*	*57·3*	*22·7*	*17·6*	*108·9*	*65·[illegible]*

surements in ctms. Ages approximate.

of head	Name	Occupation or caste	Island	Age	Remarks
I					
·7	Ahamada	Toddy Drawer	Hulule	24	Condition good.
·8	Mohammed	,, ,,	,,	22	,, thin.
·2	Ibrahim	Fisherman	,,	21	,, medium.
·3	Moussa	Toddy Drawer	,,	21	,, thin.
·2	Dongkoko	Fisherman	,,	33	,, good.
·6	Ibrahim	,,	,,	23	,, thin. Syphilitic.
·2	Hassan	Toddy Drawer	,,	32	,, good.
·8	Avakaru	Head man	,,	60	,, good. Stoops but teeth good. Deaf. (Photo.)
·9	Hassan	Fisherman	,,	30	Condition good.
·7					
·2	Mohamed	Didi	Male	25	Stout. High caste. (Photo.)
·5	Hassan	,,	,,	54	Good. ,, ,, Father of last. Velanamanihofan.
·1	Mohamed	Chief Vizier	,,	53	Good. Best caste.
·3	Manipul	Head of Mosque	,,	31	,, ,, (Photo.)
·1					
·3	Hosein	Overseer	,,	35	Good, stout. Muniku.
·3	Hassan	Singing Boy	,,	20	Poor, thin. ,,
·8	Ibrahim	Servant	,,	40	Good. Fulu. Sloping forehead.
·4					
·5	—	Fisherman	,,	22	Thin.
·0	—	,,	,,	30	Fair.
·1	—	,,	,,	25	Sparse.
·7	—	,,	,,	50	,,
·3					
·7	Mohammed	Servant	Kaharidu	35	Good. Marked stoop and broad jaws.
·3	Mohammed	,,	N. Mahlos	25	Thin.
·2	Ismail	,,	Mulaku	23	Fat. Vacant. Fool.
·9	Hassan	Sailor	Nolewangfaro	25	Good. My boys. Big Photos.
·1	Hosein	,,	,,	23	
·2					

TABLE I (*cont.*). *Maldive Islands. No. 2. Add*

Number	Height standing I	Height sitting II	Length of head III	Breadth of head IV	Face length V	Upper face length VI	Face breadth VII	Bigonial breadth VIII	Nose length IX	Breadth of nose X	Height of head XI	Height of cranium XII
26	161·7	81·8	20·7	14·8	10·8	6·5	13·4	10·9	5·0	3·8	23·1	12·8
27	169·5	82·7	18·9	14·1	11·5	6·2	14·0	11·5	4·7	3·9	22·7	13·9
28	160·1	79·9	20·9	15·2	12·3	7·2	13·0	11·4	5·5	3·7	23·7	13·2
29	158·2	79·7	20·6	15·1	11·7	6·2	13·5	12·5	4·7	4·2	23·3	13·9
30	156·6	76·5	19·8	14·9	11·2	6·3	13·7	12·0	4·3	3·9	22·9	13·0
31	165·7	82·1	19·9	14·8	12·3	6·7	13·3	10·6	5·2	3·6	22·6	13·8
32	155·7	82·2	19·4	15·9	11·6	6·5	13·8	11·9	5·9	3·4	22·0	13·0
	1127·5	*564·9*	*140·2*	*104·8*	*81·4*	*45·6*	*94·7*	*80·8*	*35·3*	*26·5*	*160·3*	*93·6*
33	172·0	82·1	19·0	16·2	11·4	6·7	14·7	12·9	5·2	4·0	23·8	15·0
34	155·5	78·9	19·5	13·8	10·5	5·9	12·7	10·6	4·4	3·7	20·6	12·6
	327·5	*161·0*	*38·5*	*30·0*	*21·9*	*12·6*	*27·4*	*23·5*	*9·6*	*7·7*	*44·4*	*27·6*
35	157·9	79·6	19·3	15·4	12·2	6·8	12·8	11·3	5·0	3·8	22·6	13·[illegible]
36	161·6	75·8	20·4	15·5	11·3	6·1	13·5	11·7	4·7	3·9	22·2	13·[illegible]
37	163·7	82·4	19·1	15·3	11·4	6·7	13·8	12·2	5·0	3·8	21·3	13·[illegible]
38	157·7	75·2	19·3	14·1	11·2	6·3	13·4	10·7	5·2	3·8	22·2	13·[illegible]
39	157·9	76·4	18·9	13·9	10·8	5·9	12·4	10·2	4·7	3·3	20·6	11·9
	798·8	*389·4*	*97·0*	*74·2*	*56·9*	*31·8*	*65·9*	*56·1*	*24·6*	*18·6*	*108·9*	*65·6*
40	162·2	81·0	19·5	14·4	10·5	5·8	13·4	11·2	4·5	3·8	20·1	12·6
41	155·9	80·1	18·8	14·0	9·8	5·7	13·3	11·4	4·2	3·7	21·6	12·9
42	163·1	80·6	19·5	14·3	11·2	6·5	13·2	11·3	4·9	3·7	21·3	13·[illegible]
43	155·7	75·4	19·0	14·3	11·4	5·9	12·9	11·4	4·5	3·5	21·4	14·[illegible]
44	146·6	73·4	19·5	15·7	10·8	5·9	13·0	11·4	4·4	3·5	20·7	13·[illegible]
45	158·0	74·8	19·4	14·4	12·1	6·7	13·4	10·1	4·9	3·5	21·0	12·[illegible]
46	160·5	80·6	19·9	15·2	12·3	7·3	13·7	11·5	5·6	3·6	23·7	13·9
47	166·4	82·7	19·1	14·4	12·1	6·6	14·2	13·2	5·3	3·9	23·4	12·[illegible]
48	161·7	80·8	19·5	13·8	11·5	6·4	12·8	11·2	5·4	3·4	21·2	13·[illegible]
49	166·5	80·5	20·0	15·6	12·0	6·6	14·0	11·9	5·3	4·2	23·7	13·[illegible]
	1596·6	*789·9*	*191·2*	*146·1*	*113·7*	*63·4*	*133·9*	*114·6*	*49·0*	*36·8*	*218·1*	*132·[illegible]*

l. 4 sets arranged according to castes.

of head II	Name	Occupation or caste	Island	Age	Remarks
5·8	Ibrahim	Didi	Addu	30	Good. Stout in face. Rich man. Marked depression right of head.
·4	Ibrahim	,,	,,	35	Good. Back of head quite flat. Base nose furrow very marked.
3·2	Hassan	,,	,,	30	Good.
5·6	Mahomed	,,	,,	35	Good. Nose very broad. Deep bridge.
5·3	Ali	,,	,,	32	Good.
5·7	Hosein	,,	,,	27	Fair, thin.
4·9	Hassan	,,	,,	30	Good. Nose and forehead level, no depression at base.
9·9					
5·6	Ahmed	Manikofanu	,,	37	Fair but thin.
4·1	Hosein	,,	,,	24	Good.
9·7					
5·6	Ali	Thackarufanu	,,	24	Fair.
7·1	Moussa	,,	,,	40	Fair. Low receding forehead.
6·2	Ali	,,	,,	42	Fair, thin.
4·5	Ibrahim	,,	,,	25	Good.
2·2	Hassan	,,	,,	24	Good.
5·6					
4·2	Mohamed	All poor men. Boatmen, fishermen and agriculturalists	,,	45	Fair.
2·7	Adam		,,	18	Thin.
5·5	Abdurahman		,,	40	Good. Angles of jaw very fat.
4·2	Ali		,,	22	Fat in lower part of face.
5·6	Hassan		,,	26	Good.
4·6	Hassan		,,	48	Good.
5·5	Hosein		,,	30	Good. Very thick lips and face looks long.
4·5	Hassan		,,	35	Stout. Head like ridge in centre; jaws square.
3·7	Ibrahim		,,	50	Moderate.
7·6	Hosein		,,	26	Good. Marked negroid features.
9·1					

TABLE I (cont.). *Minikoi Natives. All ♂. Measurements in ctms. Ages approximate.*

Number	Height standing I	Height sitting II	Length of head III	Breadth of head IV	Face length V	Upper face length VI	Face breadth VII	Bigonial breadth VIII	Nose length IX	Breadth of nose X	Height of head XI	Height of cranium XII	Circumference of head XIII	Name	Approx. age	Caste	Remarks
1	152·5	76·0	18·3	15·2	11·5	6·8	11·8	10·4	2·8	2·9	20·7	11·0	—	Hassan	53	Thackaru	Head broadest about 1½″ above ear.
2	147·6	71·5	18·6	14·3	9·7	6·2	12·1	10·5	4·2	2·8	21·7	12·8	54·3	Ismail	19	,,	Fat. Mongoloid eyes and rather high cheek bones.
3	150·9	74·7	18·6	14·5	10·7	5·4	13·2	10·9	4·0	3·4	21·0	15·1	54·7	Moussa	25	,,	Well grown and nourished.
4	155·3	75·1	17·7	14·3	10·5	6·0	13·6	10·8	4·3	3·7	21·9	12·8	55·0	Ismail	70?	,,	Claims to be about 80. Teeth still perfect.
5	164·4	83·8	19·6	13·9	10·2	6·1	12·9	10·6	3·8	3·2	21·2	12·5	53·5	Hassan Ali	25	Ravare	Large healthy man with good beard.
6	155·9	78·4	18·4	14·2	12·1	5·7	12·3	9·5	4·1	3·5	21·6	13·7	55·8	Ismail	65	Thackaru	—
7	165·4	80·3	18·1	14·2	10·7	5·6	12·9	10·8	4·1	3·2	22·2	13·5	55·5	Ali	30	Ravare	—
8	160·9	77·8	17·8	14·3	10·7	5·8	13·4	11·0	4·2	3·3	20·8	12·8	54·7	Ismail	35	,,	—
9	154·5	76·7	18·3	14·7	11·0	7·0	13·0	10·5	4·8	3·7	22·0	13·3	54·3	Mohammed	50	,,	Long beard going white.
10	156·9	77·3	16·0	13·3	10·5	6·3	13·0	8·8	4·0	2·8	20·6	12·4	50·9	Mohammed	55	,,	Head apparently slightly mis-shapen owing to a blow behind (?).
11	153·8	76·7	18·4	14·4	10·1	5·9	12·9	10·4	4·0	3·3	19·1	12·2	54·1	Hosein	40	,,	—
12	163·2	81·7	18·9	14·8	10·5	5·9	12·5	10·4	2·8	2·9	21·5	13·4	55·7	Hassan	25	,,	Well made. Very marked superciliary ridges.
13	154·6	77·3	18·2	14·1	10·5	6·1	12·5	10·5	4·3	2·8	21·3	12·7	52·2	Hosein	35	,,	
14	168·4	84·3	19·6	14·3	11·5	6·3	13·3	10·7	4·0	3·4	23·0	13·7	57·1	Ismail	30	,,	4—12 one district.
15	149·6	75·1	17·4	13·6	9·5	5·8	11·5	9·8	4·1	2·7	20·7	12·5	51·5	Hassan	21	Thackaru	13—19 ,,
16	170·0	82·1	17·7	13·9	10·0	5·9	12·8	10·5	3·9	1·8	20·4	13·2	52·7	Ali	23	,,	Castes have no great
17	160·3	79·7	18·0	15·0	10·2	5·8	12·5	10·5	3·9	2·8	20·9	13·1	55·5	Hosein	21	,,	significance in Minikoi.
18	152·5	77·2	17·7	14·0	9·0	6·0	12·9	10·9	4·3	2·9	22·1	12·6	55·5	Hosein	20	,,	
19	154·2	77·5	19·5	14·6	11·5	6·9	13·1	9·3	4·7	3·1	22·4	12·7	56·5	Ibrahim	35	,,	
20	163·6	83·5	17·7	14·0	10·5	6·2	12·8	11·1	4·4	3·1	22·1	13·5	51·8	Marikar	31	,,	Name is not a Minikoi one. I could not find out, but I suspect Malabar blood and partial descent. [Greater stature than average certainly confirms this. W. L. H. D.]

Table II. *Indices.*

No. of individual and locality Maldives		Cephalic	Altitudinal	Facial (Kollmann's)	Nasal	Gonio-Zygomatic
Hulule Island. Nos. 1 to 9	1	73·3	64·9	46·21	74·51	87·12
	2	76·4	67·4	50·00	60·04	81·75
	3	71·6	61·8	48·80	75·56	88·80
	4	76·9	69·7	50·82	74·00	81·97
	5	81·5	60·3	46·67	82·35	82·22
	6	73·7	64·2	47·20	59·57	80·16
	7	78·1	64·3	48·12	71·43	81·20
	8	80·9	72·5	50·78	60·00	90·63
	9	79·1	69·2	45·67	68·75	80·31
Male Island. High caste. Nos. 10 to 13	10	78·5	66·0	47·80	90·48	85·29
	11	77·3	73·0	48·13	84·00	94·24
	12	82·5	70·1	44·83	93·75	89·66
	13	78·0	71·7	49·29	72·55	84·51
Male Island. Intermediate caste. Nos. 14 to 16	14	77·0	60·5	44·75	79·59	81·82
	15	78·8	68·2	43·20	72·92	85·60
	16	71·3	62·9	51·13	69·23	84·21
Male Island. Low caste. Nos. 17 to 20	17	76·1	71·3	52·31	75·47	90·00
	18	76·3	72·0	53·33	63·46	80·83
	19	78·5	61·8	58·59	64·91	85·94
	20	77·9	64·2	54·33	75·51	81·89
Kaharidu	21	73·5	63·3	45·93	80·43	91·85
N. Mahlos	22	79·1	66·5	52·94	71·11	88·24
Mulaku	23	74·7	67·7	44·27	86·05	88·55
Nolewangfaro	24	71·5	68·9	46·46	81·40	92·91
,,	25	71·3	71·8	44·88	70·00	80·29
Addu Atoll. Nos. 26 to 49. Didi. Nos. 26 to 32	26	71·5	61·8	48·51	76·00	81·34
	27	74·6	73·5	44·28	82·98	82·14
	28	72·7	63·1	55·38	67·27	87·69
	29	73·3	67·4	45·93	89·36	92·59
	30	75·3	65·7	45·98	90·70	87·59
	31	74·4	69·3	50·38	69·23	79·70
	32	82·0	67·0	47·10	57·63	86·23
Manikofanu. Nos. 33, 34	33	85·3	78·9	45·58	76·92	87·76
	34	70·8	64·6	46·46	84·09	83·46
Thackarufanu. Nos. 35 to 39	35	79·8	67·9	53·13	76·00	88·28
	36	76·0	66·2	45·19	82·98	86·67
	37	80·1	69·6	48·55	76·00	88·41
	38	73·1	68·4	47·01	73·08	79·85
	39	73·5	63·0	47·58	70·21	82·26
"Poor" men. Nos. 40 to 49	40	73·8	64·6	43·28	84·44	83·58
	41	74·5	68·6	42·86	88·10	85·71
	42	73·3	68·2	49·24	75·51	85·61
	43	75·3	76·3	45·74	77·78	88·37
	44	80·5	67·7	45·38	79·55	87·69
	45	74·2	63·4	50·00	71·43	75·37
	46	76·4	69·8	53·28	64·29	83·94
	47	75·4	66·5	46·48	73·58	92·96
	48	70·8	69·2	50·00	62·96	87·50
	49	78·0	67·0	47·14	79·25	85·00

TABLE II (*cont.*).

No. of individual and locality Minikoi		Cephalic	Altitudinal	Facial (Kollmann's)	Nasal	Gonio-Zygomatic
Minikoi. Nos. 50 to 69	50	83·1	60·1	57·63	103·57	88·14
	51	76·9	68·8	51·24	66·67	86·78
	52	78·0	81·2	40·91	85·00	82·58
	53	80·8	72·3	44·11	86·05	79·41
	54	70·9	63·8	47·29	84·21	82·17
	55	77·2	74·5	46·34	85·37	77·24
	56	78·5	74·6	43·41	78·05	83·72
	57	80·3	71·9	43·28	78·57	82·09
	58	80·3	72·7	53·85	77·08	80·77
	59	83·1	77·5	48·46	70·00	68·46
	60	78·3	66·3	45·74	82·50	80·62
	61	74·7	70·9	47·20	103·57	83·20
	62	77·5	69·8	48·80	65·12	84·00
	63	73·0	69·9	47·37	85·00	80·45
	64	78·2	71·8	50·43	65·85	86·09
	65	78·5	74·6	46·09	46·15	82·03
	66	83·3	72·8	64·40	71·79	84·00
	67	79·1	71·2	46·51	67·44	84·50
	68	74·9	65·1	52·67	65·96	70·99
	69	79·1	76·3	48·82	70·45	86·72

if the Hulule men be excepted, for they are the shortest of all. But they are only nine in number.

(*b*) Head dimensions (Length, Breadth and Cranial Height): The Minikoi men provide the smallest heads, whether length, breadth or cranial height be taken. The Addu men come again into contrast, for they have the largest heads. The contrast is most marked in respect of length and it will be noted that this is in accord with the fact noted above, viz. that the men of Addu are the tallest. The other groups are again intermediate.

(*c*) Cephalic Index: The Minikoi men are more frequently (21·05 per cent.) and more intensely brachycephalic. The Addu men are more frequently and more intensely dolichocephalic. The remainder occupy an intermediate position in regard to this index.

(*d*) Altitudinal Index: The Minikoi men have higher (and therefore more spherical) heads: the Addu men are not markedly distinct from the other Maldive groups in this respect. It is to be noted that the shortness of the head in the Minikoi men, as well as their lower stature, are influential factors in the production of this result, as are also the opposite characters presented by the other groups.

(*e*) Nasal dimensions (viz. length, width), and index. The Minikoi men present a curious series of contrasts in this respect.

TABLE III.

	Minikoi, mean	Hulule, mean	Male, high caste	Male, intermediate	Male, low caste	Male, mean	Addu: Didi, mean	Addu: Manikofanu, mean	Addu: Thackarufanu, mean	Addu: Poor men, mean	Addu: Addu Atoll, mean	Misc., mean	Maldives, max.	Maldives, mean	Maldives, min.
Number	20	9	4	3	4	11	7	2	5	10	24	5	—	69	—
I. Height standing	1577	1553	1654	1593	1553	1600	1611	1638	1598	1597	1604	1581	1736	1588	1466
II. Height sitting	783	792	856	787	786	812	807	805	779	790	794	776	892	792	715
III. Cephalic length	182	189	196	194	188	192	200	193	194	194	196	193	209	190	160(174)
IV. Cephalic breadth	143	145	155	146	145	149	150	150	148	146	148	143	162	146	133(136)
V. Face length	105	109	113	112	115	113	116	110	114	114	114	110	123	110	90
VI. Upper face length	61	62	67	62	69	66	65	63	64	63	64	60	75	63	54
VII. Face breadth	128	128	141	134	126	133	135	137	132	134	134	130	147	131	115
VIII. Bigonial breadth	104	107	124	112	107	115	115	118	112	115	115	115	132	111	88
IX. Nose length	40	50	48	50	53	50	50	48	49	49	49	45	59	47	28
X. Nose breadth	31	35	41	37	37	38	38	39	37	37	37	35	45	36	18
XI. Cephalic height	214	215	233	213	221	223	229	222	218	218	222	218	250	218	191
XII. Cranial height	130	125	138	123	126	130	134	138	130	132	133	130	151	130	110
XIII. Circumference of head	543*	540	573	545	536	552	557	549	551	549	552	546	583	547†	498
INDICES—Cephalic	78·5	76·7	79·1	75·2	77·2	77·6	75·0	78·0	76·3	75·2	75·5	74·0	85·3	76·8	70·9
Altitudinal	71·4	66·1	70·4	63·4	67 0	67·7	67·0	71·5	67·0	68·0	67·8	67·3	81·2	68·4	60·0
Facial (Kollmann's)	47·6	48·4	47·5	46·3	54·7	49·6	48·2	45·9	48·5	47·3	47·8	47·7	64·4	48·1	40·8
Nasal	77·5	70·0	85·4	74·0	69·8	76·0	76·2	81·2	75·5	75·5	75·5	77·8	103·6	76·6	46·15
Gonio-Zygomatic	81·2	83·6	87·9	83·6	84·9	86·4	85·2	86·1	85·0	85·8	85·8	88·4	94·3	84·7	68·46

* 19 records only. † 68 records only.

The nasal length (Minikoi) is distinctly smaller than in the Addu group and other Maldivian islands, of which the various subgroups are not distinguishable. The nasal width is distinctly least in the Minikoi group, greatest in the Addu men and in the remainder it is intermediate. With regard to the nasal index, the Minikoi men are disposed in equal forces above and below the mean, while the Maldivian group (which is not subdivisible) shews a distinct tendency to excess on the side of the narrow noses. The seriation of these nasal characters yields some rather interesting conclusions.

(*f*) Facial dimensions (upper facial length, facial width) and facial index. The Minikoi men have the shortest and narrowest faces: the Addu men have much longer but also broader faces. Moreover the Addu men are shewn by the facial index to have relatively broader faces than the Minikoi men. The other Maldivian groups are intermediate between the Addu men and the Minikoi islanders.

5. The various conclusions set out in Section 4, may be still further summarised as follows:

(*a*) The Minikoi men are distinctly contrasted with the Addu men in eleven out of the twelve characters available for study. The exception is the head-breadth, *i.e.* the absolute dimension of that name. Of the Maldive islands, Addu atoll is the most remote from Minikoi.

(*b*) The remaining groups are intermediate between the Minikoi men and the Addu men in nine out of twelve characters.

(*c*) The Minikoi men are distinctly contrasted with the "remainder" (*i.e.* Maldive groups excepting the Addu men) in three out of the twelve characters.

(*d*) From such investigations it is fair to conclude that the Minikoi islanders really offer distinct points of contrast with the islanders of the Maldive group. In section 3 (*supra*) we have seen that the Minikoi men are more variable than the Maldive islanders. That they should be thus more variable is perhaps intelligible in view of their geographical position as compared with that of the Maldivians. This matter will be discussed a little further in the sequel.

6. The standard deviation and coefficient of variability have been determined by me for a series of some ten characters or so, and these are set forth in the accompanying table:

Only two remarks need be made in the present connection. First, the greater variability of the Minikoi men is (on the balance) confirmed. In the second place, the exclusion of the curiously formed head of No. 10 (Minikoi) has singularly little effect on the values of the means and other determinations.

7. The variability of these islanders has now to be compared

TABLE IV.

Measurement	Group	No.	Mean	σ	C	$\frac{\sigma^2}{N}$
1. Cephalic index	Maldives with Minikoi	69	77	3·47	4·5	·174
	Maldives with Minikoi (less No. 10 Minikoi)*	68	77	3·42	4·44	·172
	Minikoi men only	20	79	3·164	4·05	·501
	Minikoi (less No. 10)*	19	78	3·024	3·87	·481
	Maldives without Minikoi	49	76·2	3·53	4·64	·254
2. Nasal index	Maldives with Minikoi	69	77	10·15	13·20	1·490
	Minikoi men only	20	77·5	12·25	15·80	1·225
	Maldives only	49	76·2	8·66	11·40	1·530
3. Altitudinal index	Maldives with Minikoi	69	68	4·54	6·67	·298
4. Facial index	Maldives with Minikoi	69	48	4·10	8·54	·243
5. Stature	Maldives with Minikoi	69	1590	62·55	3·93	5·69
	Minikoi men only	20	1580	35·10	2·22	1·95
	Maldives only	49	1590	59·50	3·70	7·22
6. Cephalic length	Maldives with Minikoi	69	190	8·995	4·73	1·173
	Maldives with Minikoi (less No. 10)*	68	190	8·27	4·35	1·006
	Minikoi only	20	182	8·20	4·51	3·23
	Maldives only	49	191·2	7·26	3·80	1·07
7. Cephalic breadth	Maldives with Minikoi	69	146	6·50	4·45	·612
	Maldives with Minikoi (less No. 10)*	68	146	6·325	4·33	·558
	Minikoi only	20	143	4·37	3·056	·955
	Maldives only	49	147·2	6·75	4·59	·930
8. Cranial height	Maldives with Minikoi	69	130	7·68	5·90	·854
	Maldives with Minikoi (less No. 10)	68	130	7·70	5·092	·872
9. Nasal length	Maldives with Minikoi	69	47	5·77	12·54	·482
	Minikoi only	20	40	4·79	11·97	1·146
	Maldives only	49	49·8	4·02	8·07	·33
10. Nasal width	Maldives with Minikoi	69	36	4·45	12·36	·287
	Minikoi only	20	31	4·16	13·53	·8665
	Maldives only	49	38·04	2·99	7·85	·18
11. Upper facial length	Maldives with Minikoi	69	63	4·485	7·12	·291
12. Facial width	Maldives with Minikoi	69	131	6·306	4·81	·574

* There is a doubt about No. 10 Minikoi. The dimensions of the head are very unusual and Professor Gardiner has a note to the effect that the head in question looks as though deformed, possibly through some injury.

TABLE V.

	Group	Standard deviation	Mean value	Reference
Cephalic index	Moslems (Egypt)	2·86	74·00	Myers *
	Corsicans	2·90	75·50	Duckworth
	Amarer	3·15	77·00	Duckworth
	Minikoi	3·164	77·5	Duckworth
	Sundanese	3·18	85·53	Garrett †
	Maldives with Minikoi	3·42	77·00	Duckworth
	Javanese	3·45	85·02	Garrett †
	Copts	3·48	74·26	Myers *
	Maldives only	3·53	76·00	Duckworth
	Amarer with Jemeni	3·56	77·00	Duckworth
	Sardinians (Lanusei)	3·57	73·00 (?) (76·40)	Duckworth
	Amarer, etc.	3·72	77·00	Duckworth
	Sardinians	3·98	77·50	Duckworth
	Bishari	4·10	79·00	Chantre ‡
	Cretans	4·10	79·00	Hawes §
	Greek Youths	4·18	82·50	Duckworth
	Jemeni	4·25	77·00	Duckworth
	Moormen	4·305	79·00 (77·00)	Risley ‖
	Banjerese	4·46	81·48	Garrett †

* *Journal of the Royal Anthropological Institute*, 1903–1905-6.
† *Ibid.* 1912.
‡ Chantre, *Recherches anthropologiques en Egypte*, 1904.
§ Hawes, *Rep. British Assoc.* 1908.
‖ Risley, *Journ. Roy. Asiat. Soc. Bengal*, LXII. 1893. [The index is calculated on Flower's method and should be reduced by 2 units.]

	Group	Standard deviation	Coefficient of variation	Mean
Nasal index	Moslems of Egypt	7·67	10·12	75·83
	Moormen (Ceylon)	7·75	9·57	80·7
	Sundanese	7·76	8·93	86·92
	Banjerese	7·81	8·88	88·01
	Copts of Egypt	8·16	10·77	75·77
	Bishari	8·51	11·20	76·0
	Maldives (only)	8·66	11·40	76·2
	Javanese	9·18	10·72	85·67
	Maldives with Minikoi	10·15	13·20	77·0
	Minikoi men only	12·25	15·80	77·5
Altitudinal index	Banjerese	2·81	3·84	73·09
	Sundanese	3·22	4·27	75·31
	Javanese	3·46	4·59	75·47
	Maldives with Minikoi	4·54	6·67	68·0

TABLE V (*cont.*)

	Group	Standard deviation	Coefficient of variation	Mean
Facial index (Kollmann)	Sundanese	2·65	5·76	46·00
	Banjerese	2·92	6·26	46·58
	Copts of Egypt	3·18	6·55	48·57
	Moslems of Egypt	3·53	7·29	48·39
	Javanese	3·70	7·98	45·99
	Maldives with Minikoi	4·10	8·54	48
Stature	Minikoi only	35·10	2·22	1580
	Javanese	40·99	2·61	1571
	Banjerese	48·61	3·10	1570
	Sundanese	54·07	3·40	1591
	Maldives only	59·50	3·70	1590
	Bishari	59·90	3·63	1646
	Maldives with Minikoi	62·55	3·93	1590
Cephalic length	Javanese	4·68	2·63	178
	Sundanese	5·28	2·93	177
	Moslems of Egypt	6·09	3·13	195
	Copts of Egypt	6·13	3·17	193
	Banjerese	6·22	3·44	181
	Maldives (only)	7·26	3·80	191·2
	Minikoi (only)	8·20	4·51	182
	Maldives with Minikoi	8·27	4·35	190
Cephalic breadth	Moslems of Egypt	4·34	3·01	144
	Minikoi (only)	4·37	3·056	143
	Javanese	4·57	3·03	151
	Copts of Egypt	5·09	3·56	143
	Sundanese	5·24	3·47	151
	Maldives with Minikoi	6·325	4·33	146
	Maldives only	6·75	4·59	147
	Banjerese	6·77	4·59	147
Cranial height	Copts of Egypt	4·15	2·83	146*
	Moslems of Egypt	4·65	2·83	146*
	Banjerese	4·76	3·59	132
	Javanese	4·86	3·63	134
	Sundanese	6·62	4·97	133
	Maldives with Minikoi	7·70	5·092	130
Nasal height (or length)	Sundanese	2·39	5·30	45·1
	Banjerese	3·19	7·18	44·3
	Copts of Egypt	3·41	7·14	47·8
	Maldives (only)	4·02	8·07	49·8
	Javanese	4·33	9·56	45·18
	Minikoi (only)	4·79	11·97	40·0
	Maldives with Minikoi	5·77	12·54	47·0
Nasal width	Copts of Egypt	2·72	7·57	35·96
	Maldives (only)	2·99	7·85	38·04
	Sundanese	3·05	7·65	39·80
	Banjerese	3·50	8·76	40·00
	Minikoi (only)	4·16	13·53	40·00
	Maldives with Minikoi	4·45	12·36	36·00
	Javanese	4·70	11·88	39·53

* Auricular height not precisely comparable with other means in this section.

with that of other groups. For this purpose, I have a number of data mostly prepared by myself from the original measurements. As the data for the cephalic index are most numerous, they will be considered first. For this purpose I have collected them as shewn in Table V, where the values of the standard deviation are arranged in their sequence of increase.

The first conclusion to be drawn from Table V is that the Maldive and Minikoi men agree generally in presenting a higher degree of variability than the other groups brought into comparison with them.

But the Minikoi men are not always thus associated with the Maldive islanders. For in two important characters, viz. the stature and the width of the head, the Minikoi men are dissociated from those of the Maldive group. Moreover the Minikoi men are in these two respects more homogeneous than their neighbours.

Thirdly, this homogeneity as regards stature and cephalic breadth is in strong contrast with the great variability in respect of the nasal index shewn by the men of Minikoi.

Evidently the conclusions already formed as to the mixed character of these island populations find confirmation in Table V.

8. Before passing from the strict consideration of such numerical data, it is convenient to notice the values of the coefficient of correlation for certain pairs of dimensions. They are shewn in Table VI.

TABLE VI.

Characters compared	No. of individuals	"r"	P.E. of "r"
Maldives and Minikoi together			
1. Cephalic length and breadth	68	·484	± ·0564
2. Cephalic length : Cranial height	68	·244	± ·074
3. Cephalic breadth : Cranial height	68	·227	± ·075
4. Cephalic index and length	68	– ·520	± ·053
5. Cephalic index and breadth	68	·475	± ·056
6. Nasal length and width	69	·528	± ·051
7. Cephalic index : Nasal index	68	·0104	± ·068
Minikoi men only			
8. Nasal length and width	20	·281	± ·133

The data presented in Table VI do not differ markedly from those based upon measurements of very different origin. I have few records available for comparison, but the values set out in Table VII are not without interest. Yet they do not seem to enhance the value of "r" as a discriminating agency.

TABLE VII.

Group	Characters compared	"r"	P. E. of "r"	Reference
Maldives with Minikoi	Cephalic length and breadth	·484	± ·0564	Duckworth
Modern English I	" " "	·402	± ·019	Lee
" " II	" " "	·345	± ·019	Lee
Maldives with Minikoi	Cephalic index and length	– ·520	± ·053	Duckworth
Sardinian *crania*	" " "	– ·548	± ·075	Duckworth
English "	" " "	– ·547	?	MacDonnell
Naqada "	" " "	– ·551	± ·041	Lee

9. Thus far an endeavour has been made to deal with all Professor Gardiner's data or at least to subdivide them into two groups only, viz. the Minikoi men and the Maldivians. This was necessitated by the small number of individuals observed. For in the wider comparisons it is absolutely imperative to deal with the largest possible number in each area.

But Professor Gardiner has grouped two series of his measurements, viz. the men of Male and of Addu Atoll, according to caste. A review of Table III will shew that even with the small numbers to which each subdivision finds itself reduced, the influence of caste is quite distinct. Moreover it acts in the same direction both in Male and in Addu. For in each, the higher caste has the higher stature, and larger head-dimensions. Indeed the mean values of the circumference of the head may be regarded as an epitome of the rest of the measurements. It should be noted that fishermen are in the lowest caste and class.

The indices do not yield the same contrast, though this might be expected in regard to the nasal index at least, which indicates that the higher caste possesses paradoxically the broader nose. Yet the small number of examples must be recalled again, and this influence is doubtless much more effective in obscuring differences in indices than in the absolute dimensions.

10. The possible affinities of these natives forms the next subject of enquiry. The statistical data shew that they are not very homogeneous, although certain distinguishing characters do seem to occur within their own borders.

One feature of Professor Gardiner's lists impressed me at once. Although distinguished by caste, the natives of the Maldives and Minikoi have Moslem names. This very paradox serves, however, to indicate two out of the many possible sources of the population of these islands. In other words, Hindustan is suggested at once, and again the Moslem influence, though it may have travelled *viâ* that peninsula, need not have done so.

11. Leaving these speculations on one side for a moment, it is convenient to consider another possibility. In many islands of the south-eastern parts of Asia, indications are met with of the existence of pygmy types subjected to invasion by taller and stronger immigrants. Ceylon and the Andaman islands will serve as examples of the phenomenon in question.

Evidently it is a matter of interest to make this enquiry as regards the Maldives. It can be undertaken without prejudice in the case before us, for if Ceylon and the Andamans suggest a succession such as has been mentioned, the history of the Maldives and Minikoi goes far to discountenance the idea.

If we commence such a search, it is necessary to fix a limit of stature, this character having a prime value in the definition of pygmy types. An upper limit of 1480 mm.* will not be too great if it be understood to refer to adult males. The mean values for Andamanese and Aetas exceed this, and the same character among the Vedda is very considerably greater (1570).

(*a*) Our preliminary search yields the following results:

Adult males of stature less than 1480 mm.

1. Maldives: Hulule (No. 9), 1478.
2. " : Male (No. 18), 1474.
3. " : Addu (No. 44), 1466.
4. Minikoi: (No. 2), 1476.

Mean value (4 individuals), 1473·5 mm.

Consequently there is at least a *prima facie* case to be made out for the existence (in these islands) of a pygmy element. Of these individuals, the only information available (in addition to the other measurements) is to the effect that the three Maldive men were fishermen and therefore presumably of low caste. Two of the three are named Hassan, the name of the third is not recorded. The Minikoi man was named Ismail, and Professor Gardiner makes the following noteworthy comment, "mongoloid eyes and rather high cheek-bones"; this individual was fat, and none of the other three is described as thin or emaciated.

(*b*) The four individuals thus associated stand apart in marked contrast to the rest by reason of their small stature. The next stage in the enquiry is directed to the positions occupied by the same men in the other seriations. For a pygmy type might be expected to provide other contrasts than that of stature. A careful search through the whole range of seriations (upon which the standard deviations discussed in Table IV are based) shews me that the short-statured men are remarkable in no other respect†.

* 4 ft. 10 in.

† In other words the range of variation they yield is markedly overlapped by that given by the remainder. This of itself need not however disprove their pygmy

For in no other character are they grouped together in contrast to the other individuals. The nasal index is not far from providing an exception to this statement. But so far as the nasal index is thus concerned, its evidence is distinctly against the idea suggested by the stature of the four men. For the nasal index is in distinct contrast with that typical of a pygmy stock.

(*c*) In reference to pygmy types, a good deal of stress has been laid by some authors on the great relative length of the torso as compared with its (relative) value in the tall races. If we wish to take as a basis of comparison the percentage proportion of the torso to the stature in European races, we find that the percentage is regarded as about 52·5. According to theory, pygmy types, in some cases at least, should provide a larger number as representative of this percentage.

If we turn to the short-statured individuals of the Maldives and Minikoi we shall find this percentage represented in two cases by higher values (than 52·5), viz. 53·6 (Hulule, No. 9) and 54·1 (Male, No. 18). But on the contrary the two remaining values are 50·1 (Addu, No. 44) and 48·5 (Minikoi, No. 2) respectively. Evidently this test is of no use in the present instance; and though I have mentioned it here, I am convinced that it is not a reliable test in most if not in all instances. The examination of the existing data from various tribes scattered over all parts of the earth will soon bring this conviction home to the investigator. But where data of all kinds are as scanty as in the present instance, one must try every test that is not absolutely unreasonable.

(*d*) Except in point of stature, then, the proportions of the four small men are as variable as they could well be. In such circumstances, the onus of proof is, in my opinion, transferred to those who elect to regard these individuals as representatives of a pygmy stock. This may be the case, but, if so, the stock is not directly comparable with others generally admitted to be pygmy. If a pygmy element does exist in the Maldives and Minikoi, the data here available do not suffice for its detection, or for a demonstration of its distinctness.

The circumstances demand that this criticism should be searching: indeed I conceive that the existence of "genuine pygmy" types has been proclaimed in other instances upon a basis of evidence which is insufficient to warrant such a conclusion. In particular, I am not confident that the Vedda would survive as a pygmy type under sound criticism.

nature, for I find that in about a score of characters taken at random, the Seman (an undoubtedly pygmy stock) are overlapped by their neighbours, the South Perak Malays.

12. Three great sources of immigration into the Maldives and Minikoi can be suggested at once. These are

(i) The peninsula of Hindustan with Ceylon;
(ii) The coasts of Arabia and possibly of Africa;
(iii) The western shores of the Malay Peninsula, and the islands of the Malay Archipelago.

(i) The proximity of India and Ceylon lead naturally to the expectation that they may be called to account for some contribution at least. But again, we have to consider the seafarers of the region. For hundreds of years mariners from the West have made their way past these islands and have penetrated as far as the Pacific Ocean*. They may have come from the Arabian peninsula, and their stock might be that known as "Himyaritic." Or they may have been accompanied by the negroes of Africa, or again by Semites or Negrito-Susians from the head of the Persian Gulf.

There remains a counter-current setting westwards from the great Archipelago. For it must not be forgotten that whether as a reflux or otherwise, certain "Malayan" peoples have travelled extensively in this direction. Prichard in particular (*Nat. Hist. of Mankind*, 1844, Vol. IV. p. 190) speaks of Malay colonies on the coast of Ceylon.

It is therefore necessary to enter upon a brief consideration of each of the three possible sources in turn.

The task of comparing the natives of the Maldives and Minikoi with the various races of Southern India and Ceylon would enlarge this report to such an extent as to render it unwieldy. Only a few selected examples will be dealt with in this place.

Taking first Ceylon, the Vedda may be eliminated at once. The comparative rarity of a nasal index exceeding 82 in the islands may be taken as justification for this exclusion, and it will also rule out the Rhodias, Tamils and Singhalese. The two latter stocks are further distinguished by stature superior to that of the average man of the Maldives or Minikoi.

But there remains in Ceylon the very curious type known as that of the "Moormen" or "mariners." They present something of an enigma. I took some pains therefore to enquire into their physical characters. For these we are indebted to the late Sir H. H. Risley, who has recorded anthropometric data relating to 22 Moormen (cf. *Journ. Roy. Asiat. Soc. Bengal*, LXII. 1893, p. 33). Table VIII provides the comparison between these men and the Maldive islanders.

The concordance is admittedly small. Yet it is perhaps not

* The Maldivians themselves are said by Reclus (*Géog. Univ.*) to trade in native boats as far as Sumatra. Reclus also refers distinctly to Arabic influences among them.

altogether sufficient to justify the abandonment of the enquiry in this direction. In view, however, of the small numbers of the groups compared, the quest can hardly be pursued further here with any prospect of success. The contrasts might be attributed to the difference in circumstances. For the Maldive folk are presumably less favourably situated than the inhabitants of Ceylon. We know little of the way in which Risley obtained his records, and of the social status of his subjects. But the Moormen are characteristically Moslem, a feature already remarked as distinctive of the Maldive islanders, if their names can be held to provide evidence on this point. And again the relation of the Moormen to the Malay colonies mentioned by Prichard (v. *supra*, p. 26), remains to be investigated.

TABLE VIII.

Character	22 Moormen (Risley)	49 Maldive men (Gardiner)
Stature	1625	1590
Height sitting	815·8	795·7
Ratio: sitting height to stature=100	50·2	50·04
Head length	182 (186)*	191·2
Head breadth	144	147·2
Head height	130·2	130·0
Nasal height	47·7	49·8
Nasal width	38·5	38·04
Cephalic index	79·1 (77·1)*	76·2
Nasal index	80·7	76·2

* Flower's method. About 4 mm. to be added to the cephalic length and 2 units subtracted from the cephalic index, for comparison with groups in which the maximum cephalic length is recorded.

The Tamils of Ceylon are even taller than the Moormen, their heads are narrower and their noses are broader. The comparison with the Maldive islanders fails more definitely and conspicuously here, though it may be remarked that the Moorman probably represents a blend into which a distinct Tamil element enters.

When the Indian peninsula is considered, a vastly greater range of possibilities presents itself.

In the first place, our interest must be directed inevitably to the comparison with such "aboriginal" hill-tribes of the Nilgiris as the Irulas and Kurumbas. But in my opinion the comparison fails conspicuously. And the failure is determined chiefly by the difference in the nasal index, for in this respect the contrast is very marked between the wide nose of the hill-tribe types and the relatively narrow noses of the islanders.

Per contra, I may be allowed to make one comment in passing. The "Mongoloid" appearance of one of the Maldive men of pygmy stature should be recalled here in view of the fact that the aboriginal Kurumba of the Nilgiris is alleged by some to be of Mongolian aspect. The threads of evidence are, however, so frail that I do not venture to insist on the comparison.

Of the great host of types which still remains for discussion, I must be content to select three only for consideration. For the comparative data I am indebted to the admirable paper by the late Professor E. Schmidt*, and even more to the invaluable work by Mr Thurston (entitled *Castes and Tribes of Southern India*).

Of the three groups mentioned above, I have selected one, the Linga Banajiga, on account of the similarity in head-form and proportions which obtains between them and the Minikoi men.

The Mukkavan, another tribe of Southern India, must certainly be considered, for they are the fisher-folk of the Malabar coast, and they are also distinguished by their tendency to adopt the Moslem religion.

The Billava are not a littoral people, so far as I can learn, but they shew in their head-form so marked a tendency to brachycephaly and thus so strong a contrast with most of their neighbours that it seems well to include them in this comparison. The available data will be found arranged in Table IX.

TABLE IX.

Group	Minikoi	Maldives with Minikoi	Addu	Linga Banajiga*, Sandur	Mukkavan*	Billava*
No. of subjects	20	69	24	25	40	50
Stature	1577	1588	1604	1656	1631	1632
Head-length	182	190	196	182	190	182
Head-breadth	143	146	148	142	142	146
Cephalic index	78·5	76·8	75·5	78·3	75·1	80·1
Nasal index	77·5	77·0	75 5	74·6	81·0	72·6

* Thurston, *op. cit.*

The result of the comparison is curiously perplexing, but the one outstanding feature is the inferior stature of the islanders. Apart from this the general conclusion to be drawn is that the mainland tribes may be considered as grouped around the men of the islands, and indeed a further draft on Mr Thurston's data might be made easily to confirm this view. On the whole, too, this comparison is more apt than that already instituted (cf. Table VIII) with the Moormen.

* Schmidt, *Archiv für Anthropologie*, 1911.

It should be noted here, however, that, according to tradition, both the Mukkavan and Billava natives originally came from Ceylon. At the present time they are not widely separated, though, as has been pointed out, the Mukkavan are a seafaring folk, whereas the Billava are found inland.

(ii) When we turn to the second possible source of immigrants, viz. the western region including Somaliland and Arabia as far as the Persian Gulf, the ground is manifestly more uncertain. I will therefore content myself with the reminder that the possibility exists, and that a comparison of the Maldivians (especially the men of Addu) with the men of Yemen is not preposterously absurd. Yet the shorter stature and the greater tendency to brachycephalic heads shewn by the islanders renders the comparison unsatisfactory.

(iii) The third area to be considered may be described as the Malayan one. And it is important to note once more in this connexion that the chief difficulty hitherto encountered has depended largely on the low stature and rotundity of head met with in the islands. To match these, the South Perak Malays may be adduced at once. The Moslem names and the sporadic occurrence of "Mongolian" features are also in accord with this view. There remains the contrast in respect of the nasal index, which points to a broad nose among the Malays who are thus in contrast with the Maldivians.

But the Malay type is extraordinarily variable, so that the comparison need not be abandoned should one test (even though so important as that of the nasal index) seem to fail to provide confirmatory evidence.

Indeed there is a good deal of evidence to be brought forward on this subject, and the following notes may serve to indicate the general trend of my surmises in this connexion. It is in fact known that in the Malay Archipelago the larger islands often possess an outlying fringe of islets inhabited by native populations differing from their neighbours. A contrast in stature at least is noticeable. Dr Hose mentioned this to me in conversation, and Mr Garrett, my former pupil, has just published some notes on the Orang Balik Papan, who may serve as examples of the stunted maritime populations in question.

Further west, they are replaced by the Orang-Laut, and these again in turn by the Selungs of the Mergui Archipelago, and possibly some (though certainly not all) of the Nicobarese. In all instances the low stature*, brachycephalic head, and absence of high degrees of platyrrhiny provide just the combination of physical characters sought for. In conclusion, mention must be

* The Selungs described by Dr Anderson in 1890 are however taller than the other tribes mentioned in this connexion.

made of the Biajus or sea-gypsies of Borneo, if only on account of a custom alleged by Prichard (*Researches, etc.* Vol. v. 1847, p. 87) to be common to them and the Maldive islanders. The custom consists in the preparation and launching of a small boat as an offering to one of their deities. And even though the custom be now recognised (Skeat) as of wide dispersion in Malaysia, its practice in the Maldives would be most significant.

It remains to add that the discovery of Malayan affinities and relations may not end with the Maldive islands and their populations. For in my opinion the question may be fairly raised as to whether Malay invaders ever secured a hold in the Malabar district. We read of Malay colonies in Ceylon. We find hints of Malayan influence in the Maldives. The islanders of that group are not without resemblance to the Mukkavan, and possibly to the Billava and Linga tribes just studied. Is there any Malayan blood in the latter? I can only ask the question. The answer will depend on the study of language and customs. In regard to the latter, it is at least remarkable that the Mukkavan should make offerings to the sea, though a closed vessel and not a model boat is employed as the vehicle.

Summary.

To sum up this protracted discussion, I would conclude by recalling the great variability in physical type shewn to exist in the Maldives and Minikoi. A diversity of racial stocks is thus shewn to be probable. The seriations provide two-peaked curves in several instances, but the significance of these is not beyond question, although they may be really evidence in the same direction.

Such approaches to pygmy proportions, as can be detected, are not to be dissociated from the effects of local conditions upon nutrition, etc.

Of the possibilities in the way of invasions, I have indicated three main sources. On the whole, the resemblance to the maritime natives of Malabar is close enough to satisfy most requirements. But I feel assured that the Malabar coast is not the only source of immigrants, and in Minikoi especially I think that account must be taken of what I term generally Malayan influences. And these may have affected the Malabar natives also and even before they sent immigrants into the Maldives. It is with regret that I am compelled to make a statement which is so deficient in directness. But I do not care to lay more weight on any evidence than it can reasonably sustain, and this thought has influenced the present report. In any case the fact that Professor Gardiner has been a pioneer of anthropometric research in this little-known area is a matter upon which he is to be congratulated warmly.

Notes on the volatilization of certain binary alloys in high vacua. By A. J. Berry, B.A., of Trinity and Downing Colleges.

[*Read* 11 November 1912.]

In a previous paper (A. J. Berry, *Roy. Soc. Proc.*, 1911, **86** A, 67) it was shown that the compound $MgZn_2$ could be prepared by distillation of an alloy containing excess of zinc in an apparatus exhausted by cold charcoal. It was also shown that the compound $MgZn_2$ can itself be vaporized unchanged in a high vacuum. These observations have been repeated and confirmed and experiments have been performed on other pairs of metals in the hope of isolating intermetallic compounds.

The phenomena of the vaporization of alloys when heated in vacuo has occupied the attention of other investigators. Thus Tiede and Fischer (*Ber. Deutsch. Chem. Ges.* 1911, **44**, 1712) have effected a quantitative separation of lead and tin from an alloy of these two metals. Groves and Turner (*Trans. Chem. Soc.* 1912, **101**, 585) have examined the behaviour of a number of alloys and have classified them into five groups as follows:

Group I. The metals are non-volatile and the alloy is unaltered in weight.

Group II. The volatile metal or metals are removed and a quantitative separation results.

Group III. Any excess of volatile metal is removed and a chemical compound remains.

Group IV. Any excess of volatile metal is removed, but the residue is not a compound.

Group V. The metals composing the alloy volatilize together, their relative proportions being in part dependent on the temperature.

Experimental.

The experimental method already described (*loc. cit.*) has been employed with but slight modifications. The distillations were conducted in an electric furnace, and the charcoal was kept immersed in liquid air during the initial stages of the distillation with the object of removing gas occluded within the body of the metal.

Copper and cadmium. An alloy containing excess of cadmium was heated at a temperature of about 600° for several hours. On analysis it was found that the two metals had been separated quantitatively. A confirmatory experiment yielded identical results.

Cadmium and magnesium. On heating an alloy containing excess of cadmium in vacuo both metals volatilized together, but no definite relation between the composition of the distillate and the residue was established. From the form of the equilibrium diagram which was worked out by Grube (*Zeitsch. anorg. Chem.* 1906, **49**, 72) it is probable that the compound CdMg which forms solid solutions with both constituents would dissociate on melting. It is clear that this pair of metals belongs to Group V in Turner's classification.

Magnesium and lead. Since these two metals are both moderately volatile in high vacua at temperatures of the order of 600° according to the researches of Krafft and his collaborators, it was thought that the compound Mg_2Pb might under suitable conditions be volatilized unchanged as had already been observed in the case of the compound $MgZn_2$. The existence of the compound Mg_2Pb has been proved by the work of Grube (*Zeitsch. anorg. Chem.* 1905, **44**, 117) and confirmed by Kurnakoff and Stepanoff (*ibid.* 1905, **46**, 177). This compound does not form mixed crystals with either of its constituents. In the experiments of the present author, alloys containing the two metals in approximately equivalent proportions were distilled in vacuo at a temperature of about 680°. It was found that the distillate consisted chiefly of magnesium with mere traces of lead. In no case was it possible to isolate a homogeneous portion of the distillate for quantitative analysis. Under the microscope, steel blue crystals embedded in a matrix of magnesium or magnesium silicide from the glass were plainly visible, and these portions of the distillate underwent rapid corrosion on exposure to the air with formation of a black powder—a property of the compound Mg_2Pb noted by Grube (*loc. cit.*). It is noteworthy that the coolest portions of the tube where the distillate condensed were practically free from lead. According to Krafft and Bergfeld (*Ber. Deutsch. Chem. Ges.* 1905, **38**, 254) lead commences to volatilize at 335° in a cathode ray vacuum, while Knocke (*ibid.* 1909, **42**, 206) has shown that magnesium under similar conditions commences to volatilize at 415°. One might, therefore, expect that at a temperature of 680° both metals would readily vaporize and condense in the cooler parts of the apparatus. It must be borne in mind that magnesium vapour would diffuse nearly three times as rapidly as lead vapour, and further, although the volatilization point of lead is apparently lower than that of magnesium it does not follow that the same order would obtain with regard to the vapour pressures of these two metals at higher temperatures. In a special experiment in which 7·7 grams of assay lead were heated for about five hours at 680° in a vacuum produced by cold charcoal, it was found that only a very small

quantity of lead in the form of a thin mirror was condensed. Such an experiment might indicate that magnesium is the more volatile at that temperature, but it would be premature to conclude that this is the case in the absence of knowledge of the relative viscosities of the two vapours.

While the experiments recorded in the present communication have not been successful in effecting the isolation of intermetallic compounds of the metals under consideration, a comparison of the result obtained in the case of the magnesium-zinc series with that obtained with the magnesium-lead series is not without interest. The maximum on the freezing point curve of the magnesium-zinc series corresponding to the compound is remarkably sharp, while the maximum on the freezing point curve of the magnesium-lead system is rounded. It is clear that the less sharp the summit of the curve, the more the compound will tend to dissociate into its constituents on heating above its melting point. It is, therefore, not surprising that all the author's attempts to distil the compound Mg_2Pb resulted in showing that, in the state of vapour, this compound is largely dissociated.

On Pulsus alternans. By GEORGE RALPH MINES, M.A., Fellow of Sidney Sussex College, and Additional Demonstrator of Physiology in the University of Cambridge. (From the Physiological Laboratory, Cambridge.)

[*Read* 28 October 1912.]

[PLATE I.]

Introduction.

THE condition of *pulsus alternans*, described clinically in the first place by Traube in 1872, has long been recognised as due to alternation in the strength of the ventricular contractions. Alternation in strength of ventricular contractions of the isolated frog's heart was observed, described and explained by Gaskell in 1882.

After this, for nearly twenty years, the subject received but little attention. Within the last ten years a number of papers have appeared, dealing both with the clinical and experimental aspects of the matter. Hering (1902) and Volhard (1905) in particular have insisted on the distinction to be drawn between true *pulsus alternans*, in which the ventricular contractions are evenly spaced, and *pulsus pseudo-alternans* or *bigeminus pseudo-alternans*, in which the intervals between the beats, as well as the strengths of the beats, show alternation.

In the latter condition, the variation in size of the ventricular contractions depends immediately upon their being provoked when the ventricular muscle is in different stages of recovery from its last excitation and does not of necessity connote any abnormality in the properties of the ventricular muscle.

In the present paper I am concerned only with the condition of true *pulsus alternans*, that is to say, the condition in which the ventricular beats follow regularly-occurring auricular beats at perfectly rhythmic intervals*, but in which the musculature behaves differently in successive responses, though in the same way in alternate responses†.

The most important additions to our knowledge of this condition which have been made in recent years have resulted from the

* Under these circumstances, the pulse wave corresponding to the weak contractions arrives at the wrist a little late, so that the sphygmographic tracing shows unequal intervals between 1 and 2, and 2 and 3. Volhard shows that this is due to haemodynamic factors.

† For convenience of discussion I shall refer from time to time to a series of beats, 1, 2, 3, 4, etc., referring to the beats 1, 3, 5, etc. as the "odd series" and to the beats 2, 4, 6, etc. as the "even series."

method—fruitful in so many directions—of taking simultaneous records of the activity of the heart by two or more instruments.

In this way it has been found both clinically and in experiments on animals, that although in many instances the alternation is shown alike in pulse record, cardiogram and electrocardiogram, this is by no means invariably the case. Thus it may be well marked in the apex beat but absent from the radial pulse (Hering, 1908); it may be present in the electro-cardiogram but in opposite phase to the apex beat or to the radial pulse, so that the large excursions in the one record correspond to the small excursions in the other. There may be similar "incongruence" between apex beat and pulse wave and also between two cardiograms taken from different points on the same chest wall (Hering).

The appearance of an extra-systole, whether spontaneous or artificially provoked, may profoundly affect the course of an alternating series of beats, while progressive changes in such a series without intentional or perceptible change in the external conditions is frequently noted. The interest of all clinicians has been attracted by Mackenzie's statement that the appearance of *pulsus alternans* in a patient usually means death within two years. Such are the facts which have led Lewis (1911) in a recent review of the subject to characterise *pulsus alternans* as "one of the most mysterious and most important mechanisms of the heart with which clinicians have to deal."

The interpretation of pulsus alternans.

I have already stated that Gaskell in 1882 gave an explanation of *pulsus alternans.* Largely owing to the overwhelming interest of his later work on the tortoise heart, the full significance of Gaskell's explanation has been overlooked and another view, superficially resembling it and really forming one special case of it, has been widely adopted. The recent discoveries about *pulsus alternans* have shown this view to be inadequate.

I shall first quote Gaskell's explanation and examine the grounds on which its single assumption is founded; I shall then discuss the more recent hypothesis which makes the same fundamental assumption, but neglects its logical consequence, and finally I shall show how Gaskell's original suggestion lends itself to the interpretation of the various phenomena which at first appear so perplexing.

Gaskell's experiments on the frog's heart showed conclusively the following facts about alternation:

(1) that it depended on a local alteration in the condition of the ventricle;

(2) that it could be abolished temporarily by stimulation of the vago-sympathetic trunk;

(3) that during the course of alternation there was a relation between the beats such that if the even series got smaller, the odd series got larger, and *vice versa.*

After describing and illustrating the last point Gaskell continued as follows:

"Now we know from the experiments of Bowditch that the force of the ventricular contractions is independent of the strength of the stimulus. The explanation, therefore, of this alternation in the force of the contractions must be sought for in the muscular tissue itself, and it seems to me that the most probable explanation is that a larger amount of tissue contracts when the beats are large than when they are small, and that, therefore, in all probability, certain portions of the ventricle respond only to every second impulse, while other portions respond to every impulse. The observations of Aubert show that by the direct action of a blow a circumscribed area of the ventricular muscle can be made to remain quiescent, while the rest of the ventricle is contracting rhythmically.

"I am inclined, therefore, to suggest that, owing to some cause in the manipulation, such as cutting open the ventricle, or some other cause which affects the ventricle unequally, the excitability of the ventricular muscle is at the time not absolutely the same throughout, so that, although the impulses remain the same in strength, yet certain parts which possess a lower excitability are able to respond only to every second impulse, while the rest of the tissue responds to every impulse. In this way, if the strength of the contractions depends upon the amount of tissue contracting, we see not only that every second beat must be larger, but also that the size of each strong contraction must vary inversely as the size of each corresponding weaker contraction."

The quantitative expression for the excitability of an irritable tissue towards a particular variety of stimulus is the inverse of the strength of stimulus needed to excite it. If a piece of ventricular muscle is acted upon by stimuli, whether these be electric or whether they be auricular excitations, the muscle will respond to every stimulus, provided that a certain relation exists between the strength of each stimulus and the degree of excitability of the muscle when it arrives. But since the excitability of the muscle is greatly depressed after the beginning of each excitation, and increases after a time gradually, at first rapidly and then more slowly, it is evident that if the stimuli arrive too frequently only every alternate stimulus will find the heart muscle in a condition in which it can be excited (Hofmann, 1901). In a similar way, if stimuli of a certain strength arrive at a frequency such that each

stimulus causes a ventricular contraction, it is evident that by lowering the excitability of the muscle, the whole cycle of changes in excitability of the muscle may be carried out at a lower level than before, so that, even without supposing that the process of recovery of excitability goes on *slower* in the depressed than in the normal tissue, it is clear that it will have to go on *longer* before the excitability reaches the value at which the stimulus becomes liminal. And if the time taken to reach this value is greater than the interval between two stimuli, it is evident that every second stimulus must fail to excite. The range of muscular excitability over which this state of affairs will hold for a given strength of stimulus is considerable. In such cases the "half-rhythm" is due to the refractory phase of the muscle. In other instances it may be attributed to a property possessed by all excitable tissues—the power of summation of stimuli. If the excitability of the muscle is below a certain grade, a stimulus which is subliminal may yet so raise the excitability of the muscle as to render the same stimulus on repetition liminal. It appears certain from the experiments of von Basch (1880) and others that this is the explanation of many cases of "half rhythm." For our present discussion it matters little which of these explanations holds in any particular case, though it is important to note that the existence of these two well defined mechanisms, either of which may be responsible for "half-rhythm" and both of which are known to produce it, increases the chances of the occurrence of this type of relation between heart muscle and a rhythmic succession of stimuli in any particular instance and extends the range of modification of the excitability over which "half-rhythm" may occur. That a condition of depressed excitability affecting part of the ventricular muscle is indeed responsible for alternation, is strongly supported by the fact that alternation is removed by just those methods which improve the excitability of the heart muscle. Thus, for example, bathing with Ringer's solution* or stimulation of the vago-sympathetic, as Gaskell showed, produces just the effects demanded by the hypothesis.

Fig. 1 shows an instance in which alternation, here much more marked in the electrogram than in the mechanical record, was abolished after stimulation of the sinus venosus.

A peculiar and instructive case is that shown in fig. 2. Examination of the two upper lines of the tracing near the beginning, the records of the auricular and ventricular contractions respectively, would suggest that it is a case of partial auriculo-ventricular block. But the electrogram of the ventricle

* Cf. Mines, *Proc. Camb. Phil. Soc.* Vol. xvi. (1912), Plate 6.

(the third line of the tracing) shows that every auricular excitation reaches the ventricle, but apparently the alternate excitations spread only a very short way into the muscle, affecting so few fibres that their mechanical response is too slight to move the lever.

After stimulation of the sinus venosus, the electrical response of the alternate ventricular excitations which before were very small, greatly increases in size—indicating probably that the excitation process spreads further into the muscle, while the mechanical response makes its appearance. We have thus the appearance of a mechanical alternation as a result of stimulating the sinus venosus. This condition continues some time after the stimulation has ceased and then gradually disappears, the electric response returning finally to the condition which obtained at the beginning of the tracing. It is obvious that this was an extreme case of alternation, in which nearly the whole of the ventricular muscle failed to respond to the same alternate series of excitations. The stimulation of the intracardiac nerves reduced the grade of the alternation, making the alternate beats for a time more nearly alike in extent.

The view propounded by Hering (*loc. cit.*), by Muskens (1907) and by several other authors is that in the condition of alternation the whole musculature of the ventricle contracts in one beat, while a portion of the musculature fails to contract in the next beat, and so forth.

This view employs Gaskell's assumption,

(1) that part of the ventricular muscle has its excitability so depressed that it can respond only to every other auricular excitation, but it adds the further assumption, which, as I shall show, is not only unnecessary, but vicious, namely,

(2) that the musculature, of which the excitability is depressed, responds all of it to the *same* excitations; i.e. all of it to the first, third and fifth excitations or all of it to the second, fourth and sixth excitations, and so forth.

The effect on the contraction of the heart as a whole, of depression of excitability of a portion of the musculature such as to cause half-rhythm in this portion, will depend on the region in which this musculature is situated. If it is placed on the sole route by which excitations reach a large tract of musculature—as for example in the junctional tissue between auricles and ventricles—the effect will be to impress the half-rhythm on this tract of musculature, the excitability of which is in reality normal.

The differences known to exist in the mammalian heart between the musculature of the bundle of Stanley Kent, and that of the

auricles and ventricles, make it easy to imagine that this tissue may be affected differently from the rest of the musculature of the heart by a change of external conditions to which the whole musculature is exposed. But the ventricular muscle does not, so far as we know, present any regular differences in its various parts; though it is always to be expected that in any collection of excitable cells some will be above and some below the average of excitability. If the average excitability of the ventricular muscle is gradually depressed, some portions of it, those namely below the average excitability, will reach the condition in which they assume half-rhythm before the main part of the musculature has reached this condition. If these fibres are directly connected with each other the chances are in favour of their contracting in response to the same series of excitations, odd or even. For under these circumstances, if we consider the case of two fibres A and B, A may respond to excitation, reaching it by way of B or by perhaps one or two other routes. If in the even series of excitations of the ventricle in general, B fails to be excited, it is plain that A has a greater chance of receiving a successful stimulus during the *odd* series, when excitation arrives by way of B as well as by other routes. And, of course, where a number of contiguous fibres are concerned these may surround other fibres which are never reached by excitations except those coming by way of the affected fibres. In such cases it will be true to speak of heart block in the ventricular walls; fibres isolated in this way will of necessity follow the rhythm of their neighbours, whether their own excitability is depressed or not.

But there is no justification for the assumption that when the excitability of the ventricular muscle is gradually depressed, the fibres which are below the average excitability of the muscle at the time and in which half-rhythm develops will all be situated at a single focus. Much more likely is it that they will be distributed in various regions. And whether there are two foci or twenty, each one will respond either to the odd or to the even series of excitations. The chances are obviously against *all* of the foci responding to the odd or all to the even series. And if there are more than two foci, the chances are against their being equally divided between the two series—though both of these conditions are possible and may be expected to arrive occasionally.

To put the position tersely we may use symbols. Let V be the whole ventricular muscle and v the portion of it which is depressed in excitability so as to be capable only of the half-rhythm.

Then on the view of Muskens and Hering the series of beats runs thus,

$$V,\ V-v,\ V,\ V-v,\ \&c. \quad\ldots\ldots\ldots\ldots\ldots\ldots(1),$$

while in order to give adequate expression to the possibilities of Gaskell's hypothesis we must subdivide v, thus $v = v' + v''$. Then the series of beats will run

$$V - v',\ V - v'',\ V - v',\ V - v'',\ \&c. \ldots\ldots\ldots\ldots (2).$$

Series (1) then expresses only the special case of (2) in which v' or $v'' = 0$.

Clearly, whenever v' is greater or less than v'', as will happen in most cases, there will be alternation in the extent to which the ventricle contracts and so alternation in the pulse wave, if the circulation is intact.

But while the relative force of contraction of the ventricle in the beats $V - v'$ and $V - v''$ as registered by the suspension method or by the pulse wave, will depend chiefly on the relative *amounts of tissue* in v' and v'', the apex beat (which is due largely to a *twisting* of the ventricles) and the electrocardiogram (which is affected by the path of the excitation wave in the musculature) will depend in even greater measure on the *positions* of the portions of muscle v' and v''. Therefore, when v' is greater than v'', the pulse wave corresponding to $V - v''$ will be greater than that corresponding to $V - v'$, while the apex beat corresponding to $V - v''$ may be either greater or less than that corresponding to $V - v'$. This may be made clearer by taking an extreme case. Supposing that v' represents a fairly large area of muscle in the posterior wall of the ventricles while v'' represents a much smaller area in the anterior walls. The radial pulse wave produced by $V - v''$ will be greater than that due to $V - v'$, but the apex beat may easily be greater for $V - v'$ than for $V - v''$. Thus the larger apex beats will fall in the odd series, the larger pulse waves in the even series.

Similarly with respect to the electrocardiogram. The failure of excitation in a small region near the apex is likely to produce greater modification of the detectable electric variation of the whole heart, than is the failure of a larger amount of muscle nearer the base.

In those cases where $v' = v''$, the pulse waves or the suspension record may show no alternation, while the apex beats or the electric variations, or both, give clear evidence of alternation.

Such cases are of peculiar interest; although absolute equality of v' and v'' will be extremely rare, an approximation to this condition is encountered fairly often, so that while the beats of the ventricle are apparently only very slightly if at all different in size, the electrogram exhibits marked alternation. It was through finding several cases of this kind in the frog's heart that my interest in the subject was aroused*.

* I have already published examples in this Journal. See Vol. XVI. Plate VII. Figs. 8, 9, 10.

It is possible to interpret such curves as these on the assumption that the path followed by the excitation wave in the ventricle was different in the odd and in the even series of excitations. The mechanical record, unlike the electrical, takes no count of the order in which the various regions of the ventricle become excited.

It is interesting to note that this condition, for which I have proposed the name *pulsus alternans celatus*, is frequently followed by marked mechanical alternation. An instance of this has been shown*. Comparison of curves (*b*) and (*c*)* illustrates once more the fact discovered by Gaskell, that when the odd series gets smaller the even series gets larger. Evidently this means that a portion of the tissue which was contracting with the odd series goes over to the even series or *vice versa*. To do so, it has only to miss one beat or to take up one extra excitation.

Windle (1910) in fig. 7 of his paper gives a curve which illustrates the same point.

From what has been said, it is clear that an extra-systole of the musculature as a whole, whether provoked by an idioventricular excitation or by an artificial stimulus, will be expected to influence very materially the distribution of the tissue v' and v'' between the odd and the even series. Exactly what effect the extra-systole will produce will depend on the moment of its arrival. Theoretically it may be expected sometimes to reduce and sometimes to increase the extent of the alternation, depending on the phase in which it arrives. But it will generally produce one or the other effect, for it is only for a short period in each complete cycle of two beats that v' and v'' are *both* refractory. Supposing at a certain instant v' is refractory and v'' is not. At the beat about to arrive v'' would respond. But if at this instant an extra-systole occurs, v'' will give a premature response and will then miss either one or two natural excitations, depending on the exact time relations of the extra-systole and the natural excitations. If it misses one only it will be transferred from one series to the other.

These conclusions are precisely in accord with Windle's observations: that extra-systoles have a profound effect on the course of *pulsus alternans*, but sometimes in the direction of increasing and sometimes reducing the alternation.

Postscript. Since the above was written my attention has been drawn to another paper by Hering (*Zeitschr. f. exper. Pathol. u. Therap.* 1909, VII. p. 363). On p. 372, Hering put forward a view which is practically identical with that advocated in the present paper. He did not, however, recognise that this view was implicit in Gaskell's original statement.

Nov. 13, 1912.

* *loc. cit.* Fig. 11.

DESCRIPTION OF PLATE I.

Fig. 1. Frog's heart. Top line of record shows contractions of an auricle (down-stroke = systole). Second line of contractions of ventricle. Third line electrogram by direct derivation of base and apex of ventricle. Einthoven galvanometer. High tension of quartz fibre. Sensitiveness —about 8 mm. deflection for 1 centi-volt. Alternation, developed spontaneously. Signal line indicates stimulation of *sinus venosus* with tetanising current of moderate strength. Time in seconds.

Fig. 2. Frog's heart. Arrangement as in Fig. 1. Description in text.

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Fig. 1.

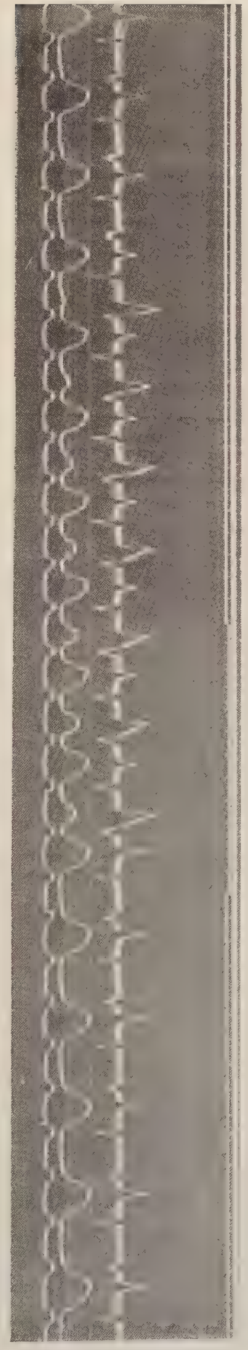


Fig. 2.

The Diffraction of Short Electromagnetic Waves by a Crystal.

By W. L. Bragg, B.A., Trinity College. (Communicated by Professor Sir J. J. Thomson.)

[*Read* 11 November 1912.]

[Plate II.]

Herren Friedrich, Knipping, and Laue have lately published a paper entitled 'Interference Phenomena with Röntgen Rays*,' the experiments which form the subject of the paper being carried out in the following way. A very narrow pencil of rays from an X-ray bulb is isolated by a series of lead screens pierced with fine holes. In the path of this beam is set a small slip of crystal, and a photographic plate is placed a few centimetres behind the crystal at right angles to the beam. When the plate is developed, there appears on it, as well as the intense spot caused by the undeviated X-rays, a series of fainter spots forming an intricate geometrical pattern. By moving the photographic plate backwards or forwards it can be seen that these spots are formed by rectilinear pencils spreading in all directions from the crystal, some of them making an angle of over 45° with the direction of the incident radiation.

When the crystal is a specimen of cubical zinc blende, and one of its three principal cubic axes is set parallel to the incident beam, the pattern of spots is symmetrical about the two remaining axes. This pattern is shown in Plate II. Laue's theory of the formation of this pattern is as follows. He considers the molecules of the crystal to form a three-dimensional grating, each molecule being capable of emitting secondary vibrations when struck by incident electromagnetic waves from the X-ray bulb. He places the molecules in the simplest possible of the three cubical point systems, that is, molecules arranged in space in a pattern whose element is a little cube of side 'a,' with a molecule at each corner. He takes coordinate axes whose origin is at a point in the crystal and which are parallel to the sides of the cubes. The incident waves are propagated in a direction parallel to the z axis, and on account of the narrowness of the beam the wave surfaces may be taken to be parallel to the xy plane. The spots are considered to be interference maxima of the waves scattered by the orderly arrangement of molecules in the crystal. In order to get an interference maximum in the direction

* *Sitzungsberichte der Königlich Bayerischen Akademie der Wissenschaften.* June 1912.

whose cosines are α, β, γ, for incident radiation of wave-length λ, the following equations must be satisfied

$$a\alpha = h_1\lambda, \quad ap = h_2\lambda, \quad a(1-\gamma) = h_3\lambda \;\ldots\ldots\ldots\ldots(1)$$

where h_1 h_2 h_3 are integers.

These equations express the condition that the secondary waves of wave-length λ from a molecule, considered for simplicity as being at the origin of coordinates, should be in phase with those from its neighbours along the three axes, and that therefore the secondary waves from all the molecules in the crystal must be in phase in the direction whose cosines are α β γ.

The distance of the crystal from the photographic plate in the experiment was 3·56 cm. The pencil of X-rays on striking the crystal had for cross-section a circle of diameter about a millimetre, and the dimensions of the spots are of the same order. The plate of crystal was only ·5 millimetre thick. It is thus easy to calculate with considerable accuracy from the position of a spot on the photographic plate the direction cosines of the pencil to which it corresponds, since the pencils of rays may be all taken as coming from the centre of the crystal. Laue found, on doing this for each spot, that as a matter of fact the values for α β $1-\gamma$ so obtained were in the numerical ratio of three small integers h_1 h_2 h_3 as they should be by equations (1).

For instance, a spot appears on the photographic plate whose coordinates referred to the x and y axes are

$$x = \cdot 28 \text{ cm.}, \quad y = 1\cdot 42 \text{ cm.}$$

The distance of the crystal from the photographic plate, 3·56 cm., gives z.

Thus since $$\alpha : \beta : \gamma :: x : y : z$$

$$\frac{\alpha}{\cdot 28} = \frac{\beta}{1\cdot 42} = \frac{\gamma}{3\cdot 56} = \frac{1}{\sqrt{(\cdot 28)^2 + (1\cdot 42)^2 + (3\cdot 56)^2}} = \frac{1}{3\cdot 83}.$$

Thus $$\frac{\alpha}{\cdot 28} = \frac{\beta}{1\cdot 42} = \frac{1-\gamma}{\cdot 27},$$

or $$\alpha : \beta : 1-\gamma :: 1 : 5 : 1.$$

Laue considers some thirteen of the most intense spots in the pattern. Owing to the high symmetry of the figure, the whole pattern is a repetition of that part of it contained in an octant. Thus these thirteen represent a very large proportion of all the spots in the figure. For these spots he obtains corresponding integers h_1 h_2 h_3 which are always small, the greatest being the number 10. But even if one confines oneself to integers less than 10, there are a great many combinations of h_1 h_2 h_3 which might

give spots on the photographic plate which are in fact not there, and there is no obvious difference between the numbers $h_1\ h_2\ h_3$ which correspond to actual spots, and those which are not represented.

To explain this Laue assumes that only a few definite wave-lengths are present in the incident radiation, and that equations (1) are merely approximately satisfied.

Considering equations (1) it is clear that when $h_1\ h_2\ h_3$ are fixed $\frac{\lambda}{a}$ can only have one value. However if $h_1\ h_2\ h_3$ are multiplied by an integral factor p, equations (1) can still be satisfied, but now by a wave-length $\frac{\lambda}{p}$. By adjusting the numbers $h_1\ h_2\ h_3$ in this way, Laue accounts for all the spots considered by means of five different wave-lengths in the incident radiation. They are

$$\lambda = \cdot 0377a$$
$$\lambda = \cdot 0563a$$
$$\lambda = \cdot 0663a$$
$$\lambda = \cdot 1051a$$
$$\lambda = \cdot 143a.$$

For instance, in the example given above, where it was found that

$$\alpha : \beta : 1-\gamma :: 1 : 5 : 1$$

these numbers are multiplied by 2, becoming 2.10.2. Then they can be assigned to a wave-length

$$\frac{\lambda}{a} = \cdot 037,$$

approximately equal to the first of those given above.

However, this explanation seems unsatisfactory. Several sets of numbers $h_1\ h_2\ h_3$ can be found giving values of $\frac{\lambda}{a}$ approximating very closely to the five values above and yet no spot in the figure corresponds to these numbers. I think it is possible to explain the formation of the interference pattern without assuming that the incident radiation consists of merely a small number of wave-lengths. The explanation which I propose, on the contrary, assumes the existence of a continuous spectrum over a wide range in the incident radiation, and the action of the crystal as a diffraction grating will be considered from a different point of view which leads to some simplification.

Regard the incident light as being composed of a number of independent pulses, much as Schuster does in his treatment of the action of an ordinary line grating. When a pulse falls on a plane it is reflected. If it falls on a number of particles scattered over a plane which are capable of acting as centres of disturbance when struck by the incident pulse, the secondary waves from them will build up a wave front, exactly as if part of the pulse had been reflected from the plane, as in Huygen's construction for a reflected wave.

The atoms composing the crystal may be arranged in a great many ways in systems of parallel planes, the simplest being the cleavage planes of the crystal. I propose to regard each interference maximum as due to the reflection of the pulses in the incident beam in one of these systems. Consider the crystal as divided up in this way into a set of parallel planes. A minute fraction of the energy of a pulse traversing the crystal will be reflected from each plane in succession, and the corresponding interference maximum will be produced by a train of reflected pulses. The pulses in the train follow each other at intervals of $2d \cos \theta$, where θ is the angle of incidence of the primary rays on the plane, d is the shortest distance between successive identical planes in the crystal. Considered thus, the crystal actually 'manufactures' light of definite wave-lengths, much as, according to Schuster, a diffraction grating does. The difference in this case lies in the extremely short length of the waves. Each incident pulse produces a train of pulses and this train is resolvable into a series of wave-lengths $\lambda, \frac{\lambda}{2}, \frac{\lambda}{3}, \frac{\lambda}{4}$ etc. where $\lambda = 2d \cos \theta$.

Though to regard the incident radiation as a series of pulses is equivalent to assuming that all wave-lengths are present in its spectrum, it is probable that the energy of the spectrum will be greater for certain wave-lengths than for others. If the curve representing the distribution of energy in the spectrum rises to a maximum for a definite λ and falls off on either side, the pulses may be supposed to have a certain average 'breadth' of the order of this wave-length. Thus it is to be expected that the intensity of the spot produced by a train of waves from a set of planes in the crystal will depend on the value of the wave-length, viz. $2d \cos \theta$. When $2d \cos \theta$ is too small the successive pulses in the train are so close that they begin to neutralize each other and when again $2d \cos \theta$ is too large the pulses follow each other at large intervals and the train contains little energy. Thus the intensity of a spot depends on the energy in the spectrum of the incident radiation characteristic of the corresponding wave-length.

Another factor may influence the intensity of the spots. Consider a beam of unit cross-section falling on the crystal. The

strength of a pulse reflected from a single plane will depend on the number of atoms in that plane which conspire in reflecting the beam. When two sets of planes are compared which produce trains of equal wave-length it is to be expected that if in one set of planes twice as many atoms reflect the beam as in the other set, the corresponding spot will be more intense. In what follows I have assumed that it is reasonable to compare sets of planes in which the same number of atoms on a plane are traversed by unit cross-section of the incident beam, and it is for this reason that I have chosen the somewhat arbitrary parameters by which the planes will be defined. They lead to an easy comparison of the effective density of atoms in the planes. The effective density is the number of atoms per unit area when the plane with the atoms on it is projected on the xy axis, perpendicular to the incident light.

Laue considers that the molecules of zinc-blende are arranged at the corners of cubes, this being the simplest of the cubical point systems. According to the theory of Pope and Barlow this is not the most probable arrangement. For an assemblage of spheres of equal volume to be in closest packing, in an arrangement exhibiting cubic symmetry, the atoms must be arranged in such a way that the element of the pattern is a cube with an atom at each corner and one at the centre of each cube face. With regard to the crystal of zinc-blende under consideration zinc and sulphur being both divalent have equal valency volumes and their arrangement is probably of this kind. It will be assumed for the present that the zinc and sulphur atoms are identical as regards their power of emitting secondary waves.

Take the origin of coordinates at the centre of any atom, the axes being parallel to the cubical axes of the crystal. The distance between successive atoms of the crystal along the axes is taken for convenience to be $2a$.

All atoms in the xz plane will have coordinates

$$pa \quad o \quad qa$$

where p and q are integers and $p+q$ is even. See fig. 1 in text.

The same holds for atoms in the yz plane. Therefore any reflecting plane may be defined by saying that it passes through the origin, and the centres of atoms

$$\begin{array}{ccc} pa & o & qa \\ o & ra & sa \end{array}$$

For instance, the plane on which the triangle OAB lies passes through the origin and

$$\begin{array}{ccc} a & o & 3a \\ a & o & a \end{array}$$

The planes can now be classified by the corresponding values of p, q, r, s as parameters.

The direction cosines of a plane $p\ q\ r\ s$ will be

$$\frac{rq}{\sqrt{p^2s^2+q^2r^2+p^2r^2}},\quad \frac{ps}{\sqrt{p^2s^2+q^2r^2+p^2r^2}},\quad \frac{-pr}{\sqrt{p^2s^2+q^2r^2+p^2r^2}}.$$

If these are called $l\ m\ n$ the direction cosines of the reflected beam are

$$2ln,\quad 2mn,\quad 2n^2-1,$$

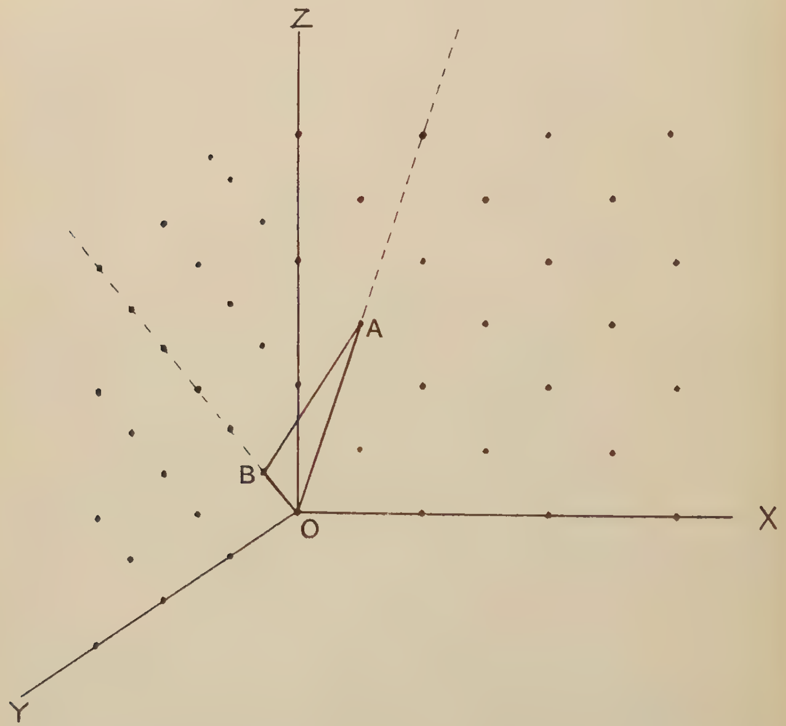


Fig. 1.

and the position of the interference maximum on the photographic plate can be found in terms of these quantities.

The corresponding wave-length is $2d\cos\theta$ where d is the perpendicular distance between successive planes. Now θ is the angle of incidence, therefore $\cos\theta = n$ above. It is easier to find the intercepts which successive planes cut off on the z axis, than their perpendicular distance apart. Calling these intercepts l, then

$$\lambda = 2d\cos\theta = 2\,.\,l\cos\theta\,.\cos\theta = 2ln^2.$$

Consider the atoms as arranged in vertical rows parallel to the z axis in the figure. A plane for which $p=1$ and $r=1$ passes through one atom in every one of these vertical rows (see fig. 3). Therefore the next plane to it passes through a set of atoms all $2a$ above the corresponding atoms in the first plane. Thus for this set of planes, $l=2a$ and the wave-length $\lambda=4an^2$. The effective density of atoms on such a set of planes is the greatest possible.

If $p=1$, $r=2$, each plane now passes through atoms in one half of the vertical rows. For instance, the plane through the origin contains no atoms in those vertical rows for which r is odd. The successive planes must cut the z axis at intervals $\frac{2a}{2}$, since the effective density of atoms in each is half as great as before and the whole number of atoms in unit volume of the crystal remains constant. Similarly if $p=1$, $r=3$ $l=\frac{2a}{3}$ and so forth.

In the general case $l=\dfrac{2a}{\text{L.C.M. of } p \text{ and } r}$.

In the tables given below planes with the same effective density of atoms on them, and therefore the same values of l, are grouped together.

The position of the spot reflected by each system of planes considered has been calculated, also the wave-length of the reflected train expressed for convenience in the form $\frac{a}{\lambda}$, and when in the photograph a spot is visible in the position calculated, its intensity is denoted by star according to an arbitrary scale.

✳ * * + •

When no spot appears in the calculated position, I have put 'invisible' opposite that plane.

There is no need to go any further than the set for which $l=\frac{2a}{4}$, to obtain all the spots in the photograph. Indeed only one spot is given by this last set.

Only one spot on the plate is to be assigned to planes of this class. It is curious that the value of $\frac{a}{\lambda}$ corresponding to this spot should be as great as 11·2. It is noticeable in the photograph that all spots at any distance from the centre of the pattern tend to become very faint, and the values of p, q, r, s which do give a spot in Table IV are the only ones to be found giving a spot at all near the centre. In the first three tables the parameters

corresponding to a value of $\frac{a}{\lambda}$ between 6 and 9 are represented by the most intense spots.

Every spot in the photograph is accounted for in the following Tables. I think it is evident that the sets of planes which actually reflect spots can be arranged in a very complete series with few or no gaps. Though at first sight it may appear that in the

TABLE I.

Planes for which $p=1$, $r=1$, $l=2a$, $\lambda=4an^2$.

p	q	r	s	$\frac{a}{\lambda}$	Intensity	h_1	h_2	h_3
1	1	1	3	2·8	*	1	3	1
1	1	1	5	6·8	✻	1	5	1
1	1	1	7	12·8	*	1	7	1
1	1	1	9	20·8	Invisible	1	9	1
1	3	1	1	2·8	*	3	1	1
1	3	1	3	4·8	*	3	3	1
1	3	1	5	8·8	✻	3	5	1
1	3	1	7	14·8	+	3	7	1
1	3	1	9	22·8	Invisible	3	9	1
1	5	1	1	6·8	✻	5	1	1
1	5	1	3	8·8	✻	5	3	1
1	5	1	5	12·8	*	5	5	1
1	5	1	7	18·8	Invisible	5	7	1
1	7	1	1	12·8	*	7	1	1
1	7	1	3	14·8	+	7	3	1
1	7	1	5	18·8	Invisible	7	5	1
1	9	1	1	20·8	Invisible	9	1	1

Range of values of $\frac{a}{\lambda}$, all possible up to 15.

tables the parameters are selected in a somewhat arbitrary way, they are in reality the simplest possible. For instance, in Table III the first values for p, q, r, s considered are 1, 1, 3, 5. This is so because '$r+s$' must be positive. If $r=1$, s must be odd.

1, 1, 3, 1 and 1, 1, 3, 3 would reflect the beam so as to miss the photographic plate. 1, 1, 3, 5 and 1, 1, 3, 7 are considered. 1, 1, 3, 9 has already been considered as 1, 1, 1, 3, and 1, 1, 3, 11 gives a value for the wave-length outside the 'visible' range.

In fig. 3, Plate II, is given a photograph of the interference pattern which Laue obtained. In fig. 4, Plate II, the key to the pattern has been drawn, showing in what planes the spots are to be considered as reflected.

TABLE II.

Planes for which L.C.M. *of* p *and* $r = 2$, $l = a$, $\lambda = 2an^2$.

p	q	r	s	$\frac{a}{\lambda}$	Intensity	h_1	h_2	h_3
1	1	2	4	3	•	2	4	2
1	1	2	8	9	✱	2	8	2
1	1	2	12	19	Invisible	2	12	2
2	4	2	0	2·5	Invisible	4	0	2
2	4	2	4	4·5	•	4	4	2
2	4	2	8	10·5	• ?	4	8	2
1	3	2	0	5	*	6	0	2
1	3	2	4	7	✱	6	4	2
1	3	2	8	13	• ?	6	8	2
2	8	2	0	8·5	•	8	0	2
2	8	2	4	10·5	• ?	8	4	2
1	5	2	0	13	Invisible	10	0	2

Consider a reflecting plane which passes through the atom at the origin and a neighbouring atom, let us suppose the atom whose coordinates are a, o, a. As the plane is turned about the line through these two points the reflected beam traces out a circular cone, which has for axis the line joining the two points and for one of its generators the incident beam. This cone cuts the photographic plate in an ellipse. If the atom through which the plane passes is in the xz plane as above, the ellipse touches the y axis on the photographic plate at the origin. Now take a plane passing through the origin and a point 0, a, $3a$. The

TABLE III.

Planes for which L.C.M. *of* p *and* $r=3$, $l=\frac{2a}{3}$, $\lambda=\frac{4an^2}{3}$.

p	q	r	s	$\frac{a}{\lambda}$	Intensity	h_1	h_2	h_3
1	1	3	5	3·6	Invisible	3	5	3
1	1	3	7	5·6	•	3	7	3
1	1	3	11	11·6	Invisible	3	11	3
3	5	3	5	4·9	Invisible	5	5	3
3	5	3	7	6·9	*	5	7	3
3	5	3	11	12·9	Invisible	5	11	3
3	7	3	1	4·9	Invisible	7	1	3
3	7	3	5	6·9	*	7	5	3
3	7	3	7	8·9	*	7	7	3
3	7	3	11	14·9	Invisible	7	11	3
1	3	3	1	7·6	*	9	1	3
1	3	3	5	9·6	+	9	5	3
1	3	3	7	11·6	Invisible	9	7	3

Range of values of $\frac{a}{\lambda}$, 5·6 = 9·6.

TABLE IV.

Planes for which the L.C.M. *of* p *and* $r=4$, $l=\frac{2a}{4}$,

$$\lambda=\frac{4an^2}{4}=an^2.$$

p	q	r	s	$\frac{a}{\lambda}$	Intensity	h_1	h_2	h_3
1	1	4	10	8·2	Invisible	4	10	4
1	1	4	14	16·2	Invisible	4	14	4
2	4	4	10	11·2	+	8	10	4
1	3	4	6	12·2	Invisible	12	6	4

locus of the reflected spot as it turns is again an ellipse, which now touches the x axis. The intersections of the two ellipses will give the position of a spot reflected by a plane passing through all three points, the origin, the point a, 0, a, and the point 0, a, $3a$.

The ellipses are drawn in the figure, and the plane corresponding to any spot can be found by noting the ellipses at the intersection of which the spot lies. Only those ellipses have been drawn which give the points in Table I. It will be seen that a very large proportion of the spots in the photograph lie at the intersection of these.

The analysis involved in this way of regarding the interference phenomena must be fundamentally the same as that employed by Laue. In fig. 1, suppose the phase difference between vibrations from successive atoms along the three axes, when waves of wave-length λ fall on the crystal, to be $2\pi h_1$, $2\pi h_2$, $2\pi h_3$. Then in order that the vibrations from those atoms, which are arranged in the figure at the centres of the cube faces, should also be in phase, one must have

$$\frac{h_1}{2}-\frac{h_3}{2}=\text{an integer}, \qquad \frac{h_2}{2}-\frac{h_3}{2}=\text{an integer}.$$

This condition is simply expressed by saying that h_1, h_2, h_3 must all be even or all odd integers. When h_1, h_2, h_3 are given, the value of λ follows from

$$\frac{\lambda}{2a}=\frac{2h_3}{\sqrt{h_1^2+h_2^2+h_3^2}},$$

since here $2a$ has been taken as the distance between neighbouring molecules along the three axes.

If the three simplest values of h_1, h_2, h_3 for a spot on the plate are not all odd, or all even, then these numbers must be doubled to make them even and the wave-length accordingly halved.

When this is done, it can be seen that for each value of h_3 there is a series of values of h_1 and h_2. These numbers all give spots in the photograph if the corresponding value of $\frac{a}{\lambda}$ lies within a certain range. The smaller the number h_1, the larger is the range of $\frac{a}{\lambda}$ for which spots are visible. Spots whose $\frac{a}{\lambda}$ lies near the extremity of the range are very faint, those whose $\frac{a}{\lambda}$ is in the middle of the range are intense. In the tables the values of h_1, h_2, h_3 corresponding to each spot are set down.

It is quite probable that the qualitative explanation put forward here to account for the intensities of the spots is not the right one, other explanations being possible. For instance, one might substitute for the factor termed 'effective density' above, one which expressed the fact that, other things being equal, spots nearer the centre of the pattern were more intense than those farther out. This, together with the right curve for the distribution of energy in the spectrum of the incident radiation, could be made to account for the intensities quite reasonably. This does not vitiate the conclusion that the spots in the pattern represent a series which is complete, and characteristic of a cubical crystalline arrangement. The other arrangements of cubical point systems cannot, as far as I can see, give such a complete series. The other possible arrangements have for elements of their pattern (1) a cube with a molecule or atom at each corner, the arrangement which Laue pictured, or (2) a cube with a molecule at each corner and one at the centre. Neither arrangement will fit the system of planes given above. It is only the third point system, the element of whose pattern has a molecule at each corner and one at the centre of each cube face, which will lend itself to the system of planes found to represent spots in the photograph.

This last system, seeing that it forms an arrangement of the closest possible packing, is according to the results of Pope and Barlow the most probable one for the cubic form of zinc sulphide.

In one of the photographs taken by Messrs Friedrich and Knipping the crystal was so oriented that the direction of the incident radiation made equal angles with the three rectangular axes of the crystal. In this case a figure is obtained in which the pattern is a repetition of the spots contained in a sector of angle $\frac{\pi}{6}$. Regarding the spots as reflections of the incident beam in planes as before, these planes can be found almost as easily as those which reflect the spots in the square pattern, and indeed in many cases the planes are identical. I will not give the calculations here, but one point is of especial interest. A photograph was taken of the crystal oriented so that the pattern obtained was perfectly symmetrical. The crystal was then tilted through 3° about a line perpendicular to the incident beam and to one of the cubical axes. This distorted the pattern considerably, but corresponding spots in the two patterns are easily to be recognised. The points which I wish to consider especially are the following.

In the first place, the spots in the distorted pattern are all displaced exactly as would be expected if they were reflections in planes fixed in the crystal. For instance, when the reflecting plane contains the line, about which the crystal was tilted through 3°, it can be ascertained that the movement of the spot

corresponds to a deviation of the reflected beam through 6°. This alone is, I think, strong evidence that the wave-length λ is elastic, and not confined to a few definite values, and that equations (1) are satisfied rigorously and not merely approximately.

Besides the distortion of the figure due to the tilting of the crystal, a very marked alteration in the intensity of the spots is to be noticed. This is especially marked for those spots which are near the centre of the pattern, but not on or near the axis about which the crystal is tilted. This is probably due to the fact that for these spots a considerable change in wave-length has taken place.

When the angle of incidence θ of the primary beam on a set of reflecting planes varies, the value of $2d\cos\theta$ is altered and the alteration for the same $\delta\theta$ is greater the greater θ is.

One spot in particular changes from being hardly visible in the symmetrical pattern to being by far the most intense when the crystal is tilted. It is the spot reflected in a plane passing through the origin and

$$3a,\ 0,\ a;\quad 0,\ 3a,\ a.$$

Planes parallel to this have for d, the shortest distance between successive planes, the value $\frac{4a}{\sqrt{11}}$. It can easily be calculated from the position of the spot that the value of $\cos\theta$ changes from ·19 to ·12 when the crystal is tilted. This corresponds to a change in the value of $\frac{a}{\lambda}$ from 4·3 to 6·5, and it was found before for the square pattern that spots corresponding to the former wave-lengths were weak, those corresponding to the latter intense.

A curious feature of the photographs may be explained by regarding the spots as formed by reflection. As the distance of the photographic plate from the crystal is altered, the shape of each individual spot varies. At first round, they become more and more elliptical as the plate is moved further away. A reason for this is found in the following. If the incident beam is not perfectly parallel, but slightly conical, rays will strike the crystal at slightly different angles. Regard the crystal as a set of reflecting planes perpendicular to the plane of the paper (fig. 2). The rays striking the reflecting planes on the upper part of the crystal on the whole meet them at a less angle of incidence than those striking the planes at the bottom; the latter are deflected more, and the rays tend on reflection to come to a focus in a horizontal line. On the other hand, rays deviating from the axial direction in a horizontal plane diverge still more after reflection. Thus as the plate is removed from the crystal, the spots up to a certain distance become more and more elliptical.

The atoms of a crystal may be arranged in 'doubly infinite' series of parallel rows, as well as in 'singly infinite' series of planes. The incident pulse falls on atom after atom in one of these rows, if the row is not parallel to the wave front, and secondary waves are emitted, one from each atom, at definite time intervals. Along any direction lying on a certain circular cone with the row of atoms as axis, these secondary waves will be all in phase, one generator of the cone being, of course, parallel to the direction of the incident radiation. If the row of atoms makes a small angle with the direction, this cone with vertex at the crystal slip may now be considered to cut the photographic plate in an almost circular ellipse passing through the big central

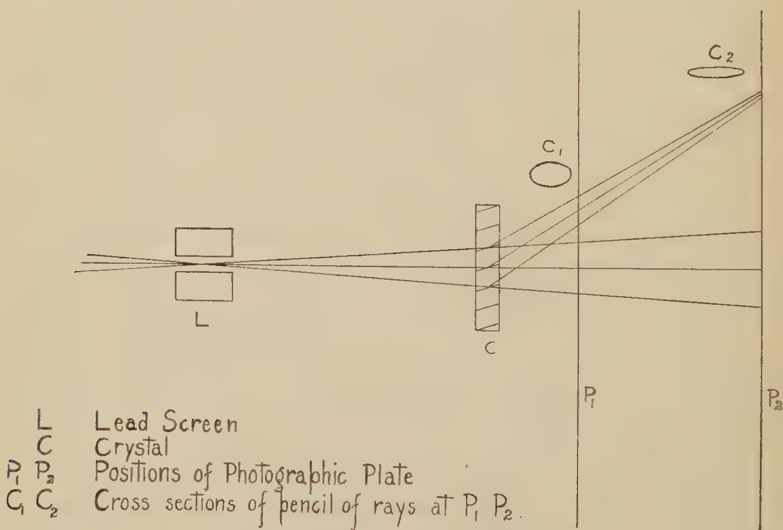


Fig. 2.

spot. Drawing the ellipses which correspond to the most densely packed rows of the crystal, a spot is to be expected at the intersection of two ellipses, for this means that pulses from a doubly infinite set of atoms are in that direction in agreement of phase. Thus it ought to be possible to arrange the spots in the photograph on these ellipses, in whatever way the crystal is oriented, and indeed they appear in all cases. They come out very strongly in the photographs taken with copper sulphate crystals.

So far it has been assumed that the atoms of zinc and sulphur act in an identical manner with regard to the production of secondary waves, but this assumption is not necessary. What is brought

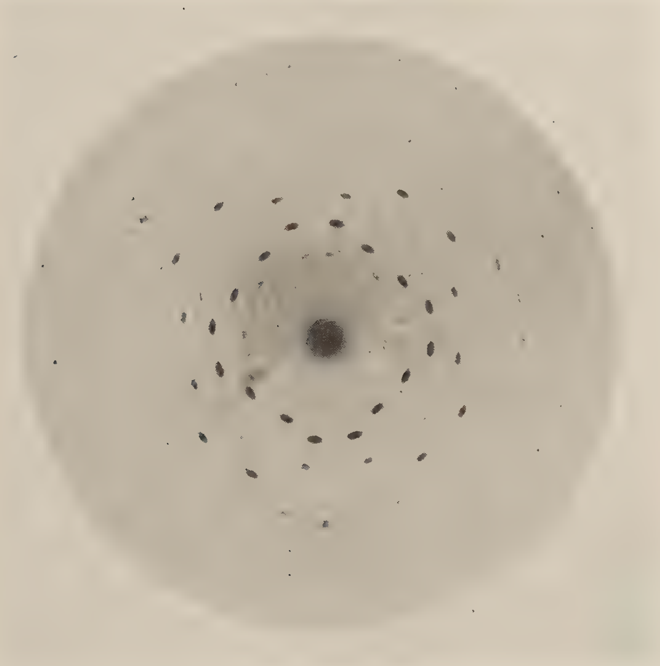


Fig. 3.

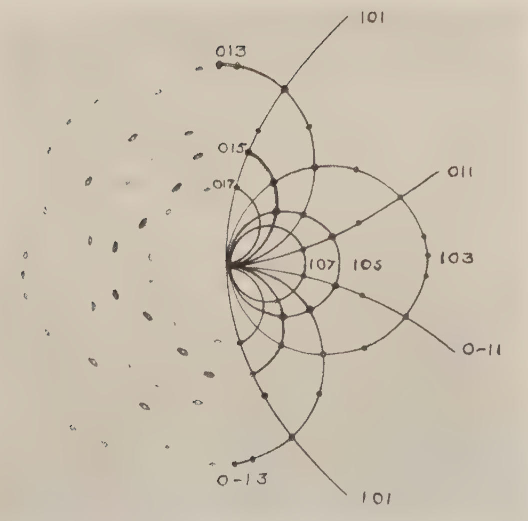


Fig. 4.

out so strongly by the analysis is this; that the point system to be considered has for element of its pattern a point at each corner of the cube and one at the centre of each cube face. In the arrangement assigned to cubical zinc sulphide and similar crystals by Pope and Barlow, this point system is characteristic of both the arrangement of the individual atoms regarded as equal spheres, and of the arrangement of atoms which are in every way identical as regards nature, orientation, and neighbours in the pattern. The atoms of zinc, for instance, in the zinc blende are grouped four together tetrahedron-wise, and as these little tetrahedra are all similarly oriented and are arranged themselves in the above point system, atoms of zinc identical in all respects will again be arranged in this point system. Which of these factors it is that decides the form of the interference pattern might be found by experiments with crystals in which the point system formed by the centres of all the atoms differs from that formed by the centres of identical atoms.

In conclusion, I wish to thank Professor Pope for his kind help and advice on the subject of crystal structure.

The Meres of Breckland. By J. E. Marr, Sc.D., F.R.S., St John's College.

[*Read* 25 November 1912.]

The sandy heaths of south-west Norfolk and north-west Suffolk form a type of physiographical feature unique in Britain. Among many points of interest in the district are several small meres, in the drainage-basin of the Little Ouse; these lie among the heaths between Croxton and Wretham, north of Thetford.

The origin of these meres has never been explained, and requires elucidation. These notes are only intended to direct attention to the question of their formation, and to offer some facts bearing upon it.

The meres above mentioned form a cluster a little way north-west of Roudham Junction. They are noticed by Mr F. J. Bennett in the Geological Survey Memoir of the district (*Geology of the Country around Attleborough, Watton, and Wymondham*, 1884, p. 17). He suggests that they started as sand and gravel pipes in the chalk, and afterwards became enlarged by chemical solution, assisted by mechanical disintegration during rise and fall of underground water.

An interesting paper on the meres by Mr W. G. Clarke appeared in the *Transactions of the Norfolk and Norwich Naturalists' Society*, Vol. VII (1903), p. 499; it contains a number of observations, but he does not give any explanation of their origin other than that offered by Mr Bennett.

The meres under notice occur in two shallow valleys lying north and south of an east-west plateau. In the northern valley lie the Devil's Punch Bowl, Fowlmere, Home Mere, and a group in Wretham Park including Mickle Mere: in the southern one are Langmere and Ringmere.

The meres possess several features in common. All are dry in seasons of little rainfall, and only under very exceptional conditions is any one completely filled to the brim, so that normally they have no outlet. The sides of some are steep, at the angle of repose of the material which composes them, and the surface

of the water usually lies at from 15 to 25 feet below the rim. The floors are flat, and the waters shallow.

They differ in size and shape, the smaller being most regular and approaching a circular outline: such are the Devil's Punch Bowl, Ringmere and many nameless ponds, the latter often only a few feet in diameter. The largest are less than half a mile in length. These are irregular, often having sinuous shore-lines, and in some, as Langmere, one diameter is longer than the other.

The meres in Wretham Park have undergone considerable modification at the hands of man, but those on the open heath are in their natural condition.

Associated with the named meres are the above-mentioned small ponds. These grade downward in size to ordinary swallow-holes, such as occur in most limestone districts, and there is little doubt that these swallow-holes mark the starting-point in the formation of the larger meres. They are situated in chalk and, as in the case of the meres which still hold water at times, near the junction of the chalk with overlying glacial deposits.

Langmere and Ringmere lie in two tributary-valleys south of the plateau. Tracing these valleys downward to their coalescence near Roudham Junction, we find remains of similar hollows, now nearly always dry, and with the sides sloping gently to the floors.

That the valleys existed before the meres is indicated by a deposit of river gravel exposed on the bank of Ringmere.

The origin of some of the meres is probably complex. The swallow-holes may have begun as sand-pipes, as suggested by Mr Bennett, or, like those of other limestone regions, may be simply due to enlargement of a place on a joint by acidulated water, or to subsidence of part of the roof of a subterranean hollow. Such a subsidence giving rise to a pit at Rockland is described by Mr Bennett (*loc. cit.* p. 21).

When sufficiently large, the surface-drainage of the little valleys situated on the glacial clays would, when reaching the chalk as it was exposed by the denudation of those clays, be carried underground through swallow-holes, but the transport of glacial clay to the floor of the swallow-hole might block it to a degree sufficient to prevent free drainage into the subterranean water-course, and hence the water would stand in the swallow-hole during wet periods. The standing water would sap the sides of the hole by solution, as suggested by Mr Bennett, slipping of the slope above would take place, as can now be seen in progress, and the hole would grow in circumference, giving rise ultimately to meres like the Punch Bowl and Ringmere. Coalescence of two or more would produce irregular meres like Langmere and

Fowlmere. Again mechanical erosion of the watercourse above the mere would widen the bed there, while that of the deserted stream below the mere would be no longer lowered, and a larger area to be filled with water would be thus produced. In other cases, direct collapse of the roofs of underground caves might initiate meres.

In hollows of other districts bearing some resemblances to those of Breckland, the upward pressure of artesian waters due to erosion of overlying impervious deposits has been regarded as the cause of formation of the hollows. The conditions in Breckland do not seem suitable for such occurrence.

The existence of deserted mere-hollows down-stream near Roudham Junction is of importance. At the time of their formation, according to the above views, the junction of glacial clays and chalk would be situated near these sites, it having been gradually driven up-stream to its present position by erosion.

When the glacial clay junction was near these hollows, their underground drainage would be partly blocked by deposit of clay on their floors. As the junction between clay and chalk was shifted up valley, the supply of clay would be stopped in the lower meres, being now deposited in newly formed upper ones. Solution would now proceed unchecked until a free underground passage for all water draining into the hollows would be established, and the meres would then be permanently dry.

In connexion with this question, however, is that of the mode of infilling of the meres, and here we find conflicting views; one, that they are filled by the rise of the saturation-level of underground water, the other that they are fed by surface streams. (See W. G. Clarke, *loc. cit.*) If the latter, the supply of water would cease when the glacial clay was denuded.

As to the age of the meres, they are clearly post-glacial in the sense that they were formed after the accumulation of the boulder-clay of the district. They may have been formed at various subsequent periods, and are probably still in process of formation. One of the meres at Wretham was drained and its floor dug out, and the results were described by Mr C. J. F. (afterwards Sir Charles) Bunbury (*Quart. Journ. Geol. Soc.* XII. 1856, p. 355). Over twenty feet of black peaty mud was found resting on a light grey sandy marl. In the mud were a number of horns of the red deer, which had been sawn off just above the brow-antlers. In connexion with this discovery Mr Skertchley (Geol. Survey Mem. *Geology of the Fenland*, p. 248), refers to the horns used for picks in the flint-mines at Grimes Graves. These mines have been hitherto referred to the Neolithic Period, but Mr Reginald Smith has recently advocated their assignment

to the Aurignacian stage of the Palaeolithic Period (*Archaeologia*, LXII. 1912, p. 109). I have recently found a station near Ringmere with implements which strongly resemble those of the last (Magdalenian) stage of the Palaeolithic Period. If these should prove to be Magdalenian, the horns found in the neighbouring mere may have been cut at the same time, for bone and horn were extensively used in the Magdalenian period.

Other remains have been found in other of the meres, and their different ages will probably be ultimately fixed on archaeological evidence.

I hope that some one may be found who will devote himself to a thorough study of these meres and their history.

Note on a Remarkable Instance of Complete Rock-disintegration by Weathering. By F. H. HATCH, Ph.D., Mem.Inst.C.E.

[*Read* 25 November 1912.]

THE material to be described occurs at Diamantina, in the province of Minas Geraes, Brazil, where it is being worked for diamonds. It consists of a conglomerate which, under the influence of weathering, has been disintegrated to the condition of a loose sandy formation capable of being dug out with a shovel at the lowest depth yet attained in the open working.

The Pebbles. The pebbles range in size from the smallest dimensions up to a maximum diameter of 3 inches. They have been worn perfectly smooth by water attrition and are rounded, generally to an ovoid shape. The materials of which they are composed are, in the order of relative abundance:

Quartzite.
Vein-quartz.
Steatite or soapstone.
Tourmaline-quartz vein-stuff.

The steatitic material, on account of its softness and consequent liability to pulverisation, rarely shows any smooth or rounded surfaces; it crumbles between the fingers to an unctuous powder, resembling French chalk. No doubt this material has been derived originally from the decomposition of an ultra-basic igneous rock in which magnesian silicates (such as olivine) predominated.

The other pebbles, viz. those of the quartzite, vein-quartz and tourmaline-quartz vein-stuff, are in a very friable condition, crushing to powder in the hand under the least pressure. This peculiar condition of pebbles of materials such as quartzite and vein-quartz, which in their normal condition are characterised by extreme hardness and unseparability, will be referred to later.

The Sand. The sand consists of a mixture of colourless quartz and of the fine powder produced by the pulverisation of the soapstone fragments. By careful washing with water the slime formed by the soapstone can be removed, leaving a white quartz sand which under the microscope is found to consist of colourless angular grains, often presenting the characteristic pyramidal faces of quartz crystals. Rounded grains are rare, and

there is little doubt that the bulk of the fragments has been derived from the disintegration of the quartzite and vein-quartz pebbles under the influence of weathering.

In order to ascertain the nature of the heavy minerals in this sand, the whole of the sample was carefully panned, and the concentrates, after drying, introduced into bromoform of the density of 2·9. By this means a separation of the minerals with a density greater than 2·9 from the quartz which floated on the surface of the liquid was effected. A minute quantity of a heavy powder was obtained, the subsequent examination of which under the microscope disclosed the following minerals:

Zircon.	Chalcopyrite.
Zinc blende.	Rutile.
Galena.	Tourmaline.
Iron pyrites.	

Of these heavy constituents, the first three constitute about 95 per cent.

In addition to the above minerals, diamonds are found in the course of the washing operations conducted at the spot. I had an opportunity of examining 40 of these stones: they ranged in weight from 0·68 to 4·42 metric carats*, and showed the characteristic rounded shape of the diamond, the prevalent forms being the octahedron, the rhombic dodecahedron, the 3-faced octahedron and the 6-faced octahedron, either alone or in combination. The octahedral faces are usually smooth and brilliant, but triangular pittings are common on them. The majority of the stones have a characteristic greenish tint, but this is only "skin deep," and does not appear in the cut stones. Examination under the microscope shows that this colouration is due to the presence of small spots and flecks of some green mineral (chlorite?) in the superficial layers of the stones. There is little to indicate the original source of these diamonds. Although found in the conglomerate they are certainly older than that rock. It appears likely that there is some community in origin between them and the soapstone, which, as already stated, probably represents a decomposed olivine-rock of igneous origin.

Origin of the Material. The material is an ancient conglomerate, that has suffered prolonged weathering. So completely have the weathering agents performed their work that not only has the cement of the conglomerate been entirely removed, but the pebbles themselves have been profoundly affected. As already stated, many of the pebbles are rounded fragments of an older quartzite formation. The material that

* 1 metric carat = 200 milligrams.

originally cemented the constituent quartz grains of these pebbles has been wholly abstracted, leaving the quartz particles as a friable, almost non-coherent mass, easily reducible to powder between the fingers. In other cases the pebbles consist of tourmaline and quartz, and were evidently derived from pre-existing veins. These also have been reduced by the same weathering action to a loose friable condition. It is clear that the fragments of both these materials could only have acquired their rounded character by water-attrition when they were in a hard, compact and firmly-knit condition. Evidence is thus afforded both of the profound nature, and the prolonged duration of the weathering to which the conglomerate must have been exposed since its first formation*.

* Compare: J. G. Branner, The Decomposition of Rocks in Brazil, *Bull. Geol. Soc. Amer.* Vol. VII. 1896, pp. 295—300.

The Variation of Magnetic Susceptibility with Temperature. Part II. On Aqueous Solutions. By A. E. OXLEY, M.Sc. (Sheffield), B.Sc. (London), B.A., Senior Scholar and Coutts Trotter Student, Trinity College.

[*Received* 4 December 1912.]

(1) *Introduction.*

IN the examination of the relation between the concentration of an aqueous solution of a salt of a ferromagnetic element and its magnetic susceptibility*, it was hoped that some light would be thrown on the existence of complex hydrates, in the case at least of very dilute solutions. That such hydrates exist is now beyond doubt. They are detected by experiments on conductivity and absorption† and are responsible for the abnormal lowering of the freezing point of the solutions. The examination referred to above gave no testimony of the existence of such hydrated systems, and the conclusion of that research was that the complex groups, which have an ion or salt molecule as nucleus, are unstable, so unstable in fact that the core rotates on the application of the magnetic field independent of the surrounding shell of water molecules.

Pascal‡ has examined the magnetic properties of solutions of metallic salts in water to find if any additive law holds, but no quantitative deductions could be made on account of the marked characteristic properties of the metallic elements.

There is considerable evidence§|| that the liquid and solid states of matter are consequences of complex aggregations of molecules and not merely of a close approach of the simple molecules considered as individual particles. In other words, the transition from the gaseous to the liquid state or from the liquid to the solid state is of a quasi-chemical rather than of a physical nature. If part of the solvent be united indeterminately with the solute, then the representation of the susceptibility of the solution as the algebraic sum of the susceptibilities of the solvent and of the solute—a common mode of representation—corresponds to no physical reality.

* A. E. Oxley, *Proc. Camb. Phil. Soc.*, Vol. XVI. p. 421, 1912.

† *Vide* H. C. Jones, "Hydrates in Aqueous Solutions." "Absorption Spectra of Solutions."

‡ *Ann. de Chim. et de Phys.*, Sér. VII. 19, p. 70, 1910.

§ For liquids, see the researches of H. C. Jones and his co-workers given in the volumes mentioned in the reference above.

|| For solids, reference may be made to numerous researches on micro-structure and on the constitution of alloys.

Assuming the existence of chemically complex systems—*chemically* complex excludes the cases of the ferromagnetic elements in which the molecular field of Weiss is not negligibly small—it has been shown* that the work of du Bois and Honda on the temperature coefficients of paramagnetic and diamagnetic substances is not inconsistent with the assumptions at the base of the Curie-Langevin theory of magnetism, providing we suppose that these assumptions apply to substances composed of simple molecules only†—such for example as exist in gases at a temperature far from the point of liquefaction. Further, it does not follow that because the differential coefficient of the susceptibility with respect to the temperature is positive for some paramagnetic elements and that the rate of variation of diamagnetic susceptibility with the temperature is not zero, that there is no distinction apart from mere positive and negative number between the nature of paramagnetism and that of diamagnetism‡.

It is important that the Curie-Langevin theory, which is the only quantitative theory of magnetism we possess, should not be discredited as failing to account for individual cases. Each element has its characteristic molecular and atomic properties, and a general theory cannot be satisfactory unless it admits of modification to suit each peculiarity possessed by that element. Take the case of tin, which is one of the most complex elements from the magnetic point of view. According to du Bois and Honda the Curie-Langevin laws of paramagnetism and diamagnetism do not apply. They find, indeed, that the susceptibility of tin changes rapidly when the density changes and also during fusion. If we could watch the rotation of the particles of tin, under the application of the magnetic field, as the temperature is increased, we should probably witness violent disturbances at the points where the constitution changes. At one temperature the groups of particles are becoming simpler, at another they are becoming more complex. So long as the constitution of an element does not alter as the temperature is changed, the magnetic properties of that element will follow the Curie-Langevin laws, the Curie constant per particle having a value dependent upon the constitution. For another stable molecular constitution which does not alter over ranges of temperature, there will be a new Curie constant per particle which will now determine the variation of the susceptibility with the temperature. In the transition stage the Curie-Langevin laws cannot alone explain the phenomena,

* A. E. Oxley, *Proc. Camb. Phil. Soc.*, Vol. XVI. p. 486, 1912.

† Or to complex groups of molecules whose constitution does not vary over wide ranges of temperature.

‡ *Loc. cit.*, p. 490.

for there is superposed upon the normal variation of the susceptibility with the temperature (due to the change in the number of particle collisions), the effect of chemical association or dissociation. In fact, over such a range of transition the susceptibility is dependent upon a summation of terms for the classes of particles of definite types, the expression for each type containing a factor regulating the rate of association or dissociation at any particular temperature during the transition.

On this view the nature of the continuity of the magnetic states of the so-called α, β, γ and δ forms of iron is apparent. The theory of Weiss*, which assumes all the particles to be of equal size, cannot apply during the stage of transition.

The rule proposed by Honda† in place of the Curie-Langevin laws is that the effect of a slight increase of the temperature on the susceptibility of an element is the same as that of a slight increase of the atomic weight. This rule is quite inadequate in the case of tin and many other elements, and even for those elements for which it does hold it is purely qualitative, and its application is therefore seriously limited.

In what follows we shall assume (1) that the paramagnetic susceptibility of a substance composed of particles which do not vary in complexity with the temperature is inversely proportional to the absolute temperature, (2) that the diamagnetic susceptibility of a substance composed of such particles is independent of the temperature.

(2) *General Theory.*

Consider the solution of a salt in water. The salt will be ionised to an extent depending upon the concentration. Moreover, there may be present complex groups of molecules—groups of water molecules, hydrated ions and hydrated molecules of undissociated salt. (Recent work has shown that in strong solutions multiply complex salt molecules exist, hydrated to an unknown extent‡.)

We shall assume that the groups of associated molecules are unstable, and that whether they are composed of similar or dissimilar molecules, the effect of a change of temperature on the group is of a similar nature in the two cases. This implies that in the groups composed of dissimilar molecules, the nucleus, whether it be an ion or a salt molecule, merely acts as a centre round which a shell of associated molecules is condensed, while the nature of the forces of association is the same as it is for a group of similar molecules.

Let there be N types of particles.

* *Comptes Rendus*, t. 144, p. 25, 1907.

† *Comptes Rendus*, t. 151, p. 511, 1910.

‡ Applebey, *Journ. Chem. Soc.*, Trans. II. p. 2000, Oct. 1910.

(a) *Paramagnetic susceptibility.*

Let n_p be the number of particles of type p per unit mass of the solution. Let ϑ be the absolute temperature and C_p the Curie constant per particle of type p. Since the susceptibility of salt solutions, even of the ferromagnetic elements, is independent of the intensity of the magnetic field and there is no hysteresis effect, it is not necessary to take into account the magnetic influences of the particles on one another.

With these assumptions we may write the specific paramagnetic susceptibility

$$\chi_P = \sum_{p=1}^{N} \frac{n_p \, . \, C_p}{\vartheta} \quad \text{......................(1).}$$

There is considerable evidence that the complex groups of molecules which are known to exist in solutions vary in composition as the temperature changes. Therefore n_p is a function of the temperature.

Write $$n_p = n_{0p} \, . \, F_p(\vartheta) \quad \text{......................(2).}$$

n_{0p} is the number of particles of type p which are found in unit mass of the solution at a temperature ϑ_0 given by

$$F_p(\vartheta_0) = 1.$$

Therefore $$\chi_P = \sum_{p=1}^{N} \frac{n_{0p} \, . \, C_p}{\vartheta} \, . \, F_p(\vartheta) \quad \text{.................(3).}$$

We shall refer to a series of researches by J. J. van Laar for the purpose of investigating the nature of the function $F_p(\vartheta)$. In his work on the theory of the liquid* and solid states, he takes into account the process of association, and shows that in the case of a substance composed of simple and double molecules, the degree of dissociation of the double molecules which exist at any temperature ϑ is given by the equation

$$\frac{\beta_2^2}{1-\beta_2^2} = C \, . \, \vartheta^{\gamma_2} \, . \, e^{-\frac{q_{02}}{R\vartheta}} \, . \, e^{-\left(P+\frac{a}{v^2}\right)\Delta b_2} \, . \, (v-b) \quad \text{......(4),}$$

where $$\left(P + \frac{a}{v^2}\right)(v-b) = (1+\beta_2) R\vartheta \quad \text{...............(4').}$$

Here β_2 is the degree of dissociation of the double molecules, $\gamma_2 R$ is the change of specific heat when one gramme molecule of double molecules passes into two gramme molecules of simple molecules, keeping the volume large and constant. Δb_2 is the accompanying change in the volume of the molecules, q_{02} is the quantity of heat absorbed in this change at the temperature $\vartheta = 0$; P, v, a, b, R and ϑ have the usual interpretation as in van der Waals' equation.

* *Arch. Teyler* (2), t. 11, troisième partie, pp. 235—331, 1909.

For water Δb_2* is negative and the equation (4) is shown graphically in fig. 1 for this case.

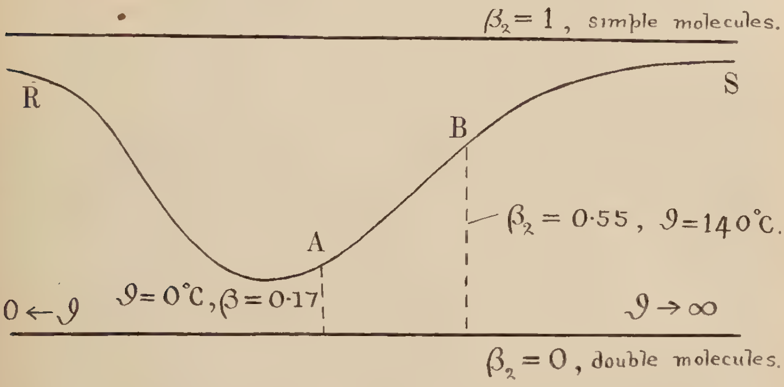


Fig. 1.

The portion *AB* of the curve, which is nearly linear, corresponds to the dissociation of double molecules into simple ones between the temperatures of 0° C. and 140° C.

The equation corresponding to (4) when the groups are on the average of p-fold complexity is shown to be †

$$\frac{\beta_p^{\,p}}{(1-\beta_p)(1+\overline{p-1}\,.\,\beta_p)^{p-1}} = C\,.\,\vartheta^{\gamma_p}\,.\,e^{-\frac{q_{0p}}{R\vartheta}}\,.\,e^{-\left(P+\frac{a}{v^2}\right).\Delta b_p}\,.\,(v-b)^{p-1} \;\ldots\ldots\ldots(5),$$

where $$\left(P+\frac{a}{v^2}\right)(v-b) = (1+\overline{p-1}\,.\,\beta_p)\,.\,R\vartheta \;\ldots\ldots\ldots(5').$$

We shall assume that the complex groups in solution are on the average of p-fold complexity. It will now be shown that between the temperatures of 0° C. and 100° C. the curves for all integral values of p are similar to the curve *RABS*, but lie everywhere above it, so that they are flatter. Near the critical temperature $\beta_p \to 1$. Substituting from (5′) in (5) and writing

$$C' = C\,.\,R^{p-1} \text{ ‡}$$

$$\frac{\beta_p^{\,p}}{1-\beta_p} = C'\,.\,\vartheta^{\gamma_p}\,.\,(v-b)^{p-1}\,.\,e^{-\frac{q_0}{R\vartheta}}\,.\,e^{-\frac{(1+\overline{p-1}\,.\,\beta_p)\,.\,\Delta b_p}{(v-b)}} \;\ldots(6).$$

* *Loc. cit.* p. 246.
† *Versl. Kon. Ak. v. Wetensch., Amsterdam, Proc.*, May 27, p. 86, 1911.
‡ *Loc. cit.* p. 98.

As the critical temperature is approached $\beta_p \rightarrow 1$, therefore

$$\frac{1}{1-\beta_p} = C' . \vartheta^{\gamma_p} . (v-b)^{p-1} . e^{-\frac{p . \Delta b_p}{(v-b)}} . e^{-\frac{q_{0p}}{R\vartheta}},$$

and

$$\frac{1}{1-\beta_2} = C' . \vartheta^{\gamma_2} (v-b) . e^{-\frac{2 . \Delta b_2}{(v-b)}} . e^{-\frac{q_{02}}{R\vartheta}}.$$

We shall assume that the amount of heat absorbed in the change of molecular complexity increases as the complexity increases, and that, further, the change of specific heat, $R\gamma_p$, and the alteration of volume which accompanies association, Δb_p, also increase numerically with the degree of complexity. Referring to the numerical values* of $(v-b)$, q_{02}, $R\vartheta$, Δb_2, it is seen that the ratio

$$\frac{1-\beta_2}{1-\beta_p} > 1 \qquad\qquad (6'),$$

on these assumptions.

Therefore near the critical temperature $\beta_p > \beta_2$.

From (6), differentiating with respect to ϑ, we find

$$\frac{\partial \beta_p}{\partial \vartheta} . \frac{p(1-\beta_p{}^2) . \beta_p{}^{p-1} + 2\beta_p{}^{p+1}}{(1-\beta_p{}^2)^2}$$

$$= C' . \gamma_p . \vartheta^{\gamma_p - 1} . (v-b)^{p-1} . e^{-(1+\overline{p-1} . \beta_p) \Delta b_p} . e^{-\frac{q_{0p}}{R\vartheta}}$$

$$- C' . \frac{p-1}{v-b} . \Delta b_p . \vartheta^{\gamma_p} . (v-b)^{p-1} . e^{-(1+\overline{p-1} . \beta_p) . \Delta b_p} . e^{-\frac{q_{0p}}{R\vartheta}} . \frac{\partial \beta_p}{\partial \vartheta}$$

$$- C' . \vartheta^{\gamma_p} (v-b)^{p-1} . e^{-(1+\overline{p-1} . \beta_p) \Delta b_p} . \frac{q_{0p}}{R\vartheta^2} . e^{-\frac{q_{0p}}{R\vartheta}},$$

neglecting the change in the value of Δb_p due to an infinitesimal change in ϑ.

If $$\frac{\partial \beta_p}{\partial \vartheta} = 0, \quad \gamma_p \vartheta_M{}^{\dagger\, \gamma_p - 1} = \vartheta_M{}^{\gamma_p} . \frac{q_{0p}}{R\vartheta_M{}^2},$$

and therefore $$\vartheta_M{}^{\ddagger} = \frac{q_{0p}}{R\gamma_p} \qquad\qquad (7).$$

Again

$$\frac{\beta_p{}^p}{1-\beta_p{}^2} = C' . \vartheta^{\gamma_p} (v-b)^{p-1} . e^{-\frac{(1+(p-1) . \beta_p) \Delta b_p}{v-b}} . e^{-\frac{q_{0p}}{R\vartheta}}.$$

* $(v-b) > 1$, $q_{02} = 5000$ gm. cals., $R\vartheta = 1380$ gm. calories for the critical temp. $\Delta b_2 = -8{\cdot}26$ c.c. if $v = 18{\cdot}0$ c.c. These values are taken from the paper referred to at the foot of p. 68.

† ϑ_M is the temperature at which the association is greatest.

‡ It should be noted that ϑ_M is not independent of p, for q_{0p} and $R\gamma_p$ are each functions of the molecular complexity. Hence the additional remarks concerning the positions of the minima.

Now β_p is a fraction, and as $p \to \infty$ the R.H.S. $\to \infty$, on the assumptions made above, because $v > b$, $\Delta b_p < 0$.

Therefore $\beta_p \to 1$ as $p \to \infty$, for all values of ϑ.

Hence for $p > 2$ the curves lie between the curve for $p = 2$ and the asymptote $\beta_p = 1$. They have each one minimum value, viz. that given by the relation (7). The portions of the curves which lie between the ordinates corresponding to 0° C. and 100° C. are very approximately linear. The exact positions of the minima for the curves of higher values of p do not matter, at least so far as the first approximation only is desired, since we have proved that the curvature of such curves is very small. On the other hand the principle of continuity demands that for the lower values of p the positions of the minima of β_p shall approximate with decreasing p to the position of the minimum for $p = 2$.

The curves must be of the form shown in fig. 2.

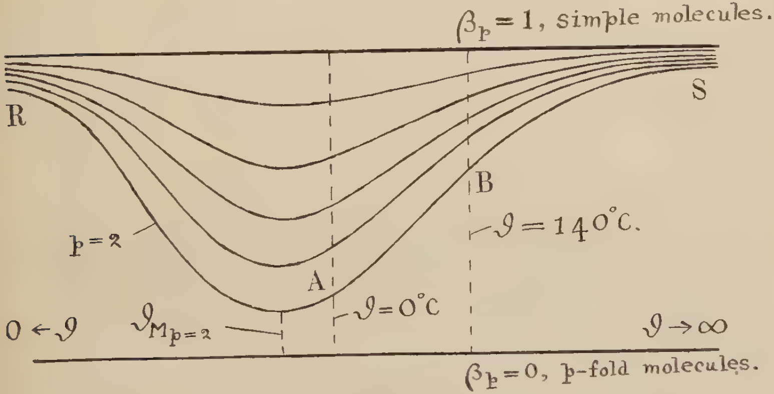


Fig. 2.

For all groups, the rate at which dissociation takes place *over the region 0° C. to 140° C.* is very approximately uniform. It is to be noted here that if we were to supercool or to superheat the solution to a sufficient degree, this step, in taking the curves as approximately linear, would no longer be justifiable, and the following reasoning holds only for solutions between the temperatures 273° and 400° absolute. It is conceivable that outside this range of temperature the formulae which will hereafter be deduced may require to be considerably modified.

We may therefore write

$$F_p(\vartheta) = \mu_p + \nu_p \,.\, \vartheta^* \quad \dots\dots\dots\dots\dots\dots\dots(8)$$

* Note added. M. M. Garver, *Journ. Phys. Chem.* Nov. 1912, p. 669. The experimental relation between polymerization and temperature is shown to be approximately linear.

for all values of p (integral) where μ_p and ν_p are independent of the temperature.

Equation (3) now becomes

$$\chi_P = \sum_{p=1}^{N} n_{0p} \,.\, C_p \,.\, \frac{\mu_p + \nu_p \,.\, \vartheta}{\vartheta}$$

$$= \sum_{p=1}^{N} \frac{n_{0p} \,.\, \mu_p \,.\, C_p}{\vartheta} + \sum_{p=1}^{N} n_{0p} \,.\, C_p \,.\, \nu_p .$$

Write $$\Sigma\, n_{0p} \,.\, C_p \,.\, \mu_p = A \quad \text{.....................(9)},$$

$$\Sigma\, n_{0p} \,.\, C_p \,.\, \nu_p = B' \quad \text{..................(10)}.$$

Then $$\chi_P = \frac{A}{\vartheta} + B' \quad \text{........................(11)},$$

where A and B' are independent of ϑ.

(b) *Diamagnetic susceptibility.*

The specific diamagnetic susceptibility of a solution may be written

$$\chi_D = \frac{1}{H} \,.\, \sum_{p=1}^{N} n_p \,.\, \delta M_p \quad \text{..................(12)},$$

where δM_p is the resultant magnetic moment produced in particles of type p, per particle, by the application of a magnetic field of intensity H. As before n_p is a linear function of the temperature.

Writing $$n_p = n_{0p} F_p(\vartheta) = n_{0p}(\mu_p + \nu_p \,.\, \vartheta),$$

we find $$\chi_D = \frac{1}{H} \,.\, \sum_{p=1}^{N} n_{0p} \,.\, \delta M_p \,.\, (\mu_p + \nu_p \,.\, \vartheta)$$

$$= \frac{1}{H} \,.\, \sum_{p=1}^{N} n_{0p} \,.\, \delta M_p \,.\, \mu_p + \frac{\vartheta}{H} \,.\, \sum_{p=1}^{N} n_{0p} \,.\, \delta M_p \,.\, \nu_p .$$

Write

$$\frac{1}{H} \,.\, \Sigma\, n_{0p} \,.\, \delta M_p \,.\, \mu_p = B'', \qquad \frac{1}{H} \,.\, \Sigma\, n_{0p} \,.\, \delta M_p \,.\, \nu_p = C.$$

Therefore $$\chi_D = B'' + C\vartheta,$$

where B'' and C are independent of ϑ.

The specific susceptibility of the solution may be written

$$\chi = \chi_P + \chi_D$$

$$= \frac{A}{\vartheta} + B + C\vartheta \quad \text{.....................(13)},$$

where $$B = B' + B'' \quad \text{..........................(14)}.$$

(3) *Application of the Equation* (13) *to Aqueous Solutions of Iron Salts.*

It is only in the cases of strongly magnetic solutions, such as those of salts of the ferromagnetic elements, that a satisfactory test of the applicability of the formula (13) can be obtained. For other salt solutions the value of the susceptibility is little different from that of water, and in measuring such small differences as those produced by change of temperature the errors are liable to be large. On the other hand, in the case of strong solutions of the salts of ferromagnetic elements, the paramagnetic susceptibility is so great compared with the variation of the diamagnetic susceptibility with the temperature that it is permissible, at least for the strong solutions, to neglect the latter. The dependence of diamagnetism on the temperature is expressed by the term $C\vartheta$ in the equation (13), and this term will accordingly be neglected. The diamagnetic susceptibility, in so far as it does not vary with the temperature, is included in the constant B. The expression connecting the susceptibility and the temperature may now be written

$$\chi = \frac{A}{\vartheta} + B \quad \text{......................(15)},$$

where $$B = (\Sigma\, n_{0p} \,.\, C_p \,.\, \nu_p) + \left(\frac{1}{H} \,.\, \Sigma\, n_{0p} \,.\, \delta M_p \,.\, \mu_p\right).$$

The relation (15) is hyperbolic. If the groups do not break up with change of temperature, all the factors ν_p in the first term of the expression for B are zero, and as in this case the second term of B represents the diamagnetic susceptibility (p const.) the above relation (15) reduces to the unmodified Curie-Langevin form. If the groups do break up with the change of temperature then the first term in the expression for B will not be zero, and the value of B may be positive or negative according as

$$\Sigma n_{0p} \,.\, C_p \,.\, \nu_p \gtrless \left| \frac{1}{H} \Sigma\, n_{0p} \,.\, \delta M_p \,.\, \mu_p \right| .$$

That the curves connecting susceptibility with temperature are approximately hyperbolae, for solutions of iron salts, can be seen from fig. 3. This diagram is reproduced through the kindness of Prof. J. S. Townsend*. As the numerical data corresponding to the curves were not published, it is impossible to test equation (15) by these observations, but from the trend of the curves it appears that an equation of this form would be suitable for the representation of Prof. Townsend's results.

* *Phil. Trans. Roy. Soc.*, Vol. 187, A. p. 547, 1896.

The same remark applies to the representation of the results of Jaeger and Meyer*, although the curvature in their diagrams appears smaller.

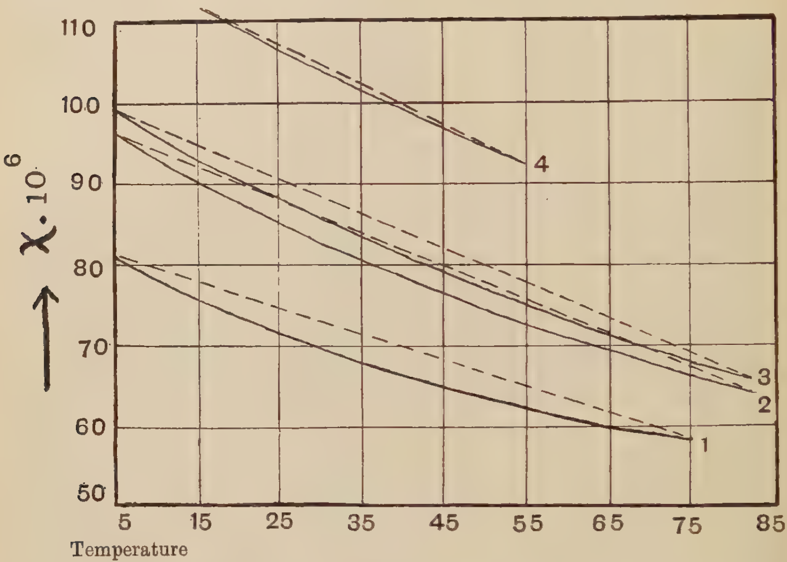


Fig. 3.

(1) Ferric Chloride ·086 gm. Iron per c.cm. (see note p. 79). (2) Ferrous Chloride. (3) Ferric Sulphate. (4) Alcoholic solution of Ferric Chloride.

In the following tables the first column represents the concentration of the solutions, used by Jaeger and Meyer; the concentrations are expressed in gramme molecules per litre. The second column gives the absolute temperature. In the third and fourth columns the values of the constants A and B are given for each concentration as worked out from the experimental values of χ given in column 5. Columns 6 and 7 show the agreement between the calculated and experimental values. For the calculations in column 6 the relation

$$\chi = \frac{A}{\vartheta} + B$$

is taken, for those in column 7 the empirical relation used by Townsend and Jaeger and Meyer is taken ($\chi = \chi_0 (1 - \epsilon . t)$).

An examination of the numbers given in columns 4, 5, 6 and 7 brings a considerable amount of information to light. It will be

* *Sitz. d. Akad. in Wien*, CVI. II A., p. 594, 1897.

observed that the representation of the susceptibility by the relation

$$\chi = \frac{A}{\vartheta} + B$$

holds good so long as we are dealing with concentrated solutions and is preferable to the linear relation

$$\chi = \chi_0 (1 - \epsilon . t).$$

But for weak solutions the linear relation is more satisfactory than the hyperbolic one. We see, on referring to p. 73, that this is precisely what would be expected. The approximate form

$$\chi = \frac{A}{\vartheta} + B$$

has been used instead of the accurate form

$$\chi = \frac{A}{\vartheta} + B + C\vartheta$$

for the representation; and the term $C\vartheta$, which deals with the variation of the diamagnetic susceptibility with the temperature, has been neglected. So long as we are dealing with strong solutions the variation of the diamagnetic susceptibility with the temperature is insignificant compared with the large value of the paramagnetic susceptibility, but for weak solutions this is not the case, and it is necessary to take into account the term $C\vartheta$.

The figures show that for the weaker solutions the calculated value of χ on the assumption that $\chi = \frac{A}{\vartheta} + B$ is in general lower than that calculated from the equation $\chi = \chi_0 (1 - \epsilon . t)$, and the experimental value lies between the two calculated values. Consider the variation of the susceptibility of water with the temperature. Although different observers have obtained different values for the absolute susceptibility of water, they are all in agreement that the susceptibility decreases as the temperature increases. This is equivalent to saying that water becomes more paramagnetic as the temperature increases. We shall take the expression

$$\chi_w = -0{\cdot}750 (1 - 0{\cdot}00164 t) 10^{-6}$$

as representing the variation of the susceptibility of water with the temperature.

$$\frac{\chi_{w0}}{\chi_{w100}} = \frac{1}{1 - 0{\cdot}164} \fallingdotseq 1{\cdot}2.$$

χ_{w100} is 20 °/。 less than χ_0. The maximum correction to be applied to the figure in column 6 in order to take this variation

of the susceptibility of water into account is $0{\cdot}07 \times 10^{-6}$, and corresponds to a temperature of 50° C. On either side of 50° C. the correction falls off and is zero at 0° C. and 100° C. due to the fact that we have made the curve pass through the extreme points. When the corrections are made there is a better agree-

Ferric Chloride Solutions.

Concn.	Tempr.	$A.10^6$	$B.10^6$	χ (Exp.)	χ (Calc.) $=A/\vartheta+B$	χ (Calc.) $=\chi_0(1-\epsilon t)$
3·62	273 + 1·7	1100·5	+ 4·18	$44{\cdot}23 \times 10^{-6}$	—	—
	11·4			42·68	42·86	43·13
	53·7			37·98	37·85	38·35
	81·9			35·18	—	—
2·32	273 + 2·1	5986·3	+ 7·51	29·27	—	—
	12·2			28·56	28·50	28·66
	32·2			27·27	27·12	27·46
	50·1			26·15	26·04	26·38
	70·1			24·85	24·96	25·07
	88·7			24·06	—	—
1·76	273 + 3·1	4930·5	+ 4·55	22·41	—	—
	16·8			21·66	21·56	21·73
	34·1			20·66	20·60	20·88
	53·0			19·41	19·67	19·95
	71·8			18·43	18·85	19·02
	89·6			18·15	—	—
1·31	273 + 3·6	3654·8	+ 2·90	16·11	—	—
	15·0			15·46	15·59	15·69
	35·1			14·93	14·76	14·98
	52·0			14·14	14·15	14·34
	70·9			13·69	13·53	13·65
	89·1			12·99	—	—
0·79	273 + 2·4	2875·3	− 0·86	9·58	—	—
	12·0			9·25	9·23	9·31
	36·5			8·50	8·44	8·61
	54·4			7·90	7·92	8·10
	71·6			7·66	7·49	7·60
	91·4			7·03	—	—
0·40	273 + 13·0	1190·4	− 0·29	3·87	—	—
	55·5			3·43	3·33	3·10
	80·0			3·08	—	—

Ferrous Sulphate Solutions.

Concn.	Tempr.	$A \,.\, 10^6$	$B \,.\, 10^6$	χ (Exp.)	χ (Calc.) $=A/\vartheta+B$	χ (Calc.) $=\chi_0\,(1-\epsilon t)$
1·09	273 + 2·3 43·9 80·3	3566·1	+ 1·286	$14{\cdot}24\times10^{-6}$ 12·57 11·38	— 12·54 —	— 12·71 —
0·88	273 + 3·2 15·0 54·0 80·8	2631·0	+ 0·90	10·43 10·04 8·93 8·34	— 10·037 8·95 —	— 10·11 9·06 —
0·53	273 + 2·4 44·2 80·1	1849·2	− 0·637	6·08 5·28 4·60	— 5·195 —	— 5·285 —
0·28	273 + 2·9 44·8 81·1	648·5	+ 0·305	2·66 2·38 2·14	— 2·35 —	— 2·38 —

Ferric Nitrate Solutions.

Concn.	Tempr.	$A \,.\, 10^6$	$B \,.\, 10^6$	χ (Exp.)	χ (Calc.) $=A/\vartheta+B$	χ (Calc.) $=\chi_0\,(1-\epsilon t)$
0·94	273 + 3·2 44·9 83·3	5934·9	− 0·161	$21{\cdot}28\times10^{-6}$ 18·70 16·46	— 18·51 —	— 18·8 —
0·48	273 + 2·0 44·3 83·7	2941·5	− 0·226	10·47 9·14 8·02	— 9·044 —	— 9·20 —

ment between the numbers in the sixth column and the experimental numbers, particularly with reference to the weaker solutions. There are a few exceptions to this statement, chief among which are the susceptibilities of a 1·76 normal solution of ferric chloride at temperatures of 53° C. and 71·8° C. These values are too high, but they are not so high as the corresponding values deduced from the linear relation, and it is possible that the irregularity is due to experimental errors in the measurement of temperature.

It is to be observed that the linear relation takes the variation of the diamagnetic susceptibility of water into account, and it is

for this reason that the linear relation is more satisfactory for low concentrations than for high concentrations.

If we take the complete expression deduced by the above theory

$$\chi = \frac{A}{\vartheta} + B + C\vartheta$$

then we have a relation between the susceptibility and the absolute temperature which holds for all concentrations, the agreement, with the exception of the two values cited above, being such as to give values within the limits of experimental error. Jaeger and Meyer do not state the amount of their experimental error, but it is difficult to get nearer to the true value than $\frac{1}{2}$ °/。 for the stronger solutions and 1 °/。 for the weaker solutions, especially when working at the higher temperatures.

The nature of the constant B is of greater importance than a mere representation of numerical values. It will be seen that this quantity, which is constant for any particular concentration for a range of temperature from 0° C. to 80° C., but varies as the concentration varies, throws some light on the variation of the constitution of the liquid state with change of temperature. The expression obtained for B (p. 73) is

$$B = (\Sigma\, n_{0p} \,.\, C_p \,.\, \nu_p) + \left(\frac{1}{H} \Sigma\, n_{0p} \,.\, \delta M_p \,.\, \mu_p\right).$$

This quantity does not vary erratically with the concentration, and it has a higher value for the higher concentrations than it has for the lower ones. Any factor of the type $n_{0p} \,.\, \mu_p$ is essentially positive, and as δM_p is negative for any particle of type p, the second term of B must necessarily be negative. Further, we know that over a wide range of concentration there is no appreciable change of diamagnetic susceptibility due to the variation of complexity of the molecular groups*, and therefore we may regard the second term in the expression for B as a negative term which admits of a negligibly small variation only, as we pass from one concentration to another. This term has a value which is very little different from the value of the susceptibility of pure water (approx. $-7{\cdot}0 \times 10^{-7}$).

The first term may be positive or negative. Any factor of the type $n_{0p} C_p$ is necessarily positive, but ν_p may be positive or negative—positive if the number of particles of type p is increasing and negative if the number is decreasing as the temperature is raised. As we have shown that the second term is nearly constant and has a small negative value we attribute the large positive values of B, for the higher concentrations, to particular values

* Townsend, *Phil. Trans. Roy. Soc.*, Vol. 187, A. p. 543, 1896.

of ν_p. For any concentration a definite selection of types of particles exists in the solution and the sum total of the products of the type $n_{0p}C_p$(const.) into the rate at which this particular type associates or dissociates as the temperature is varied is represented by the term B. Therefore variation in the value of B implies variation of the stability of the molecular complexes.

For a 2·23 normal solution of ferric chloride the constant B has a value which is 29 °/$_\circ$ of the value of the susceptibility at 50° C. (the mean temperature). The variation of the susceptibility with the temperature is well represented for this concentration by the hyperbolic relation*

$$\chi = \frac{A}{\vartheta} + B,$$

and it is remarkable that such a large positive quantity (the value of B is $7{\cdot}51 \times 10^{-6}$, that of the susceptibility at 50° C. is $26{\cdot}15 \times 10^{-6}$) which is independent of the temperature should enter into the expression for the variation of paramagnetic susceptibility with the temperature.

For solutions of ferric chloride of concentration 3·62 normal and 1·76 normal, the values of B are nearly equal, but are much less than the value for a 2·23 normal solution. It appears therefore that a solution of ferric chloride of this concentration has a unique constitution, and it is interesting to see if such a solution behaves abnormally with regard to other physical properties.

(4) *Measurements of the Viscosity of Ferric Chloride Solutions.*

The following experiments on the variation of viscosity of four solutions of ferric chloride with the temperature were carried out to investigate the complexity of strong solutions of this salt and to examine if any irregularity is shown by a 2·2 normal solution.

Experiments have been made by C. Chéneveau† and R. F. D'Arcy‡ on the viscosity of strong solutions of sulphuric acid in water, and they obtained evidence for the existence of complex groups of molecules over certain ranges of concentration.

The apparatus used in the following experiments is a modification of that used by D'Arcy and is shown in fig. 4. The ferric chloride solution was drawn into the bulb b from the receiver R, and when it had acquired the temperature of the water bath inside the calorimeter C, it was driven out through the capillary tube c'' back to the receiver R under a pressure head of water

* It should be remarked that the solution referred to contains almost exactly the same quantity of iron per c.cm. as does the solution used by Townsend, and his results are shown graphically, fig. 3, curve 1.

† *Comptes Rendus*, t. CLV., No. 2, 1912, p. 154.

‡ *Phil. Mag.* Vol. XXVIII., Ser. 5, 1889, p. 221.

supplied by the large bottles A and B. The pressure head was registered in the manometer M. The time required for the ferric chloride meniscus to fall from the mark s to the mark s' was taken by means of a stop-watch. Care was taken to ensure that the ferric chloride solution had acquired the temperature of the bath before a reading was obtained, and to secure this the bulb was filled in the following way. The solution was drawn into the bulb till the latter was about half full, and the U-tube was shaken by sliding the inlet and outlet tubes backwards and forwards through the holes in the side of the calorimeter. Meanwhile the liquid in the bath was kept well stirred. More liquid was drawn into the bulb and the shaking repeated. Finally the bulb was filled, and since the solution last admitted filled the capillary and part of the outlet tube, it soon acquired the temperature of the

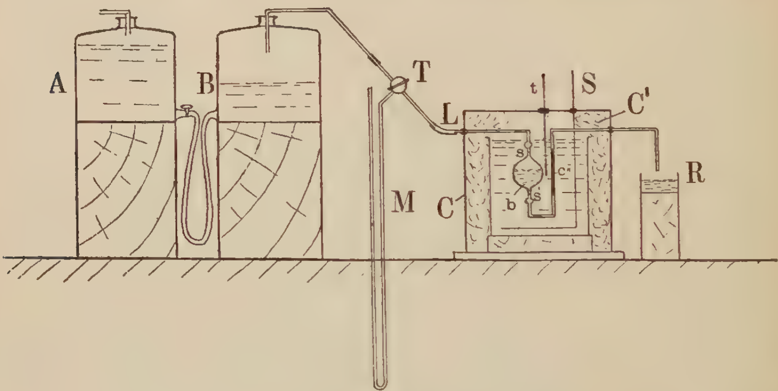


Fig. 4.

bath. An interval of about twenty minutes was usually allowed for the solution to acquire the temperature of the bath, but for the higher temperatures a much smaller interval had to suffice, on account of radiation, while at temperatures near that of the room an hour was frequently given*.

The method of taking an observation was as follows. The end of the outlet tube was kept below the level of the solution in R and the three-way cock T turned so as to shut off b and connect B with M. The pressure head was now adjusted to a certain value, approximately the same for all the observations. T was turned so as to connect B, M and b, R was lowered and the time taken as the meniscus passed the mark s.

The following observations were made:

* The small bubbles of air which were always formed in the outlet tube were removed before an observation was made.

I. *Water.*

Pressure (mm. water)	Mean temperature	Time (seconds)	Corrected Time (P. 85 mm.)
86·5	89·0	84·0	85·5
86·2	80·3	87·0	88·2
84·9	78·0	88·25	88·25
86·4	73·5	90·75	92·4
85·6	71·6	100·0	100·7
86·0	66·6	97·0	98·2
86·2	55·8	107·0	108·8
86·5	51·7	113·5	115·7
86·4	46·5	116·75	119·0
85·8	38·9	133·0	134·2
85·8	31·3	146·5	147·9
86·0	18·2	185·5	187·75
86·2	18·0	187·0	188·5
85·1	7·55	246·5	246·5
84·9	4·15	274·0	274·0
84·9	0·4	307·0	307·0
85·0	0·4	307·5	307·5

II. 1·3 *Normal Solution of Ferric Chloride.*

Pressure (mm. water)	Mean temperature	Time (seconds)	Corrected Time (P. 85 mm.)
84·7	74·5	130·0	129·4
84·2	66·3	146·25	144·6
84·8	58·1	163·0	163·0
84·0	48·1	191·5	190·6
84·6	39·1	225·0	224·0
84·8	31·1	266·25	265·6
85·0	26·4	300·0	300·0
84·9	26·0	294·25	294·25
84·6	19·6	341·75	340·2
85·4	19·6	336·5	338·9
85·7	14·8	378·6	381·5
83·3	14·6	382·0	376·0
85·4	9 75	460·0	463·0
85·6	8·55	466·5	469·5
85·6	7·5	477·5	481·0
85·5	6·0	519·5	522·5
84·7	0·2	603·0	601·0
84·8	0·2	609·0	607·6

III. 2·0 *Normal Solution* of Ferric Chloride.*

Pressure (mm. water)	Mean temperature	Time (seconds)	Corrected Time (P. 85 mm.)
85·8	84·6	133·25	134·7
85·5	76·8	147·25	148·0
85·9	70·3	162·25	164·0
85·7	69·7	161·0	162·4
84·9	64·8	181·0	181·0
84·3	61·5	200·0	198 3
85 0	54·5	213·0	213·0
84·4	46·0	258·5	256·7
85·6	44·5	250·75	252·6
84·3	36·2	317·5	314·7
85·6	33·6	325·0	327·4
85·8	27·45	377·0	380·6
85·0	19·2	488·0	488·0
85·0	18·4	479·0	479 0
85·8	18·2	484·25	489·1
85·7	14·2	568·5	573·5
86·4	12·3	625·0	635·7
86·0	11·6	633·5	637 5
86·0	11·05	647·5	651·5
85·0	8·7	704·0	704·0
86·2	8·4	710·0	718·4
85·0	7·8	742·0	742·0
85·0	6·4	742·0	742·0
85·1	4·4	795·0	794·0
84·9	0·2	900·0	902·0
84·6	0·2	900·0	896·0

On account of the deliquescent nature of ferric chloride the solutions were submitted to a chemical analysis (the permanganate method was used) to determine their concentrations. The above observations have been plotted and are shown in fig. 5. The ordinates are the times given in the last column of the tables—they are proportional to the viscosities—and the abscissae are the temperatures in degrees centigrade.

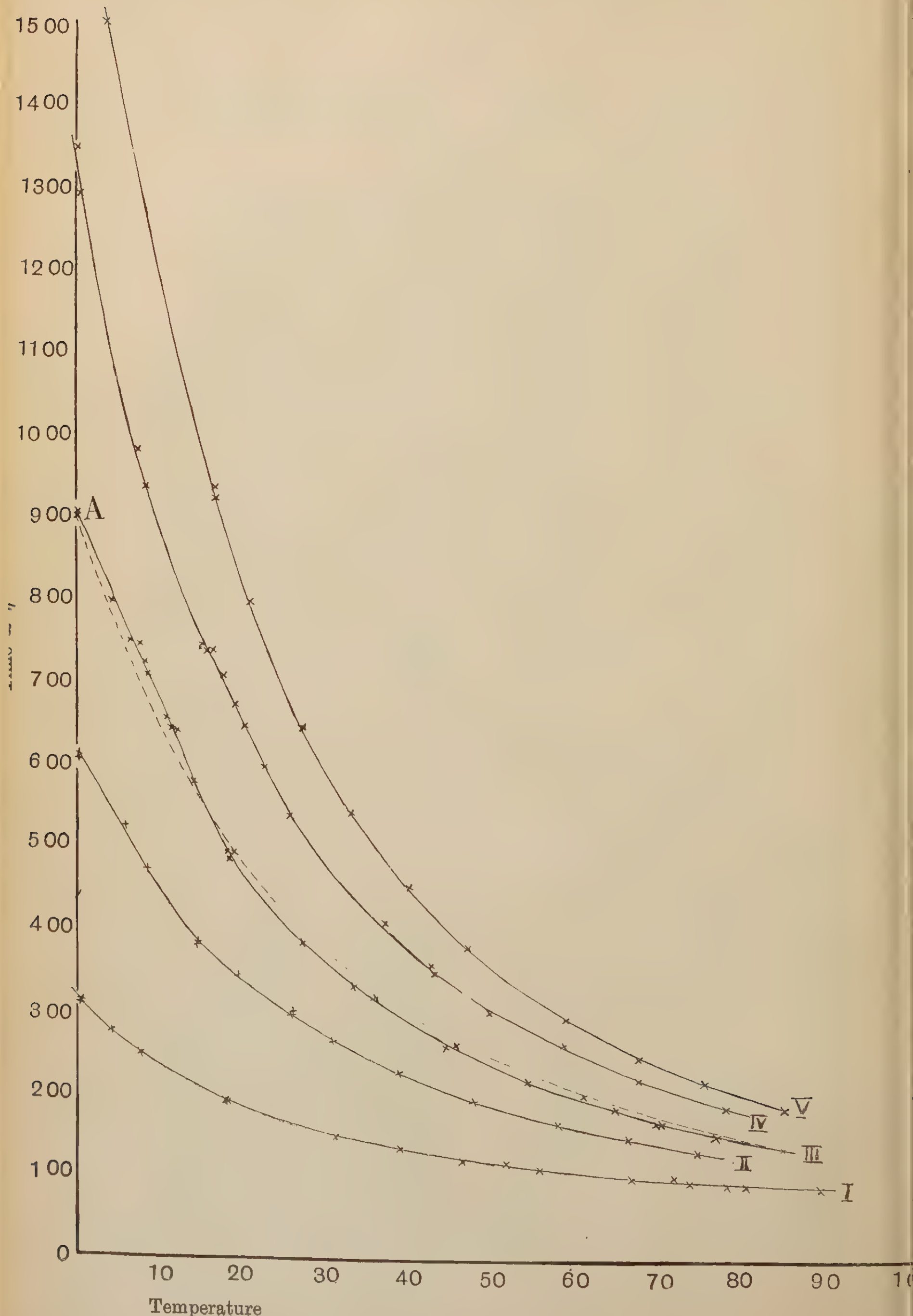
On an examination of these curves it will be observed that the curves III and IV show irregularities with respect to the curves I, II and V, which are very nearly hyperbolae. III and IV are parallel over a short range of temperature in the neighbourhood of 15° C. The dotted line shows the position of the hyperbola which passes through the end points of the curve—the position of the points *A*, and the direction of the curve at the highest

IV. 2·6 *Normal Solution of Ferric Chloride.*

Pressure (mm. water)	Mean temperature	Time (seconds)	Corrected Time (P. 85 mm.)
84·3	78·0	183·0	181·4
83·9	67·5	216·75	214·25
84·8	58·8	256·5	256·0
84·9	49·7	294·0	294·0
85·6	43·2	341·0	343·2
83·4	43·0	361·0	355·0
84·5	37·5	408·0	405·5
84·1	25·9	536·5	531·0
86·1	22·9	585·5	592·5
85·0	20·5	643·0	643·0
85·0	19·5	667·0	667·0
84·6	18·0	707·0	703·0
83·4	16·9	747·5	733·5
84·8	15·9	735·0	733·0
85·0	15·25	740·0	740·0
85·5	15·0	730·5	734·8
85·9	8·5	925·0	934·0
84·1	7·6	986·0	976·0
84·9	0·4	1290·0	1289·0
84·7	0·2	1350·0	1345·0

V. 3·22 *Normal Solution of Ferric Chloride.*

Pressure (mm. water)	Mean temperature	Time (seconds)	Corrected Time (P. 85 mm.)
86·2	84·9	179·5	181·9
86·1	75·5	209·0	211·7
86·1	67·8	239·5	242·5
86·2	59·1	284·0	288 0
86·1	47·4	372·5	377·3
86·0	40·4	441·0	446·0
86·3	33·4	527·5	535·3
86·1	27·4	631·0	639·0
85·0	21·2	792·0	792·0
85·4	17·0	924·0	930·0
86·0	17·0	906·5	916·5
85·0	3·75	1498·0	1498·0
86·5	0·45	1724·0	1739·0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90
Temperature
V
IV
III
II
I

Fig. 5.

temperature are accurately known. The deviation is admittedly small, but it is considerably greater than the error of observation. The deviation of the curve IV from a mean hyperbola is smaller than that of curve III, and although the points obtained for temperatures between 15° C. and 25° C. indicate an irregularity, yet on account of the absence of points in the neighbourhood of 12° C. it is not considered advisable to attach importance to it.

The nature of the irregularities mentioned above is brought out more clearly in fig. 6, where the ordinates represent the rate of variation of viscosity with the temperature and the abscissae are temperatures in degrees centigrade. In the following table columns 2, 3, 4, 5 and 6 give the abscissae of those points of the curves I, II, III, IV and V, where the slope has the value given in column 1.

$-\frac{\partial \eta}{\partial \vartheta}$	ϑ of curve I	ϑ of curve II	ϑ of curve III	ϑ of curve IV	ϑ of curve V
5·00	—	—	—	3·0	—
4·00	—	—	—	7·2	13·5
3·00	—	—	12·0	10·2	20·3
2·75	—	—	16·5	13·0	21·5
2·25	—	—	15·6	13·2	23·0
2·00	—	—	17·0	15·2 & 22·2	26·5
1·80	—	2·5	17·5	24·6	29·25
1·60	—	6·75	18·4	27·0	32·0
1·40	—	10·5	20·0	28·7	34·6
1·20	—	13·0	22·75	31·5	38·4
1·00	1·5	16·5	27·0	35·5	43·7
0·83	5·0	20·0	31·0	39·75	50·0
0·71	7·5	23·5	35·6	46 0	54·5
0 625	9·5	26·5	39·2	49·7	59·25
0·55	10·5	29·1	43·4	53·75	62·6
0·50	12·6	32·5	45·9	56·7	64·4
0·40	18·3	40·0	51·2	66·0	70·5
0·30	26·5	49·5	61·2	72·7	80·0
0·20	37·5	69·5	74·5	—	—
0·10	55·7	—	—	—	—

These values were measured a second time, and a good agreement was found in the two cases.

No importance is attached to the cutting of the curves III′ and IV′, since that depends upon the actual shape of the curve IV, fig. 5, in the neighbourhood of 15° C. where the number of points is not sufficient to enable an accurate value of the gradient to be

measured. The curves I′, II′ and V′, which correspond to I, II and V in fig. 5, show that the value of $-\frac{\partial\eta}{\partial\vartheta}$ steadily increases as ϑ decreases. This is not so with the curve III′. At the higher temperatures the value of $-\left.\frac{\partial\eta}{\partial\vartheta}\right|_{\mathrm{III}'}$ is low, and as a temperature

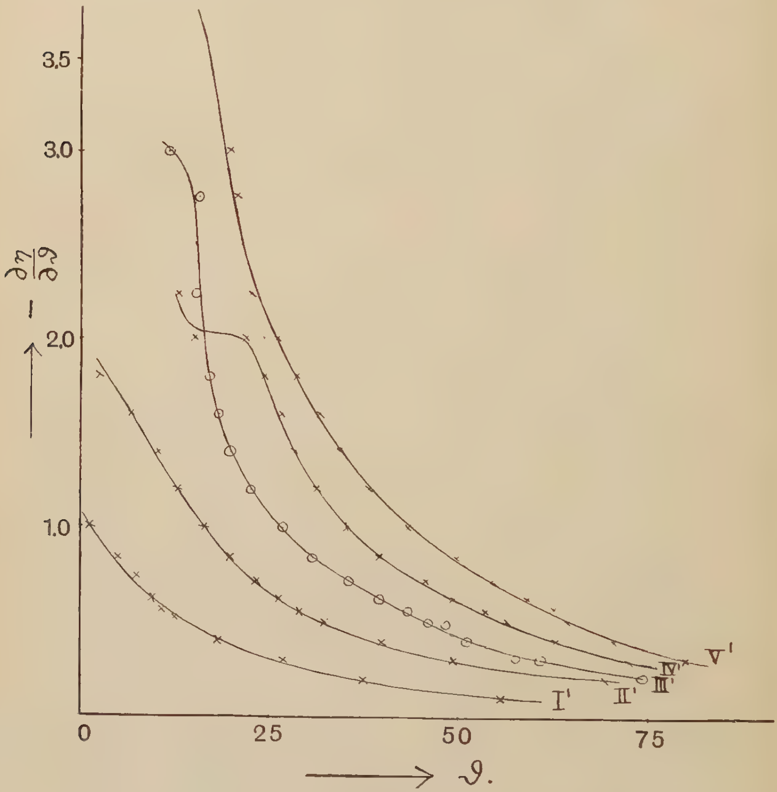

Fig. 6.

of 20° C. is approached it assumes a more normal value. Below 20° C. the value of $-\left.\frac{\partial\eta}{\partial\vartheta}\right|_{\mathrm{III}'}$ is abnormally high, and at 15° C. it has a value equal to that of the strongest solution at a slightly (4°) higher temperature. For temperatures lower than 10° C. the abnormally high value of $-\left.\frac{\partial\eta}{\partial\vartheta}\right|_{\mathrm{III}'}$ falls off.

The general shape of the curve III, fig. 5, is evidence for the existence of complex groups of particles in the solution at the

concentration 2·0 normal. At the low temperatures a dissociation of the complex particles, formed of a salt molecule or ion and its associated water molecules, takes place. At the higher temperatures the gradual recovery of the curve is probably due to the formation of a new type of complex—a solution of ferric oxychloride or ferric hydroxide in the excess of ferric chloride. All the four solutions had sufficiently high concentrations to prevent the formation of a precipitate at the higher temperatures.

It is not intended that this part of the investigation shall establish a quantitative relation between the complexity of molecular structure and the temperature. Indeed, it has been shown by Chéneveau* that the viscosity and refractivity methods used for the determination of molecular complexity in the liquid state do not give identical results, although each indicates the presence of groups composed of solute and solvent particles. The present viscosity measurements have been made with the object of verifying the prediction of the earlier part of the paper, that the high value of the constant B, in the representation of the magnetic susceptibility of a 2·23 normal solution of ferric chloride as a function of the temperature, may be connected with an abnormal variation of the viscosity of the solution. Since in a solution there are at least several types of particles, the complexity of some changing with the temperature, it seems impossible in the present state of our knowledge of the viscosity of liquids to derive from it quantitative information as to the nature of the relation between molecular complexity and temperature. For this purpose use has been made of the researches of van Laar who has considered the general theory of association in the solid and liquid states.

(5) *The Lowering of the Freezing Point of Solutions.*

There is another interesting point in connection with the value of the constant B for a 2·0 normal solution of ferric chloride. It has been shown that such a solution shows an abnormal value of the molecular lowering of the freezing point†. Since the latter quantity is directly connected with the constitution of the groups of solvent and solute particles, the molecular lowering being smaller the greater the complexity of the groups of particles, the abnormally low value of the molecular lowering produced by a 2·0 normal solution of ferric chloride indicates that the groups of particles are abnormally complex at this concentration. These

* *Comptes Rendus*, t. CLV., No. 2, 1912, p. 154.

† Jones and Getman, *Zeits. f. Phys. Chem.*, XLIX. 1904, pp. 426–433.

Note. The curve in fig. 7 of the research of Jones and Getman, relating to ferric chloride solutions, is drawn inaccurately. Prof. H. C. Jones has kindly informed me that the tabulated data are correct and an irregularity still exists for a concentration 2·0 normal, when the error of representation has been allowed for.

groups are unstable and rapidly break up as the temperature is increased. Hence ν_p is large, and it is reasonable to expect a high value of the constant B.

Another property of the constant B is that it never acquires a large negative value. Values of B have been obtained for solutions of the sulphates, chlorides and nitrates of all the ferromagnetic elements and the largest negative value found is $0{\cdot}86 \times 10^{-6}$ for a 0·79 normal solution of ferric chloride. This value is only 12 °/ₒ greater than the value of the diamagnetic susceptibility of pure water. The positive values, on the other hand, have been shown to range from zero to $7{\cdot}5 \times 10^{-6}$ a quantity which is ten times as great as the numerical value of the susceptibility of water. The preponderance of large positive values of B is to be expected if the curve of fig. 1 represents the change of molecular complexity with change of temperature.

Conclusion.

In this paper and in an earlier investigation ("The Variation of Magnetic Susceptibility with Temperature," *Proc. Camb. Phil. Soc.* Vol. XVI. p. 486) an attempt has been made to represent the variation of magnetic susceptibility with the temperature, taking into consideration the formation of complex aggregations of molecules and the effect such aggregations have in modifying the magnetic properties. It is believed that if due account be taken of the characteristic properties of substances, the work of du Bois and Honda does not prove that the foundations of the Curie-Langevin theory are unsound.

Hitherto, the relation representing the variation of the susceptibility of solutions with the temperature has been an empirical one and of the simplest form—linear. Assuming the truth of the Curie-Langevin laws for groups of particles whose complexity does not vary with the temperature, the paramagnetism of any substance will follow a simple hyperbolic law $\chi = \frac{C}{\vartheta}$ and the diamagnetism will be constant as the temperature ϑ varies. If the groups vary in complexity as the temperature changes, these laws will be modified in a manner depending on the characteristic changes of constitution possessed by the substance.

For the general case of aqueous solutions, treated in the present paper, the rate of variation of the molecular complexity with the temperature has been shown to be approximately linear. The modified relations have been applied to represent the variation of the susceptibilities of solutions of ferric chloride, ferrous sulphate

and ferric nitrate. Taking into account the variation of diamagnetic susceptibility with temperature, it has been shown that the numerical representation of the results of Jaeger and Meyer is satisfactory, and, with the exception of two cases, specially referred to, the relation proposed represents the observations to within the limits of experimental error. A solution of ferric chloride, 2·23 normal, whose susceptibility expressed by the relation $\chi = \frac{A}{\vartheta} + B$, appeared to be due to an abnormal molecular constitution, was examined in order to see if its viscosity varied abnormally as the temperature was changed. Evidence of such an abnormal variation has been obtained. The abnormal molecular constitution of such a solution is further supported by determinations of the molecular lowering of the freezing point.

The general theory developed in this paper includes the special case of water. The variation of molecular association in water with change of temperature affords a ready means of accounting for the variation of its diamagnetic susceptibility as the temperature varies, but we are not justified in taking a linear relation between the susceptibility and the temperature since we have no proof that the molecules of water do not possess a paramagnetic property smaller than the diamagnetic susceptibility and obscured by it at ordinary temperatures. Unless we suppose that the association modifies the magnetic properties of the water molecules, it is difficult to see how the diamagnetic susceptibility of water decreases as the temperature increases.

Note added Jan. 20, 1913.

The work of Dr G. Piaggesi (*Il Nuovo Cimento*, Ser. v, T. iv, 1902, p. 247) has, until now, escaped my notice. Piaggesi shows that for a given concentration of an aqueous solution of ferric chloride, ferrous sulphate, ferric nitrate, manganese chloride, manganese sulphate, manganese nitrate, cobalt chloride and nickel chloride, the product of susceptibility and absolute temperature is nearly constant.

Some Experiments on the Electrical Discharge in Helium and Neon. By HERBERT EDMESTON WATSON, B.Sc. (Lond.), 1851 Exhibition Scholar, Trinity College, Cambridge. (Communicated by Professor Sir J. J. Thomson.)

[*Read* 11 November 1912.]

THE investigation which is about to be described had as its original object the determination of the cathode fall and the spark potential (or minimum potential necessary to produce a spark between two electrodes) in all the inactive gases. Unfortunately, before it could be completed, the author was obliged to leave England, but as several results of interest had already been obtained, it seemed advisable to give a brief account of them.

The spark potential in helium has already been carefully investigated by the Hon. R. J. Strutt (*Phil. Trans.* A. 1900, 193, p. 379). His experiments differ, however, from those about to be described, in that, in the former, the electrodes were brass plates 0·755 mm. apart, a Wimshurst machine was used for producing the required potential difference, mercury vapour was allowed access to the sparking tube, and no allowance seems to have been made for the lag which is a very pronounced feature of the phenomenon under consideration. His results will be referred to later.

In 1904, Ritter (*Ann. d. Physik*, IV. 1904, 14, p. 118) examined the discharge in helium between a steel plate and sphere for pressures between 100 mm. and one atmosphere. The gas admittedly contained argon, and probably, judging by the results, other impurities as well, although it is difficult to compare experiments carried out under widely differing circumstances. In any case, the investigation has no bearing on the present work.

Bouty (*Ann. Chim. Phys.* 1911, 23, p. 5) has made numerous experiments on the *cohésion diélectrique* or rate of change of the spark potential with pressure, for helium and neon, and has recognised the need for the extreme purity of the gases under examination. His results are of great interest and will be mentioned later.

Apparatus.

The annexed diagram shows the general arrangement of the apparatus used. The current was obtained from a battery B of small cells giving a maximum E.M.F. of 1000 volts, 20 cells B' connected with the others in series were also connected with the

terminals of a 2000 ohm resistance C, contact with which could be made at 100 different points by a sliding arm D. The current was regulated by an adjustable water resistance A.

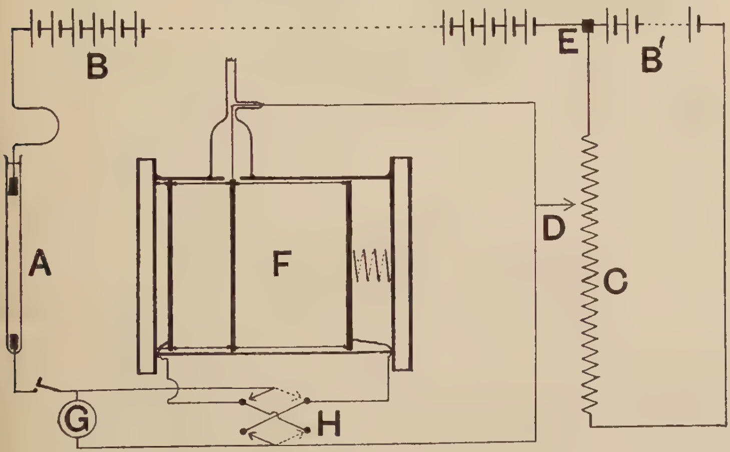

Fig. 1.

The discharge tube F was made from a piece of wide glass tubing 5·5 cm. in diameter, ground flat at the ends, and closed with sealing-wax. Needless to say it was perfectly air-tight. It contained three plane aluminium electrodes tightly fitting the tube, and held apart at the edges by frames of very thin glass rod constructed so that the plates were exactly 1·5 and 3 cm. away from each other. They were held in position by a spring at one end. An aperture for the admission of gas and the leads to the centre electrode, which were soldered to its edge, was drilled opposite the latter, and covered with a dome of glass ground to fit the tube.

The electrodes were very carefully cleaned with glass paper, and the middle one used as cathode. The two others could be made the anode in turn by means of the tipping key H, the one not in use being simultaneously connected to the cathode, which was itself at a potential only a few volts removed from that of the earth, E.

An accurate Weston voltmeter G was connected between the electrodes, and since its resistance was comparable with that of the liquid resistance A, the potential difference across the tube could be varied by altering A. The movement of D afforded a fine adjustment. A telephone was included in the circuit to enable any intermittency in the discharge to be detected.

In making an experiment upon the spark potential, the voltage was increased slowly until the discharge passed through the tube, and the potential difference noted. This was found to be independent of the current passing when once established, a fact which was determined by using different voltages on the main battery B, and consequently different values of A; the fact that the voltmeter was in parallel with the tube was also found to be without influence, for the spark potential was unchanged if the voltmeter was removed and the number of cells in the battery reduced until there were only sufficient to give the required voltage.

As noticed by other observers, there was a distinct lag in the lighting up of the tube after the potential difference was applied. This was most marked in the case of neon, and was frequently a minute in duration. In the present experiments, which were carried out in an indifferently lighted room, no external stimulus was given to start the discharge, but the potential difference was applied for three minutes, and if no discharge occurred in this time, it was considered that the sparking potential was not attained. This method gave very consistent results, the difference between successive values at the same pressure being nearly always less than one volt, and hardly noticeable on the voltmeter. It could be detected, however, by the variation in the position of the sliding contact D. Three observations were made at each pressure for each side of the cathode. It was rather remarkable that after a change of pressure, the first value of the spark potential was always two or three volts below those obtained subsequently. The figures given below refer to the steady values, as the first discharge potential was uncertain.

The gases were those used in a previous investigation on electrical discharge (*Roy. Soc. Proc.* A. 1912, 86, p. 168), and were very carefully purified before every experiment. They were first mixed with oxygen, and phosphorus ignited in the mixture to remove hydrogen, a very convenient method when dealing with small quantities of gas. The residue was then transferred to a tube containing charcoal immersed in liquid air, and allowed to stand for at least half an hour, the gas in the connecting tubes being pumped away. After this treatment the spectrum is perfectly clear from foreign lines, except those of mercury, at 100 mm. pressure. The gas was admitted directly from the charcoal tube to the discharge tube, passing however through a U tube immersed in liquid air on the way in order to remove mercury vapour. There is little doubt that mercury vapour exerts a considerable influence on the spark potential, especially when aluminium electrodes are used, and for this reason the discharge tube was shut off from the rest of the apparatus, and

no gas of any kind ever admitted except after freezing in liquid air.

Before starting the experiments some helium was admitted to the tube and a current run for some weeks, the gas being frequently renewed until no change in its spectrum was observed.

As observed by Strutt (*loc. cit.*), the minimum spark potential is a very delicate criterion of the purity of the gas. It was also found in the present experiments that the minimum voltage necessary to maintain the current when once started (a quantity which will be discussed later) was likewise very sensitive to the presence of impurities, and as this was constant over a wide range of pressure it afforded a more convenient method of detecting their presence.

Some of the results obtained are given below, p is the pressure in mm. of mercury (measured by a McLeod gauge below 9 mm.), V the spark potential between the electrodes 1·5 cm. apart, and V' between those 3 cm. apart.

In the case of helium the figures are for two very pure samples of gas. After the first series of readings at comparatively high pressures had been taken, the tube was pumped out and more helium admitted directly from the charcoal, values for this gas being marked with an asterisk. It will be observed that the first of these for the nearer pair of electrodes is 180. As a matter of fact, immediately the gas had been introduced into the tube, one reading of 170 was obtained, and 180 between the other pair of electrodes, 180 and 214 being recorded a minute or two later. It is difficult to account for this, as the value 184 was found in a very similar experiment, when the gas was probably just as pure. There may possibly have been some ionising agent in the vicinity at the time. Such a low value was never obtained again, and it seems probable that the correct figure is not far removed from 184.

The case of neon is of special interest as at comparatively high pressures it allows an electrical discharge to pass far more readily than any other gas at the same pressure. This was observed in 1909 by Dr Collie and the author, and some rough measurements were made but not published. Soon afterwards M. Bouty (*loc. cit.*) determined the *cohésion diélectrique* which is a measure of the readiness with which a spark will pass, and found it to be at least 6·1, and probably 5·6, the figures for helium and air being 18·3 and 419, respectively. Hence it seemed not unlikely that the minimum spark potential would also be less than that of helium, and it was rather surprising to find that this was not the case, as may be seen from the figures in the table. Unfortunately the experiments are not quite complete, especially at high pressures, but the minimum spark potential has been examined carefully, and certainly seems to be higher than that of helium.

The first set of figures, which is not reproduced in full for the lower pressures, was obtained from an experiment with a sample of gas which was originally very pure, but which had been pumped into a test tube and allowed to stand a few days before use. Even

Helium			Neon		
p	V	V'	p	V	V'
56·3	387	616	32·8	267	372
48·7	375	568	27·7	251	350
38·8	343	505	24·0	243	327
31·7	322	455	17·9	228	299
25·2	304	414	13·2	219	273
20·6	293	385	99·3	215	261
14·4	271	351	—	—	—
11·8	264	335	2·81	204 †	241
9·1	246	324	1·28	216	231 †
6·75	227	309	10·1 §	200	240
5·34	212	295	7·98	200	240
4·44	205	276	5·76	203	240
3·18	188	245	4·42	200	240
2·45	186	225	2·65	200	232
2·43*	180	214	2·19	200	230
1 87	186	213	1·60	204	224
1·86*	190	206	1·23	218	224
1·31*	216	201	0·96	240	229
1·15	221	198	1·87 §	211	228
1·09*	249	210	1·43	215	229
0·79*	390	230	1·01	242	235
0·69*	>1000	258	0·77	285	232
0·495*		309	0·59	358	231
0·475*		332	0·45	450	246
0·44*		360	0·345	?725	290
0·355*		473	0·27	?790	360
0·262*		750	0·21	>1000	435
			0·16		650
			0·125		830

† Minimum value. § Fresh gas.

though all the apparatus and the test tube had been previously well rinsed with the gas, it is likely that some impurity was introduced, as the minimum values for V were 204 and 231 respectively, those obtained later with purer gas being 200 and 224. By analogy with helium, however, it is probable that the

figures at higher pressures for the slightly impure gas are not very far wrong. In the case of the second sample, the constancy of the spark potential over a wide range of pressure was surprising, although it was probably illusory, and caused by the gradual contamination of the gas. It is rather remarkable, however, that another sample of exactly the same gas admitted immediately afterwards directly from the charcoal at what was assumed to be approximately the critical pressure for minimum spark potential, gave minimum values of 206 and 228.

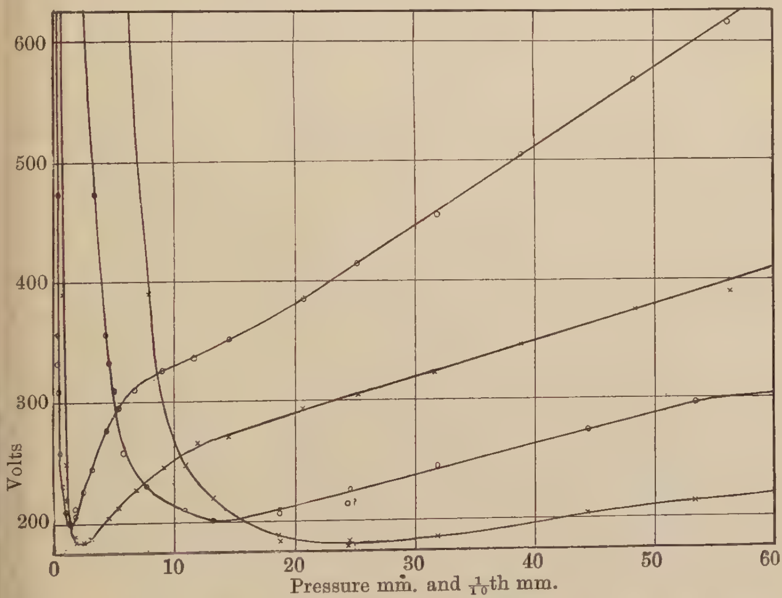

Fig. 2. The above curves show the relation between spark potential and pressure in helium for plane electrodes 15 mm. and 30 mm. apart, the upper curve being for the latter. The curves on the right correspond to the others, but the horizontal scale is ten times as great to show the shape at low pressures.

Fig. 2 shows two typical curves for helium. It may be mentioned here that the electrodes in the present experiments were placed at a considerable distance, in order that the phenomenon under consideration might be studied through a wide range without unduly increasing the pressure. Paschen's law states that the spark potential depends only upon the mass of gas between unit area of the electrodes, and consequently, if the electrodes are 15 mm. apart, it will be the same as it would be in a tube with electrodes 1 mm. apart and containing gas at 15 times the pressure. When using a tube of small dimensions the volume of the dead space becomes relatively large, and the total quantity

of gas required is more. This is always to be avoided, as the purification of large quantities of gas is a difficult and uncertain operation. The whole of the present experiments were carried out with less than 15 c.c. of each gas.

There is, of course, the objection that the field obtained in this way is not uniform, and indeed, there is about 20 volts difference between the minima obtained with the two pairs of electrodes. At the same time, as will be seen from the figure, the form of curve is not altered, and indeed, if the horizontal scale of the upper curve be doubled (the distance of the electrodes for which this was obtained was double that of the other pair) and the whole moved downwards, the two nearly coincide. It was intended to make more experiments with the electrodes quite close, but up to the present this has not been possible. It seems unlikely, however, that the true spark potential for a uniform field between infinite planes is more than two or three volts lower than the minima already obtained.

On examining the curves, it will at once be seen that they are of a type not previously observed. Below the critical pressure of minimum spark potential they are approximately hyperbolic, and then for a short distance linear, as is usually the case, but at a slightly higher pressure there is a break, and the curves turn over, and again become very nearly linear. The same thing occurs in the case of neon. With the closer pair of electrodes, this break begins at a pressure of 10 mm. This would correspond with a pressure of 200 mm. in Strutt's experiments, and is beyond the range studied by him. The bend occurred at the same point for all samples of gas even when distinctly impure, and so is not likely to be caused by impurities. It may however be due to the irregularity of the field, but this in turn is improbable, owing to the similar shape of the two curves.

At high pressures there was little difference in the spark potential when a small trace of impurity was present, but the most minute amount affected to a large degree the depth of the dip in the curve and consequently the minimum spark potential.

The pressures of minimum spark potential were, as nearly as could be judged, 2·4 and 2·8 mm. for the closer pair of electrodes in helium and neon respectively. The latter figure is probably low, and consequently these pressures are by no means proportional to the mean free paths of the molecules of the gases (cf. Sir J. J. Thomson, *Conduction of Electricity through Gases*, 2nd edition, p. 450) which are in the ratio 1 : 0·704 (Rankine, *Roy. Soc. Proc.* A. 1910, 83, p. 524).

Since the electrodes in the present case were 20 times the distance of those used by Strutt, it might be expected by Paschen's law, that the slope of his curves above the critical pressure would

be about 1/20th of that of the lower ones in fig. 2. As far as can be judged from his diagram, 20 times the steepness of his flattest curve is very slightly steeper than the first and corresponding portion of my curve, and in fact, the agreement is as good as can be expected. One remarkable effect observed by Strutt, namely that the spark potential was lowered by vigorous sparking, or by standing, was not noticed at all in the present experiments, and in fact the reverse was the case. It has already been mentioned that the first discharge passed more easily than the subsequent ones, and if the gas was left standing for a few hours the spark potential invariably rose slightly. It is quite possible that the reverse effect may be produced by mercury vapour, which in my experiments was completely excluded.

With regard to Bouty's experiments, it is difficult to compare the slopes of the curves obtained, as all the conditions in the two sets of experiments differ so widely. Considering the closer pair of electrodes in my experiments, the rate of change of spark potential with pressure at high pressures where the curve has assumed its final definite linear form is 30 volts per cm. of mercury in the case of helium, and 22 in the case of neon. These figures should be proportional to the *cohésions diélectriques* of Bouty which are 18·3 for helium and probably less than 6·0 for neon. It will be seen that this is not the case, but on the other hand, for pressures not so far removed from the critical pressure, the gradient for helium is about 90 volts per cm. while, as far as can be judged, that for neon is not much greater than at higher pressures. The ratio for this part of the curve would therefore be nearer to the value found by Bouty.

The Cathode Fall.

It has recently been shown by Aston (*Roy. Soc. Proc.* A. 1911, 84, p. 526), that the supposed anode fall of potential measured by many observers is probably an entirely illusory phenomenon induced by the methods of measurement, and he concludes that, provided there is no positive column, the cathode fall, or difference in potential between the cathode and the negative glow, is practically equal to the total voltage across the tube provided that the current is not large enough for the whole surface of the cathode to be covered with the glow. The potential fall in the negative glow itself is negligible. In the light of these results, the hypothesis put forward by Strutt as a theoretical reason for the cathode fall being equal to the minimum spark potential, does not seem very plausible, and is in no way consistent with my experimental results. Strutt considers that the rise in spark

potential at pressures above the critical pressure is due to the fact that a positive column of gradually increasing length has to be maintained, while below this pressure, the negative glow is crushed. Except at very high pressures, far above the critical pressure, no positive column appeared in any of my experiments, and it has been shown that moving the anode up and down in the negative glow has no appreciable effect upon the voltage unless the dark space is reached. Moreover, the present experiments tend to show that the cathode fall is not equal to the minimum spark potential at least for helium and neon, for the voltage across the tube invariably fell when the current started.

Assuming the above mentioned results, it follows that the cathode fall is equal to the minimum voltage at which the current will pass when once started, and can be easily measured by increasing the resistance in the circuit until the discharge ceases, and measuring the voltage. This is very constant over a wide range of pressure, but varies greatly as already mentioned with the purity of the gas, and also with the material of the cathode. In what follows, whenever cathode fall is mentioned, it is to be understood that it is measured by the minimum running voltage.

In the case of the tube already mentioned, with aluminium cathode, the value of this quantity was 164 volts for helium, and 170 for neon, figures which are 20 and 30 volts respectively lower than the minimum spark potentials. It is perhaps worthy of note that they correspond approximately to the calculated fall in potential in the Aston dark space for these gases, namely, 20 and 40 volts (*Roy. Soc. Proc.* A. 1907, 80, p. 45; 1911, 84, p. 535; 1912, 86, p. 172). A short consideration of the theory put forward in these papers to explain the existence of this dark space, will show that, if in a gas with no Aston dark space the cathode fall is equal to the minimum spark potential, then in the gases which do possess it, the cathode fall should be less than the minimum spark potential by the amount of the voltage drop in this region.

Experiments with other Electrodes.

A number of other experimental tubes were constructed, some of which were as follows.

1. A short tube about 4 cm. in diameter with a plane circular aluminium electrode touching the sides, and about 4 cm. from the rounded end which contained sodium potassium alloy introduced after complete evacuation of the tube. Some of the alloy was shaken on to the aluminium to which it adhered. A series of spark potentials for neon from a pressure of 11·3 mm. downwards was taken with this electrode as cathode, and a curve exactly

similar to those in fig. 2 obtained. The bend occurred at about 6 mm. pressure, and the minimum spark potential was 150 volts at 1·5 mm. pressure. The *cathode fall was* 85 *volts.* Mey found 78·5 for helium (*Verh. deut. physikal. Ges.* 1903, 5, p. 72).

2. A similar tube with an aluminium disk upon which were a few drops of sodium potassium alloy as one electrode, and a clean aluminium wire normal to it and 1·5 cm. away as the other. The first neon introduced was not perfectly pure and the surface of the alloy tarnished. When in this condition at 20 mm. pressure a 300 volt alternating current was completely rectified. After shaking to clean the surface of the alloy, and refilling with pure neon at 3·5 mm. the spark potential with plane as cathode was 145 and the cathode fall 85. When the tube was filled for the first time, the minimum spark potential with the wire as cathode was 155 and the cathode fall 130, on the second filling the latter had risen to 142. These figures are much lower than those obtained in the original spark potential tube, and it seems quite possible that they are due to the freshness of the electrode which had never been used before and was probably coated with a film of oxide, or was evolving hydrogen. The gas in the present case would also, if possible, be purer.

3. A tube 22 cm. long and 2·5 cm. wide, with an aluminium strip running the whole length as cathode. A sodium rivet was fixed in this, but by the time the tube was filled it was slightly tarnished. The anode was a wire in the form of an inverted U passing along each side of the strip. This tube was filled with neon at varying pressures, and showed a minimum spark potential of 209 at 5 mm. It was then refilled at this pressure and used for experiments which will be described later.

4. A tube 14 cm. long, and 2 cm. wide with two copper sheets in the same plane along its axis and their ends 3 mm. apart as electrodes. Minimum spark potential 270 volts at 6·5 mm. pressure of neon, and cathode fall 221 volts.

5. A tube the same size as the above with two square carbon rods 5 mm. apart at the ends as electrodes. Constant results could not be obtained even after frequent refilling and passing heavy currents. The lowest values obtained for the spark potential and cathode fall were 360 and 217 volts respectively.

6. A tube of the same size with an aluminium plate covered with sheets of very slightly tarnished calcium as cathode. U shaped anode. Minimum spark potential 200, cathode fall 150 volts.

7. Similar tube with spirals of magnesium wire as electrodes. Results very inconsistent. A value of 229 for minimum spark potential was obtained, and one of 150 for cathode fall.

Abnormal forms of Discharge.

Soon after starting the experiments on spark potential, a rather remarkable occurrence was observed. It has been stated that to determine the discharge point, the potential was gradually raised. On several occasions it was noticed that before the discharge took place, a glow appeared over the surface of the anode and increased in brilliancy as the potential was raised. Suddenly the discharge would assume its normal form with Aston dark space, Crookes dark space and negative glow. This *anode glow* was about 5 mm. thick, and although of quite a different order of brightness to the normal discharge, was distinctly visible in a well lighted room. It was more conspicuous in neon than in helium, as would be expected from the extreme readiness with which the former gas glows under the least electrical excitation. In helium it could not be determined whether the glow appeared suddenly or gradually, but in neon it seemed to be produced at a definite potential. For pressures above the minimum spark potential pressure this occurred at voltages often below the minimum spark potential itself, and always below the normal spark potential. For instance, in one case with helium at 4 mm. pressure, the glow appeared at a potential difference of 185 volts, while the true spark potential was 205, while in another with neon at 33 mm. the glow started at 165 volts and the spark potential was 267. Below the critical pressure, the voltage at which the glow formed seemed identical with the spark potential at that pressure.

It must be emphasised that this glow discharge is not a part of the normal discharge occurring at a voltage insufficient to produce the latter, such as takes place from a Wehnelt cathode or in the discharge from points (*Cond. of Electricity through Gases*, 2nd edn. pp. 479, 513), but an entirely different and alternative form of discharge. It was quite impossible in any experiment to determine *a priori* which form would make its appearance, but at low pressures the glow seemed to be the more stable form.

If the glow appeared, the potential could be raised far above the normal spark potential without any change occurring, but if the rise was sufficient, then the ordinary discharge would appear, although the potential at which this happened was by no means definite. Thus, in a sample of neon, at 10 mm. pressure, the glow formed at 199 volts, and the normal discharge appeared at voltages varying from 272 to 330. The spark potential was 200.

Owing to these circumstances the measurement of the spark potential was a matter of some difficulty. It was found, however, that if the normal discharge was once obtained, it tended to

reappear when the potential was reduced below the spark potential and then raised; consequently, in practice, the potential was raised slightly above the spark potential, causing the glow to appear, the voltmeter was then momentarily disconnected, a proceeding which largely increased the potential across the tube (as may be seen from fig. 1) and started the normal discharge. The current was at once cut off and the potential reduced to just below the spark potential. On again switching on the current and raising the potential, the tube sometimes lit with the normal discharge, but very often the glow reappeared and the process had to be repeated.

Unfortunately it has been so far impossible to make any measurements of the currents passing through the tube under these varied conditions, and it is not easy to ascribe any reason for the glow discharge. It has however been remarked by the author in conjunction with F. W. Aston (*Roy. Soc. Proc.* A. 1912, 86, p. 176), that helium appears to conduct the discharge in two different ways, and the occurrences under consideration appear to be a manifestation of the same phenomenon. It seems that an explanation might be afforded by the rather crude assumption that if the gas be considered analogous to a metallic conductor, it can have two (or perhaps more) different resistances. In this case the normal discharge would be the one corresponding to the low resistance, and the glow discharge to the high resistance, for the latter is exactly similar to the normal discharge when the external resistance is largely increased. The glow resembles the negative glow and appears to be on the anode because of the great size of the dark space owing to the low current density; moreover four or five Aston dark spaces are often seen at high pressures exactly as in the normal discharge. It is noteworthy, however, that the glow discharge could not be produced from the normal one by decreasing the current, although at low pressures the change often occurred spontaneously when no alteration was made in the external conditions.

On a few other occasions, yet another type of discharge appeared. If the potential was very slowly raised when the glow discharge was taking place, the glow, which was approximately a plane disk over the anode, became unstable, and bulging out at the centre, moved bodily over to the cathode where it assumed the form of a paraboloid having its vertex towards the cathode, and separated from it by a small dark space. No change in intensity occurred during this process until the potential was raised sufficiently to produce the normal discharge. I can offer no explanation for this.

An interesting type of discharge could also be produced as follows. When the current was passing through the gas, the

voltmeter being disconnected, the pressure was reduced until the dark space just reached the anode. If then the pressure was very slightly further reduced or the current density lowered by increasing the external resistance, the current ceased to pass continuously, and passed in flashes. By careful adjustment these could be made to succeed each other with great rapidity, or at regular intervals as far apart as quarter of a minute. This discharge was previously noted in the experiments carried out in conjunction with F. W. Aston (*loc. cit.*), and affords a beautiful illustration of the well known although rarely concisely stated fact, that if the anode be brought inside the Crookes dark space, no current will pass unless an enormously increased voltage is applied. In the present case matters are so adjusted that if the current were to pass continuously, it would be so weak that the dark space would be longer than the distance between the electrodes. Consequently this cannot occur. Increasing the current density, however, shortens the dark space, and a large current can pass. To effect this, the energy seems to store itself up until sufficient is accumulated to pass over in a burst. As the condenser capacity of the vacuum tube must be quite small, it is difficult to tell how this storage takes place, and it is remarkable that the flashes can be obtained at such long intervals as those already mentioned. A further investigation of this point would most certainly prove of great interest, and might throw valuable light on the phenomenon of initial discharge in gases at low pressure.

Another form of discharge was observed which was probably similar to that from a Wehnelt cathode. Its characteristic was that it occurred from a single point on the cathode, and it was induced by traces of impurity on the surface. It only appeared once in the carefully cleaned tube used for spark potentials, and was exceedingly persistent in tube no. 7, with magnesium electrodes. Its appearance in the case of neon was extraordinarily beautiful, and resembled nothing so much as the sun just setting in a perfectly clear atmosphere. Its brilliance was dazzling as may well be imagined from the fact that currents of the order of 50 milliamperes frequently passed from this one point.

The voltage when this discharge was passing was very low, some figures being: with helium in spark potential tube 154 volts, neon in tube no. 3, 17·5 mm. pressure, aluminium wire cathode, 109 volts; neon in tube no. 7, magnesium electrodes, 75 volts at 18 mm. pressure, and 110 at 3 mm.; neon in tube no. 2 aluminium wire cathode 1 mm. pressure, 85 volts. These values did not vary greatly with the current and seemed to bear no relation to one another. As with the other abnormal forms of discharge, it was impossible to tell when this form would make its appearance.

Fatigue of the Electrodes.

Some experiments were carried out to determine the effect on the electrodes of prolonged passage of the current. Tube no. 3 with a large sodium and aluminium electrode was originally made for this purpose. No convenient source of direct current being available it was connected to the 200 volt alternating mains with one carbon lamp in series. The current passing was $\frac{1}{16}$th amp. and was sufficient to cover only about half the cathode with glow. This glow, however, gradually spread, and at the end of 44 hours covered the whole electrode. The tube was about as hot as an ordinary carbon filament lamp. After about 10 hours further running, most of the discharge took place from the sodium, although there was still a certain amount from the aluminium. The experiment was continued, but no further changes occurred. The initial voltage across the tube was 143, corresponding to a maximum of 200, and this rose to 154, the glow also diminishing greatly in intensity.

From this it may readily be seen that after some time, current passed less easily from a given portion of the electrode, and moved towards an unused portion. Finally when the whole of the aluminium was "exhausted" the discharge started from the sodium which was slightly oxidised and evidently less favourable to its passage than fresh aluminium.

A peculiar effect was observed with this tube. When the current was cut off with a one-pole switch which disconnected the large sheet electrode, the gas round the wire electrode continued to glow. Presumably the leakage across the switch or its capacity allowed sufficient current to pass to produce luminosity. This is another striking demonstration of the minute currents which suffice to show their presence in neon, and a tube of this nature might well be used in testing for leaks.

Fatigue was especially noticeable in the tube no. 5 with carbon electrodes. For instance, after running about 30 milliamperes through one electrode at 2·1 mm. pressure, the spark potential rose from 400 to 530, and the running potential from 255 to 300. The tube then completely rectified a 300 volt alternating current. In this case a large amount of gas was evolved, and this appears to facilitate the passage of the current. If this is the case, a new aluminium electrode which evolves much hydrogen might be expected to give rise to a lower cathode fall than an old one as was observed to be the case in one experiment.

Experiments were made later on a small tube with two sheet aluminium electrodes in the same plane, and filled with neon at 5 mm. pressure. This was connected to a 240 volt main through

a lamp. After five or six hours the glow became far paler, and it was found that if the current were switched off and then on again, the lamp would not light up, although it would do so if the current were reversed. This seems to be an effect similar to that in connexion with the initial spark potential. It occurred in the case of both electrodes, and can hardly be ascribed to true fatigue, since, if the current passed only for a small fraction of a second, the tube would not again light up. Reversal of the current, and possibly alteration of pressure, seems to produce some molecular rearrangement of the surface of the cathode.

This tube also showed signs of true fatigue, and after about 60 hours would not light at all with the potential difference in question.

It will thus be seen that with helium and neon there are considerable fatigue effects similar to the well known phenomenon of photo-electric fatigue, and probably arising from the same cause.

Intensity of the Illumination.

Anyone who has seen a discharge through pure neon will be aware of the intense brilliancy which characterises it. The brightness of the negative glow on a flat electrode at pressures in the neighbourhood of 10 mm. is remarkable, and it was thought that the efficiency of this light might be very high, and moreover, as it appeared likely before starting the experiments that the potential difference necessary to start and maintain a current would be very low, it seemed not improbable that it would be possible to make an efficient lamp which would run from an ordinary lighting circuit. Nearly all the experiments so far described were carried out with this end in view, and this accounts for any apparent lack of connection between them, and in many cases their incompleteness.

A number of very rough measurements of intensity were made with a grease spot photometer, the voltage and current being measured simultaneously, but it is not proposed to give these in detail. One very interesting fact was that the intensity of the light when measured was found to be far less than when estimated. Some tubes which were dazzlingly bright were of considerably less than one candle power, and the estimated efficiency was correspondingly reduced.

There appeared in all cases to be a maximum efficiency for medium currents, which were of the order of 30 milliamperes for the small tubes already described. The pressure exerted a great influence, for example, with copper electrodes, at 3 mm. pressure

the efficiency was about 90 watts per candle, while at 12·7 mm. it was 20. A similar tube with aluminium electrodes at the former pressure had an efficiency of about 40 w.p.c. More satisfactory results were obtained with tube no. 2 with a sodium potassium electrode. Here the efficiency was increased to 4 w.p.c. for pressures between 10 and 16 mm., the voltage across the tube being 90, but this of course is far above the limit required for practical illumination. Experiments were started with a number of other metals as cathodes, but were unavoidably broken off.

The above figures were only obtained with gas of the utmost purity free from mercury vapour, and after a short time, in spite of all precautions, it invariably became contaminated, and the brightness diminished. This was less noticeable when the cathode was of sodium potassium alloy owing to the absorption of the impurity, but this liquid is most inconvenient as it tends to soil the glass, and it is almost impossible to coat a metallic surface evenly with it.

While these experiments were in progress, I was informed privately that similar work was being carried on by M. Claude in Paris. Some of his results have now been published (*C. R.* 1910, 151, p. 1122; 1911, 152, p. 1377), and are much more satisfactory from a practical point of view, an efficiency of 0·64 watts per candle having been attained. In his experiments no attempt was made to keep the voltage low, 1000 volts being used, and the light appears to be that from the positive column, although this is not precisely stated. In my experiments the light was derived from the negative glow, and the amount of energy lost in the dark space is many times the amount which produces luminosity. On the other hand, when high voltages are utilised, the dark space loss becomes negligible.

Physiological Effects.

The light given out by helium and neon under the influence of the electric discharge is of a peculiar nature. It has already been mentioned that the apparent brightness is far in excess of the actual intensity, but what is more remarkable is the acute physiological effect of such a feeble illuminant. In the foregoing experiments it was found that if the light from one of these tubes emitting say $\frac{1}{10}$th of a candle power, was allowed to fall on the unprotected eyes for about a quarter of a minute, a violent headache was the result, followed by temporary blindness, while on one occasion, looking at a tube of one candle power for one or two seconds produced pain in the eyes, and a total inability to read for two days, and these effects were not confined to one

individual. In consequence, during the whole of the experiments it was necessary to wear very dark brown glasses.

The same effects have been noticed with argon, krypton and xenon although the light emitted was of the faintest kind. The most obvious suggestion is that ultra-violet light is the cause, but an examination of the spectra of these gases shows that the intensity of the ultra-violet spectrum as measured photographically is certainly less in all cases than that of the visible spectrum, and indeed, in krypton, there are practically no bright ultra-violet lines.

A property common to these gases is that their spectra all consist of a comparatively small number of intensely bright lines, and it would be interesting to know if these could possibly affect the eye to the same extent as a continuous spectrum emitting for instance the same amount of energy per $\frac{1}{10}$th A.U. as is given out by one line of the gas spectrum. The total energy of the continuous spectrum would naturally be far greater, but for certain colour senses the two would be equal, and an equal strain on some portion of the eye might result.

In conclusion, it may be mentioned that of the several points which have been dealt with in this paper, there is not one, our knowledge of which is in the least degree complete, and a further investigation would certainly in every case be amply repaid.

I should like to take this opportunity of conveying my very best thanks to Sir J. J. Thomson in whose laboratory these experiments were carried out, for the great interest he has at all times shown in them.

Summary.

1. Measurements have been made of the spark potentials at different pressures, between plane aluminium electrodes in very pure helium and neon, no external exciting agent being used.

2. In the case of electrodes 15 mm. apart, the normal minimum spark potentials excluding a lower value obtained on the first passage of the current, were found to be 184 and 200 volts for helium and neon respectively, the corresponding pressures being 2·4 and 2·8 mm. respectively. For a perfectly uniform field the potential differences might possibly be two to three volts less.

3. At pressures higher than those given above, the curve connecting spark potential and pressure did not assume a linear form until the pressure rose to about 10 mm. After this point, the gradients were 30 volts per cm. pressure for helium, and 22 volts per cm. for neon (electrodes 15 mm. apart).

4. Special attention was paid to the lag, the purity of the gas, and the exclusion of mercury vapour.

5. The cathode fall with aluminium electrodes which have been well "run" is at most 164 volts for helium and 170 volts for neon. Consequently it does not appear to be equal to the minimum spark potential.

6. With a cathode of sodium potassium alloy in neon, the minimum spark potential is near to and not greater than 145 volts, and the cathode fall is 85 volts.

7. The cathode fall in neon with cathodes of copper, carbon, magnesium and calcium is approximately 221, 217, 150, and 150 volts respectively.

8. Four abnormal and alternative forms of discharge were observed.

(*a*) Corresponding to a state of very high resistance in the gas under examination.

(*b*) A variation of the above corresponding to a transition stage between it and the normal discharge.

(*c*) An intermittent discharge, the frequency of which could be controlled.

(*d*) Corresponding to the discharge from a Wehnelt cathode.

9. Experiments were made on the fatigue of the electrodes which was found to be considerable, and of two kinds. Firstly a true fatigue, and secondly a reluctance to allow the current to start for a second time when one discharge had already passed.

10. Measurements were made on the efficiency of the light from the negative glow. This was found to be less than was expected.

11. Peculiar physiological effects were observed, corresponding to arc blindness produced by light of very feeble intensity.

Some Diophantine Impossibilities. By H. C. POCKLINGTON, M.A., St John's College.

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1. THE object of this paper is mainly to discuss some equations obtained by equating a quadratic function of x^2 and y^2 to a square, either to show that they are impossible in integers or rational fractions (§ 3—§ 10), or to completely find their solutions in integers (§ 12). Two theorems on arithmetical progressions of which given terms are to be squares are given in §§ 6, 11, and the impossibility of $x^{2n} + y^{2n} = z^2$ is discussed in § 13.

2. We make use principally of three lemmas. The first is that if $xy = z^n$, and x is prime to y, then both x and y are nth powers. The proof is obvious on expressing z as a product of primes and their powers. For each prime that occurs in z must occur in x or y, but not in both, and clearly occurs raised to a power the index of which is some multiple of n. If $n = 2$ we have an alternative proof in Euclid IX. 2, and if n is a power of 2 we can get our result by a repeated application of Euclid's proposition. We have assumed that x and y are positive. If they can be negative there is the alternative $x = -u^n$, $y = -v^n$. By a repeated application we see that if $xyz = w^n$, then x, y and z are nth powers.

We shall also require the following application of the lemma. Let $x^2 + Ny^2 = z^2$, where either x or z is prime to Ny. Then x is prime to z, for if p is a prime common divisor of x and z it divides Ny^2 and hence Ny. Hence the greatest common divisor of $z + x$ and $z - x$ is 1 or 2. First suppose that it is 1, i.e. that Ny is odd. Then, writing the given equation $Ny^2 = (z + x)(z - x)$, we see that $z + x = lu$, $z - x = mv$, where $lm = N$ and lu is prime to mv. We now have $y^2 = uv$ whence $u = \xi^2$, $v = \eta^2$ and so

$$x = (l\xi^2 - m\eta^2)/2,\quad y = \xi\eta,\quad z = (l\xi^2 + m\eta^2)/2.$$

Next suppose that the greatest common divisor is 2, i.e. that Ny^2 is divisible by 4. Then if y is even we have

$$N(y/2)^2 = \tfrac{1}{2}(z + x) \cdot \tfrac{1}{2}(z - x),$$

where $(z + x)/2$ is prime to $(z - x)/2$. Treating this as before we find $x = l\xi^2 - m\eta^2$, $y = 2\xi\eta$, $z = l\xi^2 + m\eta^2$. If y is odd, N must be divisible by 8, and $Ny^2/4 = \frac{1}{2}(z + x) \cdot \frac{1}{2}(z - x)$, whence as before we find $x = l\xi^2 - m\eta^2$, $y = \xi\eta$, $z = l\xi^2 + m\eta^2$, where $l\xi$ is prime to $m\eta$ and $lm = N/4$.

In the case of $x^2+y^2=z^2$ we see by taking remainders to modulus 4 that x or y is even. Supposing that y is even the solution can be put into either of the equivalent forms $x=u^2-v^2$, $y=2uv$, $z=u^2+v^2$ or $x=uv$, $y=(u^2-v^2)/2$, $z=(u^2+v^2)/2$. We have assumed that z is positive. If it can be negative, we must prefix the ambiguous sign to its value. The proof given above shows that it is necessary that x, y, z should be of the forms found; we easily see by Algebra that it is also sufficient.

Our second lemma is that if $xy=uv$ then $x=\alpha\beta$, $y=\gamma\delta$, $u=\alpha\gamma$, $v=\beta\delta$. This also may be proved by expressing x and y as products of primes and their powers, or more readily by noticing that a solution is to take α to be the greatest common divisor of x and u, and put $\beta=x/\alpha$, $\gamma=u/\alpha$, which substituted in $xy=uv$ gives $\beta y=\gamma v$ with β prime to γ. Hence β divides v. Take $\delta=v/\beta$, then this equation gives $y=\beta\delta$, and the lemma is proved, for we have already arranged that $x=\alpha\beta$, $u=\alpha\gamma$, $v=\beta\delta$. It is clear that the lemma still holds if x, y, u, v may be negative. The extension to the product of m indeterminates equated to the product of n others is obvious.

For an example take the equation $x^2+y^2=u^2+v^2$. By finding the remainder to modulus 4 we see that x and y are both even, both odd, or even and odd, according as u and v are. Hence we may suppose x to be that one of the first pair that has the same parity as u. Then $\frac{1}{2}(x+u)\cdot\frac{1}{2}(x-u)=\frac{1}{2}(v+y)\cdot\frac{1}{2}(v-y)$, where $(x+u)/2$, etc. are integral. Applying the lemma we have

$$(x+u)/2=\alpha\beta,\ (x-u)/2=\gamma\delta,\ (v+y)/2=\alpha\gamma,\ (v-y)/2=\beta\delta,$$

whence $x=\alpha\beta+\gamma\delta$, $y=\alpha\gamma-\beta\delta$, $u=\alpha\beta-\gamma\delta$, $v=\alpha\gamma+\beta\delta$, which gives a complete solution of the problem to find a sum of two squares equal to the sum of other two squares. These values of x and y give $x^2+y^2=(\alpha\beta+\gamma\delta)^2+(\alpha\gamma-\beta\delta)^2=(\alpha^2+\delta^2)(\beta^2+\gamma^2)$, and we easily see that δ is the only one of the four that can vanish, so that only the first bracket can be unity. This requires that $x=u$, $y=v$, so that the squares are the same. This is the well-known theorem that if a number can be expressed in more than one way as the sum of two squares, it is composite*.

We can deduce the solution of $x^2+y^2=2(u^2+v^2)$. For it is clear that x and y are either both even or both odd, so that $\xi=(x+y)/2$ and $\eta=(x-y)/2$ are integral. Substituting for x and y, the equation reduces to the form just discussed,

$$\xi^2+\eta^2=u^2+v^2.$$

* The more general case $x^2\pm Ny^2=u^2\pm Nv^2$ can be treated in the same way by the extension of the lemma. The equation $x^3+y^3=u^3+v^3$ was solved by Bachet (in his *Diophantus*). See also Fermat's note (p. 133). The equation $x^4+y^4=u^4+v^4$ also can be solved generally, one solution being

$$x=12231,\ y=2903,\ u=10381,\ v=10203.$$

From this we easily solve completely the problem of finding the parallelograms that have their sides and diagonals integral. The solution is that the sides are $\alpha\beta-\gamma\delta$ and $\alpha\gamma+\beta\delta$, and the diagonals $\alpha\beta+\gamma\delta+\alpha\gamma-\beta\delta$ and $\alpha\beta+\gamma\delta-\alpha\gamma+\beta\delta$. We may without loss of generality take α to be positive, and then in order that x, y, u, v may be positive and form a real parallelogram we must have β, γ, δ positive, and $\alpha>\delta$, $\beta>\gamma$.

The third lemma is that if $xy=uv$ and x is prime to v and y to u, then $x=\pm u$, $y=\pm v$. For from the first condition x divides u, and from the second u divides x.

3. Consider $x^4-py^4=z^2$, where p is a prime of the form $8m+3$, and suppose that we have that solution in which xy has the least value. Then x is prime to y. It is also prime to p, for otherwise z^2 is divisible by p and hence by p^2, so that py^4 is divisible by p^2, which is impossible, as y, being prime to x, is not divisible by p. Also y is even, for otherwise we should have $z^2\equiv 5$ or 6, mod. 8. Hence $x^2=u^2+pv^2$, $y^2=2uv$, where u is prime to pv and one of them is even. If u is even, v is odd and $x^2\equiv 3$, mod. 4, which is impossible, so that v is even. Hence $\pm u=\xi^2-p\eta^2$, $v=2\xi\eta$, where ξ is prime to $p\eta$ and one of them is even, which gives $y^2=\pm 4\xi\eta(\xi^2-p\eta^2)$. Hence $\xi=\alpha^2$, $\eta=\beta^2$, $\xi^2-p\eta^2=\pm\gamma^2$ or $\alpha^4-p\beta^4=\pm\gamma^2$, and α or β is even. Hence $\pm\gamma^2\equiv 1$, mod. 4, so that the upper sign must be taken, and the equation is of the same form as that considered. Also

$$\alpha^2\beta^2=\xi\eta<y^2<x^2y^2,$$

that is we have found a solution in which xy has a value less than the least that it can have, and we infer that the equation considered is impossible. In the same way we show that the equation $x^4+2y^4=z^2$ is impossible, and the impossibility of $x^4-8y^4=z^2$ can be proved similarly. The complete solutions of $x^4-2y^4=z^2$ and $2x^4-y^4=z^2$ can be found.

4. The equation $x^4+y^4=nz^2$ is impossible if n (supposed not divisible by any square) contains an odd prime not of the form $8m+1$, for if it has a solution it has one in which x is prime to y, and x^4+y^4 is then not divisible by any odd prime that is not of this form. If $n=17$ there is a solution $x=2$, $y=1$ and we can prove by Fermat's* method that the number of distinct solutions is infinite.

If possible let x, y, z be that solution of the equation

$$x^4-y^4=pz^2,$$

where p is a prime of the form $8m+3$, in which xy has the least value. Then as before x is prime to pz and z is even. Hence

* "Doctrinae Analyticae Inventum novum" (prefixed to S. Fermat's edition of *Diophantus*), p. 26 *et seq.*

$x^2 = u^2 + pv^2$, $\pm y^2 = u^2 - pv^2$, where u is prime to pv and one is even. But by taking residues to modulus 4 we see from the first equation that v is even, and that the upper sign must be taken in the second. Hence solving, $2pv^2 = x^2 - y^2$, $2u^2 = x^2 + y^2$. Writing the second equation $\left(\frac{x+y}{2}\right)^2 + \left(\frac{x-y}{2}\right)^2 = u^2$, where $x - y > 0$, we have $(x \pm y)/2 = \xi^2 - \eta^2$, $(x \mp y)/2 = 2\xi\eta$, whence

$$p(v/2)^2 = \xi\eta(\xi^2 - \eta^2).$$

Now ξ is prime to η and so each is prime to $\xi^2 - \eta^2$. Hence $\xi = p\alpha^2$, $\eta = \beta^2$, $\xi^2 - \eta^2 = \gamma^2$ or $\xi = \alpha^2$, $\eta = p\beta^2$, $\xi^2 - \eta^2 = \gamma^2$ or $\xi = \alpha^2$, $\eta = \beta^2$, $\xi^2 - \eta^2 = p\gamma^2$ according to the factor that is divisible by p, and in each case α, β, γ are prime to each other. These give $p^2\alpha^4 - \beta^4 = \gamma^2$, $\alpha^4 - p^2\beta^4 = \gamma^2$ or $\alpha^4 - \beta^4 = p\gamma^2$ respectively. The first of these is impossible for p cannot divide $\beta^4 + \gamma^2$. The third is of the form of the hypothesis, and $\alpha^2\beta^2 \not> \alpha^2\beta^2\gamma^2 < v^2 < x^2 < x^2y^2$ which contradicts the assumption that xy had the least possible value. In the second α is prime to p, for if not we can write the equation $p^2\alpha'^4 - \beta^4 = \gamma'^2$ so that β is divisible by p, which is impossible as α is prime to β. Hence as α or β is even,

$$\alpha^2 \pm \gamma = \lambda^4, \quad \alpha^2 \mp \gamma = p^2\mu^4,$$

and adding we have $\lambda^4 + p^2\mu^4 = 2\alpha^2$, which is impossible, for p does not divide α and 2 is a non-residue of p. Hence the given equation is impossible. We have proved one case of the theorem that $x^4 - p^2y^4 = z^2$ is impossible, and the other case gives

$$x^2 = u^2 + v^2, \quad py^2 = u^2 - v^2,$$

so that $u^4 - v^4 = px^2y^2$, which is impossible. Hence $x^4 - p^2y^4 = z^2$ is impossible.

We have also proved that $xy(x^2 - y^2) = pz^2$ is impossible if x is prime to y, and we easily see that this restriction can be removed, hence the area of an integral right-angled triangle cannot be the product of a square and a prime of the form $8m + 3$.

5. If $x^4 - x^2y^2 + y^4 = z^2$ has any solution it has one in which x is prime to y (and $x \neq y$ unless $x = y = 1$). Firstly let x or y be even, say y, and suppose that we have that solution in which xy has the least value, subject to y even. Writing the equation in the form $(x^2 - y^2)^2 + x^2y^2 = z^2$ we see that $x^2 - y^2$ is prime to xy, so that $x^2 - y^2 = u^2 - v^2$, $xy = 2uv$, where the first equation shows that v is even. Dividing the last equation by 2 and using the second lemma we have $x = \alpha\beta$, $y = 2\gamma\delta$, $u = \alpha\gamma$, $v = \beta\delta$, where α, β, γ, δ are prime to each other in pairs, for x is prime to y, and u to v. Also α, β, γ are odd and δ even. Substituting in the previous equation we have $\beta^2(\alpha^2 + \delta^2) = \gamma^2(\alpha^2 + 4\delta^2)$. Now β is prime to γ and as α is prime to δ the greatest common divisor of

$\alpha^2+\delta^2$ and $\alpha^2+4\delta^2$ is some divisor of the determinant formed from their coefficients, which is 3. But 3 does not divide $\alpha^2+\delta^2$, hence the brackets are prime to each other, and applying the third lemma we have $\alpha^2+\delta^2=\gamma^2$, $\alpha^2+4\delta^2=\beta^2$. The last gives $\alpha=\xi^2-\eta^2$, $\delta=\xi\eta$ which substituted in the previous one gives $\xi^4-\xi^2\eta^2+\eta^4=\gamma^2$. This is of the original form, for ξ or η is even. Also $\xi\eta=\delta<2\gamma\delta<y<xy$, which contradicts the assumption, and we conclude that the equation is impossible if x or y is even.

Secondly if x and y are both odd, we again write the equation $(x^2-y^2)^2+x^2y^2=z^2$ and unless $x-y=0$ we find $x^2-y^2=2uv$, $xy=u^2-v^2$, whence

$$u^4-u^2v^2+v^4=(u^2-v^2)^2+u^2v^2=(x^2+y^2)^2/4,$$

which is of the form just proved to be impossible, for u is prime to v, one of them is even, and $(x^2+y^2)/2$ is integral. Hence $x^4-x^2y^2+y^4=z^2$ is impossible unless $x=y$.

From this we can prove that an integral triangle with an angle of 60° cannot be equal in area to an equilateral triangle, unless it is itself equilateral. For this requires

$$x^2-xy+y^2=z^2,\quad xy=w^2,$$

where x, y are the sides about the angle of 60°. If these equations have a solution they have one in which x is prime to y, and then the second equation shows that $x=u^2$, $y=v^2$, and the first becomes $u^4-u^2v^2+v^4=z^2$, which is impossible unless $u=v$, that is, $x=y$.

6. If we attempt the second part of the discussion of

$$x^4-x^2y^2+y^4=z^2$$

by the same method as was used for the first part we get

$$2\alpha^2+\delta^2=3\gamma^2,\quad \alpha^2+2\delta^2=3\beta^2,$$

whence α^2, γ^2, β^2, δ^2 are in arithmetical progression. It is possible but troublesome to go on and complete the proof. As however we have proved the theorem in question we can invert the present proof and show that four squares cannot be in arithmetical progression. For if x^2, y^2, z^2, w^2 are such squares, we have $2x^2z^2-2y^2w^2=x^2y^2-z^2w^2$. Hence putting

$$u=xz,\quad v=yw,\quad \xi=(xy+zw)/2,\quad \eta=(xy-zw)/2,$$

which we easily see to be integral, we have

$$u^2-v^2=2\xi\eta,\quad uv=\xi^2-\eta^2,$$

and hence $u^4-u^2v^2+v^4=(\xi^2+\eta^2)^2$. Also unless the squares are all equal (when they can hardly be a progression) we have $y>x$, $w>z$ or $y<x$, $w<z$, so that u and v are unequal, and the

last equation is impossible. Hence four squares cannot be in arithmetical progression*.

Also $x^4-4x^3y+2x^2y^2+4xy^3+y^4$, $x^4+2x^2y^2+y^4$,

$x^4+4x^3y+2x^2y^2-4xy^3+y^4$ and $x^4+8x^3y+2x^2y^2-8xy^3+y^4$

are in arithmetical progression, and the first three are squares. Hence we conclude that the last expression cannot represent a square†.

If $x^4+14x^2y^2+y^4=z^2$ has any solution it has one in which x is prime to y. If the indeterminates are odd and even we have $(x^2-y^2)^2+16x^2y^2=z^2$, whence $x^2-y^2=u^2-v^2$, $2xy=uv$ and so $u^4-u^2v^2+v^4=(x^2+y^2)^2$, which is impossible, for u and v are odd and even and hence unequal. If x and y are odd and unequal, we write the equation $\left(\frac{x^2-y^2}{4}\right)^2+x^2y^2=\left(\frac{z}{4}\right)^2$, whence

$$(x^2-y^2)/4=2uv, \quad xy=u^2-v^2,$$

so that $u^4+14u^2v^2+v^4=(x^2+y^2)^2/4$, where u is prime to v and one of them is even, which we have just shown to be impossible. Hence $x^4+14x^2y^2+y^4=z^2$ is impossible unless $x=y$.

7. If possible let $(x^2+y^2)^2-Nx^2y^2=z^2$, where N is odd, not of the form $8n+3$ and not divisible by any prime of the form $4n+1$, and moreover $N-4$ is an odd power of a prime‡ (it cannot be an even power for N cannot be the sum of two squares), and let us suppose that we have that solution in which xy has the least value. Then x is prime to y, and by taking the remainders to modulus 8 we see that x and y are not both odd. Let y be the even one. Also from its composition N is prime to x^2+y^2, hence the terms of the equation are prime to each other, and from the first lemma $x^2+y^2=lu^2+mv^2$, $xy=2uv$, where $lm=N$ and l is prime to m. Applying the second lemma to the second equation we get $x=\alpha\beta$, $y=2\gamma\delta$, $u=\alpha\gamma$, $v=\beta\delta$, where α and β are odd, γ or δ is even. Since x is prime to y we have α prime to γ, δ and β also prime to γ, δ, and since lu is prime to mv, we have α prime to β and γ to δ, and also l prime to β, δ and m to α, γ. Substituting in the previous equation we get

$$\alpha^2(\beta^2-l\gamma^2)=\delta^2(m\beta^2-4\gamma^2).$$

Now α is prime to δ, and since β is prime to γ, the greatest common divisor of the two brackets divides

$$\begin{vmatrix} 1, & -l \\ m, & -4 \end{vmatrix} = N-4=p^{2k+1}$$

* Fermat, "Inventum novum," p. 20.

† *Ibid.* p. 36.

‡ If $N=1$, then $N-4=-3$. This case is included in the proof, taking $p=3$ later.

say, and hence must be a power of p, say p^λ, where $\lambda \not> 2k+1$. Dividing by p^λ and using the third lemma we get

$$\beta^2 - l\gamma^2 = \pm p^\lambda \delta^2, \quad m\beta^2 - 4\gamma^2 = \pm p^\lambda \alpha^2.$$

First suppose that λ is even. If we take the lower sign the first equation shows that $l = 1$, for otherwise l would contain a prime of the form $4n+3$ and could not divide $\beta^2 + p^\lambda\delta^2$ which is a sum of two squares the first of which is prime to l. Hence $m = N$. The first equation has become $\gamma^2 = \beta^2 + p^\lambda\delta^2$, whence γ is odd, and as α and β are odd the second equation gives

$$N - 4 \equiv -1, \text{ mod. } 8,$$

which is contrary to the hypothesis.

If we take the upper sign the second equation shows that $m = 1$, for otherwise it would contain a prime of the form $4n+3$ and so could not divide $4\gamma^2 + p^\lambda\alpha^2$, where 2γ is prime to m. Hence $l = N$ and the second equation, $\beta^2 = p^\lambda\alpha^2 + 4\gamma^2$, gives

$$\beta = \xi^2 + \eta^2, \quad \gamma = \xi\eta,$$

and on substituting in the first we get $(\xi^2+\eta^2)^2 - N\xi^2\eta^2 = p^\lambda\delta^2$. This is of the original form, for λ being even, p^λ is a square. Also $\xi\eta = \gamma < 2\gamma\delta < y < xy$, which contradicts the assumption.

Next suppose that λ is odd. Solving the equations for β and γ we have $\alpha^2 - m\delta^2 = \pm p^\mu\gamma^2$, $l\alpha^2 - 4\delta^2 = \pm p^\mu\beta^2$, where $\mu = 2h + 1 - \lambda$ and is even. These equations are of the same form as those just considered, with l and m, α and β, γ and δ interchanged, and in precisely the same way we show that the lower sign gives a contradiction of the hypothesis, and the upper sign gives an equation of the original form with a smaller value of the product of the indeterminates, which contradicts the assumption. Hence the equation is impossible under the restriction on N stated in the hypothesis.

The most important case of the proposition is that of $N = 1$ or that $x^4 + x^2y^2 + y^4 = z^2$ is impossible. Hence, just as in § 5, an integral triangle with an angle of 120° cannot be equal in area to an equilateral triangle. Also

$$x^4 + x^2y^2 + y^4 = (x^2 + xy + y^2)(x^2 - xy + y^2),$$

so that both brackets cannot be squares. Hence a parallelogram with integral sides and diagonals cannot have an angle of 60° between the sides or between the diagonals.

If $x^4 - 14x^2y^2 + y^4 = z^2$ has any solution it has one in which x is prime to y. If one of these is even we write the equation $(x^2+y^2)^2 - 16x^2y^2 = z^2$, whence $x^2 + y^2 = u^2 + v^2$, $2xy = uv$, and so $u^4 + u^2v^2 + v^4 = (x^2-y^2)^2$, which is impossible. If x and y are both odd we write the equation

$$\left(\frac{x^2+y^2}{2}\right)^2 - 4x^2y^2 = z^2/4,$$

and as $(x^2+y^2)/2$ is odd we have $z^2/4 \equiv 5$, mod. 8, which is impossible. Hence the proposed equation is impossible.

8. If possible let $(x^2+y^2)^2+Nx^2y^2=z^2$, where N is odd, not of the form $8n+5$, and not divisible by any prime of the form $4n+1$, and $N+4$ is an odd power of a prime (it cannot be an even power, for it is odd and not $\equiv 1$, mod. 8). As before we take the solution for which xy is least, and see that x is prime to y and one of them even, say y, and that N is prime to x^2+y^2. Hence $x^2+y^2=lu^2-mv^2$, $xy=2uv$. As before the last gives $x=\alpha\beta$, $y=2\gamma\delta$, $u=\alpha\gamma$, $v=\beta\delta$, where α and β are odd, α, β, γ, δ prime to each other in pairs, and l prime to β and δ, m prime to α and γ. Substituting in the previous equation we have

$$\alpha^2(l\gamma^2-\beta^2)=\delta^2(4\gamma^2+m\beta^2).$$

The determinant of the coefficients in the brackets is

$$N+4=p^{2k+1},$$

say, and hence the greatest common divisor of the brackets is p^λ, where $\lambda \not> 2k+1$. Proceeding as before we have $l\gamma^2-\beta^2=p^\lambda\delta^2$, $4\gamma^2+m\beta^2=p^\lambda\alpha^2$. First suppose that λ is even. Then the first equation shows that $l=1$ (and so $m=N$) and that γ is odd. Hence as α and β are odd the second gives $N+4\equiv 1$, mod. 8, which is contrary to the hypothesis. Next suppose that λ is odd. Solving for β and γ we find $l\alpha^2-4\delta^2=p^\mu\beta^2$, $\alpha^2+m\delta^2=p^\mu\gamma^2$, where $\mu=2k+1-\lambda$ and is even. The first equation shows that $l=1$ (and so $m=N$) and then gives $\alpha=\xi^2+\eta^2$, $\delta=\xi\eta$, and on substituting in the second we get $(\xi^2+\eta^2)^2+N\xi^2\eta^2=p^\mu\gamma^2=$ square. This equation is of the original form, and $\xi\eta=\delta<y<xy$, which is contrary to the assumption. Hence the equation in question is impossible.

In a closely similar way we can prove the impossibility of $(x^2+y^2)^2-2Nx^2y^2=z^2$, where N is of the form $8m+7$, is divisible only by primes of the form $8m+7$, and $N-2$ is an odd power of a prime; that of $(x^2+y^2)+2Nx^2y^2=z^2$, where $N+2$ is an odd power of a prime and N is of the form $8m+1$ and either divisible only by primes of the form $8m+3$ or only by those of the form $8m+7$; that of $(x^2+y^2)^2-8Nx^2y^2=z^2$, where N is of the form $4m+3$, is divisible only by primes of that form, and $2N-1$ is an odd power of a prime; and that of $(x^2+y^2)^2+8Nx^2y^2$, where N is of the form $4m+1$, is divisible only by primes of the form $4m+3$, and $2N+1$ is an odd power of a prime. (We easily see that in each case the power is necessarily odd.)

9. The general method can also be applied to many other cases, the necessary exclusion of some alternatives being effected by noticing that an equation such as $a\alpha^2+\delta^2=b\beta^2$ can only hold if some such condition as that $-a$ and b are quadratic residues

of each other is satisfied. It is sometimes convenient to commence by writing the equation in the form $(x^2-y^2)^2 \pm Nx^2y^2 = z^2$, and then we must examine the equation to see if $x=y$ gives a solution. Sometimes we have to use the method developed in § 12, second case. Collecting results, we have $x^4+nx^2y^2+y^4=z^2$ impossible if n is 0, 1, 3, 4, 5, 6, 7 (unless $x=y$), 9, 10, 11, 14 (unless $x=y$), 15, 18, 19, 20, 21, 22, 25, 28, 29, 35, 45. 51, 59, 65, 69, 74, 81, 91, and if $-n$ is 1 (unless $x=y$), 3, 5, 6, 7, 8, 10, 12, 14, 17, 18, 19, 20, 21, 22, 23, 24, 27, 29, 31, 45, 54, 55, 60, 61, 69, 75. If n lies between -30 and 30 the equation can be solved except in the cases just given. If $n=-15$ the least solution is $x=95$, $y=24$, but in the other cases the values of x and y in the least solution do not exceed 6.

10. We now propose to prove the impossibility of

$$(x^2-5y^2)^2+128x^2y^2=z^2,$$

not so much for its own sake as because of a deduction we make from it. As before, if it has a solution, take that in which xy has the least value. Then x is prime to y. If x is divisible by 5 let it be $5x'$, then the equation becomes

$$(y^2-5x'^2)^2+128y^2x'^2=(z/5)^2,$$

which is of the same form as the original equation but with a smaller value of the product of the indeterminates. Hence x is prime to 5 and x^2-5y^2 to x^2y^2.

Firstly, suppose that x is even and y odd. Then $x^2-5y^2=2v^2-u^2$, $4xy=uv$, where u is odd and v even, $x=\alpha\beta$, $y=\gamma\delta$, $u=\alpha\gamma$, $v=4\beta\delta$ and $\alpha^2(\beta^2+\gamma^2)=\delta^2(32\beta^2+5\gamma^2)$, where β is even and γ odd. The only possible greatest common divisor of the brackets is unity, so that $\alpha^2=32\beta^2+5\gamma^2$, which is impossible to modulus 8.

Secondly, suppose that x is odd, and y even. Then

$$x^2-5y^2=u^2-2v^2,\ \ 4xy=uv,$$

where u is odd and v even, $x=\alpha\beta$, $y=\gamma\delta$, $u=\alpha\gamma$, $v=4\beta\delta$, where α, β, γ are odd and δ even and $\beta^2(\alpha^2+32\delta^2)=\gamma^2(\alpha^2+5\delta^2)$. The greatest common divisor of the brackets divides 27, hence $\alpha^2+32\delta^2=\gamma^2$ or $=9\gamma^2$, $\alpha^2+5\delta^2=\beta^2$ or $=9\beta^2$, the other cases being excluded by using the modulus 8. Since δ is even the second of these equations gives in either case $\pm\alpha=\xi^2-5\eta^2$, $\delta=2\xi\eta$, where ξ or η is even, and the first becomes $(\xi^2-5\eta^2)^2+128\xi^2\eta^2=\text{sq.}$ This is of the original form, and $2\xi\eta=\delta \not> y < 2xy$, which is impossible since we have chosen a solution for which xy has the least possible value. Hence the equation has no solution in which x and y are one even and the other odd, or in which both are even but containing 2 to different powers.

Lastly, let both x and y be odd. Then $x^2 - 5y^2$ is divisible by 4, the quotient being odd. Writing the equation

$$\left(\frac{x^2-5y^2}{4}\right)^2 + 8x^2y^2 = \text{sq.},$$

we have $(x^2-5y^2)/4 = \pm(u^2-2v^2)$, $xy = uv$, where u and v are odd, and $x = \alpha\beta$, $y = \gamma\delta$, $u = \alpha\gamma$, $v = \beta\delta$, where α, β, γ, δ are odd. Taking the lower sign in the first equation we have

$$\alpha^2(\beta^2+4\gamma^2) = \delta^2(8\beta^2+5\gamma^2).$$

The greatest common divisor of the brackets must be unity, so that $\alpha^2 = 8\beta^2 + 5\gamma^2$, which is impossible to modulus 8. Taking now the upper sign we have $\beta^2(\alpha^2+8\delta^2) = \gamma^2(4\alpha^2+5\delta^2)$. The greatest common divisor of the brackets must divide 27, hence $\alpha^2 + 8\delta^2 = \gamma^2$ or $= 9\gamma^2$, $4\alpha^2 + 5\delta^2 = \beta^2$ or $9\beta^2$, the other cases being excluded by using the modulus 8. The second of these gives $\pm\alpha = (\xi^2 - 5\eta^2)/2$, $\delta = \xi\eta$, where ξ and η are odd, and the other becomes

$$\left(\frac{\xi^2-5\eta^2}{4}\right)^2 + 8\xi^2\eta^2 = \text{sq.},$$

which is of the original form. Also $\xi\eta = \delta = xy/\alpha\beta\gamma < xy$ unless $\alpha = \beta = \gamma = 1$ numerically. But then $x = 1$, $y = \delta$, $u = 1$, $v = \delta$ and $(x^2-5y^2)/4 = u^2 - 2v^2$ gives $\delta^2 = 1$, so that our solution is $x = y = 1$ numerically. Hence if we have any solution in which x and y are odd and not both $= \pm 1$, and x is prime to $5y$, we can find another satisfying the same condition, in which the product of the indeterminates has a smaller value. This is impossible. Hence the given equation can be satisfied only if $x = y$ or $x = 5y$ numerically, or if x or y vanishes.

11. We deduce that the first, second, fifth and tenth terms of an arithmetical progression cannot all be squares, unless the first term is zero or all are equal. For if possible let the squares be those of δ, γ, β, α (supposed positive). Then

$$\alpha^2\beta^2 - 5\gamma^2\delta^2 = 4\alpha^2\gamma^2 - 8\beta^2\delta^2$$

and if $x = \alpha\beta$, $y = \gamma\delta$, $u = \alpha\gamma$, $v = \beta\delta$, we have $x^2 - 5y^2 = 4(u^2 - 2v^2)$, $xy = uv$ and $(x^2-5y^2)^2 + 128x^2y^2 = 16(u^2+2v^2)^2$. This is impossible except in four cases. If $x = y$ we have $\alpha\beta = \gamma\delta$, which gives $-4\alpha^2\beta^2 = 4\alpha^2\gamma^2 - 8\beta^2\delta^2$. Eliminating γ this gives $\alpha^2\delta^2 = 2\delta^4 - \alpha^4$, and so $\alpha^2 = \delta^2$, whence the squares are all equal. If $x = 5y$ we have $\alpha\beta = 5\gamma\delta$, which gives $5\gamma^2\delta^2 = \alpha^2\gamma^2 - 2\beta^2\delta^2$. Eliminating α this gives $5\beta^2\gamma^2 = 25\gamma^4 - 2\beta^4$, which has no solution in integers. If $x = 0$ or $y = 0$, one of the numbers α, β, γ, δ must vanish, and we easily see that it must be δ. Hence we have the theorem stated above.

12. Some equations of the type considered can be solved completely. We choose as an example $x^4 - 4x^2y^2 + y^4 = z^2$ and confine our attention to the case of x prime to y, as it is clear that when all such solutions are known the others can be immediately derived from them. We cannot have both x and y odd. Suppose y to be even. Then writing the equation $(x^2 + y^2)^2 - 6x^2y^2 = z^2$ we have $x^2 + y^2$ odd, and either

$$x^2 + y^2 = u^2 + 6v^2, \quad xy = 2uv,$$

with u odd and (using modulus 4) v even, or

$$x^2 + y^2 = 3u^2 + 2v^2, \quad xy = 2uv,$$

with u odd and (using modulus 4) v also odd. In the first case $x^2 + y^2 \equiv 1$, mod. 8, in the second case $x^2 + y^2 \equiv 5$, mod. 8.

In the first case $x = \alpha\beta$, $y = 2\gamma\delta$, $u = \alpha\gamma$, $v = \beta\delta$, with α, β, γ odd and δ even. The other equation now is $\beta^2(\alpha^2 - 6\delta^2) = \gamma^2(\alpha^2 - 4\delta^2)$, which gives $\alpha^2 - 6\delta^2 = \gamma^2$, $\alpha^2 - 4\delta^2 = \beta^2$, the alternatives being impossible to modulus 8. From the second of these $\alpha = \xi^2 + \eta^2$, $\delta = \xi\eta$, and the first becomes $(\xi^2 + \eta^2)^2 - 6\xi^2\eta^2 = \gamma^2$. Also

$$\xi\eta = \delta < 2\alpha\beta\gamma\delta < xy.$$

Of course ξ is prime to η.

In the second case $x = \alpha\beta$, $y = 2\gamma\delta$, $u = \alpha\gamma$, $v = \beta\delta$, but α, β, γ, δ are all odd. The other equation now is $\alpha^2(\beta^2 - 3\gamma^2) = 2\delta^2(\beta^2 - 2\gamma^2)$, which gives $3\gamma^2 - \beta^2 = 2\delta^2$, $2\gamma^2 - \beta^2 = \alpha^2$, the alternatives being impossible to modulus 8. These give

$$(\beta + \alpha)(\beta - \alpha) = 4(\gamma + \delta)(\gamma - \delta).$$

We suppose none of the factors to vanish. Each of the factors is even, one of those on the left is not divisible by 4, and the other must be divisible by 8. Supposing* it to be $\beta + \alpha$ we have

$$\frac{\beta + \alpha}{2} = 4ab, \quad \frac{\beta - \alpha}{2} = cd, \quad \frac{\gamma + \delta}{2} = ac, \quad \frac{\gamma - \delta}{2} = bd,$$

where a, b, c, d are mutually prime (for α, β, γ, δ are), and c and d are odd. Substituting in $2\gamma^2 - \beta^2 = \alpha^2$, we have

$$a^2(16b^2 - c^2) - 2ad \cdot bc + d^2(c^2 - b^2) = 0.$$

But as a/d is rational we have

$$b^2c^2 + 16(b^2 - c^2)(b^2 - c^2) = \text{sq.} = \zeta^2,$$

say. Putting $2b = \eta$, $c = \xi$, this is $\xi^4 - 4\xi^2\eta^2 + \eta^4 = \zeta^2$, where ξ is prime to η. Also numerically

$$\xi\eta = 2bc < 4abcd < \gamma^2 - \delta^2 < \gamma^2 \text{ or } < \delta^2,$$
$$\xi\eta < 16abcd < \beta^2 - \alpha^2 < \beta^2 \text{ or } < \alpha^2.$$

* If it is $\beta - \alpha$ we put $(\beta + \alpha)/2 = bd$, $(\beta - \alpha)/2 = 4ab$, $(\gamma + \delta)/2 = ac$, $(\gamma - \delta)/2 = bd$ and get the same result as that given in the text.

Therefore we have $\xi\eta$ either $< \gamma\beta$ or $< \gamma\alpha$ or $< \delta\beta$ or $< \delta\alpha$, so that $\xi\eta < \alpha\beta\gamma\delta < xy$. But if one of the factors does vanish we find on working back that numerically α, β, γ, δ are all equal and $x = 2y$.

Hence if we are given any solution in which x is prime to y except the solution (1, 2) we can deduce a smaller one, and repeating the process must ultimately reach the excepted solution (1, 2). Hence we can reach any solution by retracing our steps from this one. Let then $\xi\eta$ satisfy $\xi^4 - 4\xi^2\eta^2 + \eta^4 = \zeta^2$, where η is even and prime to ξ. In retracing the work of the first case we see that each number can be determined, and that α is prime to δ, then that α, β, γ, δ are mutually prime in pairs, and finally that x is prime to y. In retracing the work of the second case we have b, c and ζ mutually prime and c odd. Then

$$a/d = (bc \pm \zeta)/(16b^2 - c^2).$$

If we take a/d to be in its lowest terms we have a prime to b, c and d, and d odd and prime to b and c. Then $(\beta + \alpha)/2$ is prime to $(\beta - \alpha)/2$ and α is prime to β, both being odd. The previous equations now show that α, β, γ, δ are all odd and mutually prime, so that x is prime to y. The ambiguous sign in the value of a/d may be taken either way unless $b = c = \zeta = 1$ numerically, and the values of a/d are not reciprocals of each other. Hence in general we can deduce three solutions from any given one, but only two can be deduced from the solution (1, 2).

All these solutions are different. For if in retracing our steps we arrive at the same solution in different ways, then in the direct work there would be two alternative ways of proceeding. But this does not happen, for the case that the work falls in can be determined by the remainder of $x^2 + y^2$ to divisor 8, and in each case each step is uniquely determined.

By a method similar in principle but differing in detail we can find the complete solution of $2x^4 - y^4 = z^2$. From the least solution (1, 1) we can deduce one other, but from any other we can deduce two new ones, and by continuing the process can reach any solution. In the case of $y^4 - 2x^4 = z^2$ we deduce one solution from each solution of the allied equation $2x^4 - y^4 = z^2$ and from any solution of the equation itself we can deduce one other, and continuing can reach any solution. The arrangement of the primitive solutions is not a dichotomously or trichotomously branched one, as before, but consists of an infinite number of unbranched sequences.

13. If the equation $x^{2n} + y^{2n} = z^2$ has any solution it has one in which x is prime to y, one of them, say y, being even. Then $x^n = u^2 - v^2$, $y^n = 2uv$, and if u is even $u + v = \alpha^n$, $u - v = \beta^n$,

$u=2^{n-1}\gamma^n$, $v=\delta^n$, or if u is odd $u+v=\alpha^n$, $u-v=\beta^n$, $u=\gamma^n$, $v=2^{n-1}\delta^n$, whence $\alpha^n+\beta^n=(2\gamma)^n$ or $\alpha^n-\beta^n=(2\delta)^n$ respectively. Hence $x^{2n}+y^{2n}=z^2$ is impossible for all values of n for which $x^n+y^n=z^n$ is impossible, that is according to a statement of Fermat *, for all values of n greater than 2, or according to a proof by Kummer †, for all values of n that are divisible by any odd prime less than 100. Our equation is also impossible if n is even.

14. We can of course apply the method of this paper to $x^n+y^n=z^2$, but we get no results other than those found by Abel ‡. We notice, however, that if it has any solution it has one where x is prime to y. Hence $(xyz)^n=x^ny^n(x^n+y^n)$ is of the form $w^n=uv(u+v)$ with u prime to v, so that the problem reduces to that of proving that an nth power cannot be represented primitively in a certain binary cubic form. We can now apply the criterion that the number can be represented, but the resulting condition is so complex that further progress appears impossible.

15. *Summary.* We have proved or given a method for proving that the following equations have no solutions in which x, y, z are rational fractions or integers other than zero:— $x^4-py^4=z^2$, where p is a prime of the form $8m+3$; $x^4+2y^4=z^2$; $x^4-8y^4=z^2$; $x^4-y^4=pz^2$, where p is a prime of the form $8m+3$; $(x^2+y^2)^2-Nx^2y^2=z^2$, where N is odd, not of the form $8m+3$ and not divisible by any prime of the form $4m+1$, and $N-4$ is an odd power of a prime (including the case $N=1$);

$$(x^2+y^2)^2+Nx^2y^2=z^2,$$

where N is odd, not of the form $8m+5$ and not divisible by any prime of the form $4m+1$, and $N+4$ is an odd power of a prime; $(x^2+y^2)^2-2Nx^2y^2=z^2$, where N is of the form $8m+7$ and is divisible only by primes of the form $8m+7$, and $N-2$ is an odd power of a prime; $(x^2+y^2)^2+2Nx^2y^2=z^2$, where $N+2$ is an odd power of a prime and N is of the form $8m+1$ and either divisible only by primes of the form $8m+3$ or only by those of the form $8m+7$; $(x^2+y^2)^2-8Nx^2y^2=z^2$, where N is of the form $4m+3$, is divisible only by primes of that form, and $2N-1$ is an odd power of a prime; $(x^2+y^2)^2+8Nx^2y^2=z^2$, where N is of the form $4m+1$ and divisible only by primes of the form $4m+3$, and $2N+1$ is an odd power of a prime; $x^4-nx^2y^2+y^4=z^2$ for the following values of n not included in the general results given above, $n=17, 18, 20, 23, 24, 27$; $x^4+nx^2y^2+y^4=z^2$ for the following values of n not included in the general results given

* *Diophantus*, p. 61.

† See H. J. S. Smith, *Brit. Assoc. Report*, 1860 (Oxford), p. 151.

‡ N. H. Abel, *Oeuvres Complètes* (Christiania, 1839), vol. II. p. 264.

above, $n = 15, 19, 22, 91$; $x^{2n} + y^{2n} = z^2$ for all values of n for which the equation $x^n + y^n = z^n$ has no solution in integers other than zero. We have also indicated the proof that the following equations have no solutions in which x, y, z are rational fractions or integers other than zero, x and y being arithmetically unequal; $x^4 + nx^2y^2 + y^4 = z^2$ if $n = 7, 14$; $x^4 - nx^2y^2 + y^4 = z^2$ if $n = 1$; and proved the impossibility of $(x^2 - 5y^2)^2 + 128x^2y^2 = z^2$ if x, y, z are rational fractions or integers other than zero and $x \neq y \neq 5y$ numerically. We have proved that four consecutive terms of an arithmetical progression cannot each be square, unless all are equal; and that the first, second, fifth and tenth terms cannot each be square unless the first is zero or all are equal. We have solved the equation $x^4 - 4x^2y^2 + y^4 = z^2$ completely and given the nature of the complete solution of $2x^4 - y^4 = z^2$ and $y^4 - 2x^4 = z^2$. We have also shown that Fermat's impossibility may be made to depend on the properties of a binary cubic form.

On the earlier Mesozoic Floras of New Zealand. By E. A. NEWELL ARBER, Sc.D., F.G.S., Trinity College, Cambridge, Demonstrator in Palæobotany.

[*Read* 25 November 1912.]

THE presence of the "Terra Nova," the ship of Capt. Scott's Second Antarctic Expedition (which is still at work in the far South), during the winter months of the last two years in New Zealand waters has led to the collection of materials which are likely to add considerably to our knowledge of the earlier fossil floras of that island. For the seizing of this opportunity we are indebted to Mr D. G. Lillie, B.A., of St John's College, a member of the Biological Staff of Capt. Scott's expedition. Mr Lillie, who previous to his departure for the South had had considerable experience of the art of fossil plant collecting in this country, has set himself the task of making fresh collections by modern methods from the oldest (in a geological sense) plant-bearing beds in New Zealand. He has also been the means whereby the best of the fossil plant material, already collected by the Survey and other institutions in New Zealand, has been sent on loan to Cambridge to undergo a thorough examination for the first time. I thus hope within the next few years to offer a fairly complete account of the older Mesozoic floras of these islands.

In the present communication I have a two-fold object. In the first place it is proposed to make some brief remarks on the nature of the new material collected last year by Mr Lillie, in conjunction with Mr R. Speight, F.G.S., of the Canterbury College, Christchurch, New Zealand, from the celebrated Mount Potts beds, the geological age of which has remained so long in doubt. In the second it is proposed to revise our present knowledge of the pre-Cretaceous floras of New Zealand.

The Mount Potts Beds.

Since the discovery in 1878 of the fossil flora of the Mount Potts beds by McKay, a controversy has raged, on and off, in New Zealand, as to the character and geological age of the flora of these beds in the Rangitata Valley, Ashburton County (Canterbury), while in Europe subsequent uncertainty has also continued until this day on these points. In that year it was asserted, by Hector, that *Glossopteris* occurred in these beds, and the impression has come

to be widely held that a typical Glossopteris flora would one day be forthcoming in this locality. This is really a matter of great importance, for we have long been uncertain whether New Zealand, in Permo-Carboniferous times, did, or did not, form part of the great Southern continent of "Gondwana-land." If, as now seems almost certainly the case, New Zealand formed no part of that continent, then we are face to face with a fact of far-reaching geological interest.

Mr Lillie's collections from Mount Potts show that the age of the flora is either late Triassic (Rhætic) or very early Jurassic. *Glossopteris* itself does not occur, though a fern-like frond of somewhat similar habit, but without a reticulate lateral nervation, is present. This plant which has in the past been mistaken for *Glossopteris* is a member of a new genus. The other plants associated are all of the Mesophytic type, such as *Thinnfeldia*, *Cladophlebis* and *Baiera*, and no typical Permo-Carboniferous species are represented.

So far no Palæozoic plants of any kind are known from New Zealand; which is a most remarkable fact. A Geological Survey of the islands has been in existence for nearly half a century, and there is little doubt that by this time the main features of the geology of the islands are pretty well known. It is thus unlikely, though not of course beyond the range of probability, that any great series of Palæozoic plant-bearing beds remains to be discovered, and, bearing in mind the very wide extent of the Glossopteris beds in Australia and other parts of Gondwana-land, it is still more unlikely now that such rocks remain undiscovered in New Zealand. Many new localities for fossil plants will doubtless be discovered in the future, but at present it seems likely that they will only furnish Mesozoic or Tertiary plants.

A Review of our present knowledge of the earlier Mesozoic floras of New Zealand.

At the present time our knowledge of the fossil floras of New Zealand remains in an extremely unsatisfactory condition. It is true that a number of Tertiary plants, and possibly also some of Cretaceous age, were described and figured by Ettingshausen* many years ago. From this evidence Ettingshausen drew several startling conclusions with regard to the origin of the present flora of New Zealand, conclusions which are now generally discredited, and which tended at one time to throw considerable doubt on the value of the study of fossil plants. The systematic portion of this

* Ettingshausen, *Denkschr. K. Akad. Wissen., Wien* (Math.-Natur. Klasse), Vol. LIII. Pt. I. p. 143, 1887; see also *Geol. Mag.*, Dec. 3, Vol. IV. p. 363, 1887, and *Trans. N. Zealand Inst.* Vol. XXIII. p. 237, 1887.

work, however, remains as one of the very few real contributions to the study of the fossil floras of New Zealand.

It has been known, however, for more than half a century, that New Zealand is rich in Mesozoic floras of pre-Cretaceous age. Large collections of these plants were gathered together from time to time by the officers of the Geological Survey of the islands and by others, and several half-hearted attempts, at one time or another, have been made to describe these floras, attempts which generally ended in long lists of valueless *nomina nuda* (see p. 130). As the literature on the subject of these pre-Cretaceous floras stands at present, it consists of little more than strings of names, applied to fossils which have never been described or figured, names which are therefore meaningless.

There have, however, been some exceptions. Through the good offices of Mr Lillie, Mr P. G. Morgan, the Director of the New Zealand Geological Survey, has kindly sent me on loan all the previously figured specimens in the Collection of the Survey. I have also gratefully to acknowledge my indebtedness in this matter to Mr J. Allan Thomson, Palæontologist to the Survey, for facilitating the loan of these specimens, and for much information as to the localities and to the literature published in New Zealand.

The first step in the revision of these fossil floras has naturally been a re-examination of the specimens which have hitherto been regarded as types. I propose here briefly to review these specimens with the object of sorting out those which are of real value, and of further compiling lists of imperfect determinations and of *nomina nuda* (p. 130). Such names cannot unfortunately be entirely ignored, and the lists at the conclusion of this paper have at least the melancholy interest that they include terms which should never again be applied to any fossil or living plants.

Osmundaceous Stems.

There are two fossil plants of which our knowledge is on altogether a different plane to that of any other plants from New Zealand. These are the two Osmundaceous Fern-stems, obtained from the Jurassic rocks near Gore, Otago District, *Osmundites Gibbiana*, K. and G. V., and *O. Dunlopi*, K. and G. V., so named in honour of their discoverers, Mr Robert Gibb, and Mr R. Dunlop respectively. These are the only petrified plant remains, with the exception of certain post-Jurassic woods, so far known from New Zealand. They have recently been fully described by Dr Kidston and Prof. Gwynne-Vaughan * conjointly.

* Kidston and Gwynne-Vaughan, *Trans. Roy. Soc. Edinb.* Vol. XLV. Pt. III. p. 759, 1907.

Unger Records (1864).

The earliest descriptions and figures of Mesozoic plants from New Zealand are those given by Unger in the "Paläontologie von Neu-Seeland" by Hochstetter and others (Novara-Expedition, *Geol. Theil*, 1 Band, 2 Abtheil., 1864). They are few in number. The present whereabouts of these specimens is unknown to me, and I have consequently not been able to examine them.

Polypodium hochstetteri, Unger (Plate II). This plant is no doubt a *Cladophlebis*, allied to *C. australis* (Morr.). The chief difference between the Australian and New Zealand species appears to be that the lateral nerves in the former case fork twice, as a rule, whereas, in the New Zealand plant, they are shown as only occasionally forking a second time. As is well known, the species of this genus are exceedingly difficult to discriminate, and in the absence of any personal knowledge of Unger's type, I am, for the present at least, inclined to include the New Zealand plant in the older species *C. australis* (Morr.). In the similar British plant, *C. denticulata* (Brongn.) the lateral nerves appear as a rule to fork only once. For the present I am inclined to regard *C. denticulata* (Brongn.) and *C. australis* (Morris) as distinct species. Unger's plant was obtained from the Kalkmergelbänken (Calcareous marls) of the West Coast of the province of Auckland, south of the estuary of the Waikato river.

Asplenium palæopteris Unger (Plate I, figs. 4—8). This plant appears to be a *Sphenopteris*, and, were the fructification known, one would rather expect that it would be referred to the genus *Coniopteris*. It appears to be a distinct species, though it bears some slight resemblance to the Jurassic frond *Sphenopteris Murrayana* (Brongn.). On the other hand Professor Seward* regards it as identical with the Wealden frond, *Sphenopteris Fittoni*, Seward. Whatever the correct genus may be, Unger's specific name can hardly stand, for it had been previously used by Geinitz in 1855 as a generic term, and I am quite in agreement with the rule that the same term should not be used both as a generic and specific name.

I do not, however, propose to invent a new specific name for this plant at present, for I have yet some hopes of rediscovering Unger's types. I will, therefore, simply term it *Sphenopteris sp.* It was obtained from coal-bearing beds on the West Coast of the province of Auckland, between the estuary of Waikato and the harbour of Whaingaroa.

Unger also figures three obscure specimens from the coal-bearing beds of Pakawau in Massacre Bay, in the province of

* Seward, *The Wealden Flora*, Part I. p. xxxiii (Brit. Mus. Cat.), 1894.

Nelson. Those described as *Phönicites ? sp.* and *Equisetites ? sp.* are too imperfect even for generic determination. That figured as *Neuropteris sp.* is possibly a fragment of a frond of *Cladophlebis.*

Hector's Records (1870—1886).

The late Sir James Hector apparently made more than one attempt at an account of the fossil floras of New Zealand, but, for some reason or other, they all ended, with one exception, in long lists of *nomina nuda.* His "Catalogue of the Colonial Museum, Wellington," 1870, his Progress Report for 1878 (New Zealand Geological Survey, p. viii), his paper in the *Trans. and Proc. New Zealand Inst.* (Vol. XI. p. 536), 1879, and his Appendix to the Official Catalogue of the New Zealand Court, International Exhibition, Sydney, 1879 (being the first edition of his Handbook of New Zealand), all contain long lists, for the most part of new specific names, but without any figures or descriptions. The last but one of these papers was meant to serve as a "prodromus of a work on the Fossil Flora of New Zealand containing descriptions and figures of about one hundred species." This work however never appeared*.

In the "Detailed Catalogue and Guide to the Geological Exhibits of the New Zealand Court of the Indian and Colonial Exhibition" (London) of 1886, Hector figures (pp. 65—66, figs. 30 and 30 A) a number of plants without descriptions, in addition to another long list (pp. 31—32) of *nomina nuda.*

Despite the lack of descriptions and the rough nature of the figures, I am inclined to accept most of the specimens figured as types duly published, and as species to be reckoned with, though as I shall show here the majority of the names are synonyms of plants previously described. I have seen most of these types, and I now include a brief revision of this flora here.

The first eight specimens were derived from the Clent Hills (Ashburton County) in the province of Canterbury.

Asplenites rhomboides. Fig. 30 (1) of Hector's Catalogue is an inaccurate drawing of a very small fragment of a *Thinnfeldia,* somewhat recalling *Thinnfeldia argentinica* (Gein.) from Argentina. For the present it may be referred to as *Thinnfeldia sp.* Hector's specific name cannot stand in relation to *Thinnfeldia,* since it approaches too closely Ettingshausen's *T. rhomboidalis.*

* There are in existence, however, in New Zealand a large number of copies of lithographed plates, many of which contain figures of fossil plants, and these no doubt represent the beginnings of this work. No scientific names appear on the plates and further none of them were ever published, and thus it is quite impossible to regard these figures as having any scientific status as regards priority of nomenclature.

Pecopteris acuta. Fig. 30 (2) of Hector's Catalogue is a small fragment of a frond of *Cladophlebis,* far too incomplete to merit specific distinction, and is thus best termed *Cladophlebis sp.* The specific name 'acuta' could not stand in any case, as the term *Pecopteris acuta* had been used long previously by Brongniart for another plant.

Pecopteris linearis. Fig. 30 (3) of the same work is a reduced and very inaccurate drawing of a frond probably identical with *Cladophlebis australis* (Morris).

Vertebraria novæ-zealandiæ. Fig. 30 (4) is a restored sketch of a very obscure specimen which it is quite impossible to determine even generically. It certainly has nothing whatever in common with *Vertebraria,* and this name must in future be excluded from lists of the fossil plants of New Zealand.

Taxites maitai. Fig. 30 (5) is founded on a very small fragment of Coniferous branch, probably identical with *Palissya conferta* (Old. and Morr.) first figured in 1862.

Pecopteris ovata. Fig. 30 (6) is again a small fragment of a frond of *Cladophlebis,* and, although I have not seen the original specimen, I have no doubt that it is best described as *C. sp.* This name *P. ovata* was also used long previously for a quite different plant by Brongniart.

Pecopteris obtusata. Fig. 30 A (1) appears to represent a distinct species of *Cladophlebis,* which will have to be renamed later on. Hector's name *P. obtusata* had already been occupied by a quite different plant, described by Presl in 1833.

Camptopteris incisa. Fig. 30 A (8) almost certainly represents a small portion of the frond of *Dictyophyllum acutilobum* (Braun).

Next, we have six specimens from the Mataura Falls, in the province of Southland.

Macrotæniopteris lata. Fig. 30 A (4) of Hector's Catalogue is undoubtedly *Tæniopteris crassinervis* (Feistm.) first described from the Rajmahal Group of India in 1877. It is certainly not *Tæniopteris lata* (Old. and Morr.).

Lomarites pectenata. Fig. 30 A (5) represents a species of *Gleichenites,* allied to, but perhaps specifically distinct from, the Indian *Gleichenites gleichenoides* (Old. and Morr.) and may for the present be regarded as a distinct species *Gleichenites pectinata* (Hector).

Taxites manawao. Fig. 30 A (6) represents a plant which appears to be undoubtedly *Pagiophyllum peregrinum* (L. and H.), well known in the earlier Mesozoic rocks of England and the Continent.

Pterophyllum matauriensis. Fig. 30 A (7) appears to me to be a distinct species, though somewhat similar to some of the

Pterophyllums of the Jurassic rocks of the Rajmahal Hills of India. This species may thus stand for the present.

Sphenopteris asplenoides. Fig. 30 A (8) represents a small fragment of a *Sphenopteris* frond, the original of which I have not seen. The specific name cannot in any case be retained, for it had been previously applied to another plant by Sternberg, as far back as 1826.

Taxites kahikatea. Fig. 30 A (11) represents a small fragment of a Coniferous branch, the type of which appears to have been lost. Judging by the figure, it would appear to be impossible to refer this plant with certainty to any genus, and thus I should, for the present at any rate, be inclined to omit this determination from any list of New Zealand fossils.

Lastly we have three species from Waikawa.

Taxites manawao. Fig. 30 A (2) represents a specimen which I have not seen, but which I imagine is probably identical with *Palissya tenuifolia* (McCoy) first described from New South Wales in 1847. This name (see above) was applied to two quite distinct plants by Hector in this Catalogue.

Pecopteris grandis. Fig. 30 A (3), the original of which I have not seen, no doubt represents a portion of a frond of *Cladophlebis australis* (Morr.).

Asplenites palæopteris, Unger. Fig. 30 A (10), which again I have not seen the original of, is a fragment of a *Sphenopteris*, which bears a considerable resemblance to fig. 6, Plate I of Unger's memoir, and is very possibly correctly referred to Unger's species by Hector. Unger's plant has been already discussed on p. 125.

We see therefore that even if we are inclined to accept 15 of the 16 plants, first figured but not described by Hector in 1886, as published types, six of them had been previously recorded from other regions, and at the most only six others are likely to stand as first records.

Ettingshausen's Records (1887).

In 1887 Ettingshausen* discussed the floras of five localities, Mount Potts, the Malvern Hills, Haast Gully (Clent Hills), Mataura and Waikawa, and included long lists consisting for the most part of new species. All these are however *nomina nuda*, and as regards the pre-Cretaceous floras this paper is best neglected. Some of these names had been previously published in the preceding year in a paper by Haast†.

* *Vide ante.*

† Haast, *Trans. and Proc. New Zeal. Inst.* Vol. XIX. for 1886, p. 449, 1887.

Crié's Records (1888).

In 1888 Crié* published a short note on a comparison of the earlier Mesozoic floras of New Zealand, Australia and India. He also instituted a considerable number of new names, all of which are *nomina nuda.*

Since 1888, no further contributions to this subject have been made, so far as I am aware, with the exception of the recent work of Kidston and Gwynne-Vaughan already mentioned (p. 124).

The following list sums up our present knowledge of the pre-Cretaceous floras of New Zealand, and includes only ten species, with the addition of three or four other types for which new specific names must be found. I hope before very long to publish a full account, with figures, of these plants, and of other specimens from the same localities.

A List of the Valid Species (known in 1912) from the Pre-Cretaceous rocks of New Zealand.

Cladophlebis australis (Morris).
Dictyophyllum acutilobum (Braun).
Gleichenites pectinata (Hector).
Osmundites Dunlopi K. and G. V.
Osmundites Gibbiana K. and G. V.
Palissya conferta (Old. and Morr.).
Palissya tenuifolia (McCoy).
Pagiophyllum peregrinum (L. and H.).
Pterophyllum matauriensis Hector.
Tæniopteris crassinervis (Feist.).

The occurrence of new species of the genera *Thinnfeldia, Cladophlebis,* and *Sphenopteris* has been also noted.

In conclusion, I add lists of names applied to figured specimens of New Zealand fossil plants, which are synonyms etc., and also a list of *nomina nuda.*

A list of names applied to figured specimens of Pre-Cretaceous plants from New Zealand, which are rejected as being either synonyms or names previously occupied or names founded on imperfect materials.

Asplenites palæopteris Hector.
Asplenites rhomboides Hector.
Asplenium palæopteris Unger.
Camptopteris incisa Hector.
Pecopteris acuta Hector.
Pecopteris grandis Hector.
Pecopteris hochstetteri Hector.
Pecopteris linearis Hector.
Pecopteris obtusata Hector.

* Crié, *Compt. Rend.* Vol. CVII. p. 1014, 1888.

Pecopteris ovata Hector.
Polypodium hochstetteri Unger.
Sphenopteris asplenoides Hector.
Taxites kahikatea Hector.
Taxites maitai Hector.
Taxites manawao Hector.
Vertebraria novæ-zealandiæ Hector.

A List of *Nomina Nuda* applied to Pre-Cretaceous Fossil Plants from New Zealand, with an indication of the place of publication of the earliest references to the names.

* First mentioned in Hector, *Cat. Colonial Mus.* Wellington, 1870, pp. 199—201.

† First mentioned in Hector, Rep. Geol. Explorations (1877—78), *Geol. Surv. N. Zeal.* 1878, p. viii.

‡ First mentioned in Hector, *Trans. and Proc. N. Zeal. Inst.* Vol. XI. p. 536, 1879.

§ First mentioned in Hector, *Official Cat. N. Zeal. Court*, Intern. Exhib. Sydney, 1879, Appendix, pp. 48—49.

|| First mentioned in Hector's *Detail. Catal. N. Zeal. Court*, Indian and Colon. Exhib. London, 1886, pp. 31—32.

¶ First mentioned by Ettingshausen in Haast, *Trans. and Proc. N. Zeal. Inst.* for 1886, p. 449, 1887.

** First mentioned in Ettingshausen, *Denkschr. K. Akad. Wissen., Wien* (Math.-Nat. Klasse), Vol. LIII. p. 143, 1887.

†† First mentioned in Crié, *Compt. Rend.* Vol. CVII. p. 1014, 1888.

Alethopteris Hochstetteri Hector†.
Alethopteris insignis Hector†.
Araucarioxylon australe Crié††.
Asplenites cuneata Hector§.
Asplenites distans Hector§.
Asplenites oblonga Hector§.
Asplenium palæo-dura Ettingshausen¶.
Asplenium Ungeri Ettingshausen¶.
Asterophyllites clentii Hector§.
Baiera australis Ettingshausen**.
Camptopteris haastii Ettingshausen**.
Camptopteris novæ-zealandiæ Hector†.
Cyperites wiwi Hector§.
Dictyophyllum huttonianum Crié††.
Equisetum microdon Ettingshausen**.
Gleichenia waitia Hector||.
Glossopteris haastii Hector§.
Hymenophyllites australis Ettingshausen**.
Lycopodites palæo-selaginella Ettingshausen**.
Macrotæniopteris affinis Ettingshausen**.
Macrotæniopteris zeelandica Crié††.
Neuropteris stricta Hector†.
Nilssonia zeelandica Ettingshausen**.

Noeggerathia valida Hector‖.
Oleandridum distans Hector§.
Oleandridum huttoni Hector†.
Oleandridum matauriense Hector§.
Oleandridum obtusatum Hector§.
Oleandridum stipulatum Hector§.
Oleandridum tæniopteroide Hector§.
Oleandridum vittatum Hector†.
Palæozamia mataurensis Hector‡.
Palissya australis Crié††.
Palissya podocarpioides Ettingshausen¶.
Pecopteris distans Hector*.
Pecopteris gracilis Hector*
Pecopteris grandis Hector*.
Pecopteris haastii Hector§.
Pecopteris ligulatus Hector*.
Pecopteris obliqua Hector§.
Pecopteris oblongis Hector§.
Pecopteris proxima Ettingshausen**.
Pecopteris serratus Hector*.
Pecopteris stricta Hector§.
Pisoniaphyllites novæ-zealandiæ Hector§.
Podozamites malvernicus Ettingshausen**.
Protocladus lingua Ettingshausen**.
Psaronius mataurensis Crié††.
Pterophyllum dieffenbachi, Ettingshausen**.
Pterophyllum grandis Hector‖.
Sphenopteris amissa Ettingshausen**.
Sphenopteris clentiana Ettingshausen**.
Sphenopteris lomaroides Hector§.
Tæniopteris gramineus Hector*.
Tæniopteris Huttoni Hector‖.
Tæniopteris linearis Hector*.
Tæniopteris lomariopsis Ettingshausen**.
Tæniopteris mataurіensis Hector‖.
Tæniopteris pseudo-simplex Ettingshausen**.
Tæniopteris pseudo-vittata Ettingshausen¶.
Tæniopteris obtusatus Hector*.
Tæniopteris robustus Hector*.
Tæniopteris tetranervis Hector‖.
Taxites manoao Hector‖.
Taxites miro Hector§.
Taxites totara Hector§.
Taxites totaranui Hector‖.
Thinnfeldia australis Ettingshausen**.
Tympanophora paradoxus Hector‖.
Zamites Etheridgei Crié††.
Zamites mataurensis Ettingshausen**.

The Mineral Composition of some Cambridgeshire Sands and Gravels. By R. H. RASTALL, M.A., Christ's College.

[*Read* 25 November 1912.]

IT has been recognized of late years that a detailed and careful study of the mineral composition of sands and gravels often yields valuable information as to the sources from which the material was derived and frequently throws much light on the geographical and other conditions prevailing at the time of deposition*.

Some time ago I had occasion to apply this method of research to the Lower Greensand of Bedfordshire, Cambridgeshire and Norfolk. The results of this investigation have not yet been published, but they were of some interest, and it occurred to me that it might be useful to compare with them the mineral composition of some of the Pleistocene and Recent deposits of the same district, in order to see whether it was possible to trace the derivation of material from the Lower Greensand as well as from far-travelled glacial deposits.

A careful examination of numerous specimens of the so-called Neocomian sands showed that all possessed certain peculiar features in common; of these features the most important for the present purpose are the abundance of kyanite, staurolite and tourmaline, and the complete or almost complete absence of garnet, amphiboles and pyroxenes. This is not the place to discuss the source from which these materials were derived: it must suffice here to say that they were certainly not of local origin, but must have come from some distant land-area not now exposed at the surface. The presence of tourmaline is not of much significance, since this mineral is very stable and resistant, and is common in nearly all sandy sediments, being often passed on with little change from one formation to another. But the freshness and angularity of the chips and crystals of kyanite and staurolite in the Neocomian sands suggest that they were then recently separated from the parent rock, and not derived from some older sediment.

* Thomas, 'The Petrography of the New Red Sandstone of the West of England,' *Quart. Journ. Geol. Soc.* Vol. xv. 1909, pp. 229—245.

The Collection and Preparation of the Material.

The specimens on which the present paper is founded were collected from deposits of several different ages, as follows*:

Surface Deposits.
The Gravels of the Present River System.
The Gravels of the Ancient River System.
The Plateau Gravels.

Most of the samples were collected by myself, but for some specimens of blown sands and other surface deposits from the Breckland I am indebted to Dr Marr. Whenever possible a fairly large quantity of the sand was collected from different points in the same bed so as to obtain a really representative sample. After drying, the material was passed through a sieve, to remove large stones, and the finer portion reserved for chemical and mineralogical investigation.

The samples were first washed by shaking with water to remove the fine muddy material; after several repetitions of this process the sand was well boiled with water, and then dried. In this state it was then usually ready for examination with a pocket-lens. Very commonly, however, many of the grains were still covered with a pellicle of calcium carbonate or iron oxide, and further treatment was required to render the constituents visible.

Methods of Investigation.

In order to get rid of the calcium carbonate, most of which was in the form of minute particles of Chalk, the samples were then treated with dilute hydrochloric acid. When the effervescence had subsided the washing and drying were repeated and notes were then made on the general appearance of the samples.

If the sand appeared to be highly ferruginous, it was then boiled for some time with fairly strong hydrochloric acid. This removed the iron oxides almost completely, and after again boiling with water, thorough washing and drying, the sand was ready for detailed examination. No doubt this treatment would also remove certain unstable minerals, if present, but from a comparison of samples, some of which had been treated with strong acid, and others not, it appears that the loss was not serious. Most of the important minerals which are likely to occur in sands are not affected by this treatment, and it is obvious that

* Penning and Jukes-Brown, 'The Geology of the Neighbourhood of Cambridge' (Expl. quarter-sheet 51 S.W.), *Mem. Geol. Surv.* 1881, pp. 82—109. Marr and Shipley, 'The Natural History of Cambridgeshire,' *Brit. Ass. Handbook*, 1904, pp. 42—50. Rastall, 'The Geology of Cambridgeshire, Bedfordshire and West Norfolk,' *Geol. Ass. Jubilee volume*, 1909, pp. 173—177.

a mineral grain covered by an opaque skin of haematite or limonite cannot be identified, and is therefore useless, unless this skin is removed. It is necessary therefore to run the risk of decomposing some of the less stable minerals.

The separation of the heavier constituents was effected by the heavy-liquid method, the particular liquid used being bromoform, which when pure has a density of about 2·93. This removes all the quartz and felspar and nearly all the glauconite. The density of this latter mineral, however, appears to be variable since some green grains occasionally sank in bromoform. According to the best authorities the density of glauconite is about 2·3 only. It is evident therefore that this point requires further investigation.

It is unnecessary here to describe the precise form of apparatus employed, which was of the most simple nature. A very little experience showed that any attempt at a quantitative determination by the heavy-liquid method must be absolutely unreliable, owing to the impossibility of ensuring a complete separation. In some cases it was found necessary to treat the heavy residues again with acid in order to get rid of the pellicle of iron oxide which is such a persistent feature in these sands: in a few instances this was unnecessary. A part of each residue was then mounted in Canada Balsam in the usual way for microscopic examination, and the rest reserved for any other tests required.

Detailed Description of the Sands.

A large number of samples were examined by the above methods, but many of them were very much alike in their general characters, and it is unnecessary to describe them all. The following cases are therefore selected as typical examples of the sands and finer portions of the gravels of different ages.

I. *The Plateau Gravels.*

Pit on Golf-Links, summit of Gog-Magog Hills, 200 ft. above O.D.

The beds exposed in this pit consist partly of gravel and partly of a very fine-grained sand in thick beds, which often show marked contortion. The larger constituents of the gravel comprise a great variety of far-travelled rocks, and from pits closely adjoining the present one, which were open a few years ago, a large number of interesting rock-types were recorded, including abundant igneous rocks, both Scotch and Scandinavian, Carboniferous Limestone and Millstone Grit, several varieties of Jurassic sediments, Carstone and Hunstanton Red Rock, together with shells of *Gryphæa* bored

by *Pholas* or some other rock-borer. The presence of these and of the Cretaceous rocks of Norfolk type is of special significance, as indicating derivation of the material from the Wash district. Rounded pebbles of Chalk are abundant, but they are believed to be of Yorkshire type and the large grey tabular flints are much more like those of Lincolnshire than of any more southern locality*.

The bed of sand, about 5 feet thick, which is now being worked for use on the golf-links, is of very fine texture, marly and rather tenacious, of a yellow-brown colour. It differs much in appearance from other sands of the district. It effervesces only feebly with dilute acid, but very large quantities of fine mud are removed by boiling with water and subsequent washing. Chips of white flint are common, but the most notable constituent seen on macroscopic examination is mica, both white and brown, together with glauconite. As will appear later, mica is almost completely absent from the other sands investigated.

From a first separation, without preliminary treatment with acid, the heavy residue was very large, and mostly of a brown colour, owing to the presence of a coating of limonite. In this state identification of the grains was impossible, and the residue had to be digested with acid and again separated.

On examining a microscope-slide of the ultimate residue the first feature to be noticed is the small average size of the grains, which are much less than those of the other gravels of the Cambridge district, and can only be compared in this respect with the blown sands from the neighbourhood of Brandon, to be afterwards described.

The following is a list of the principal minerals identified: hornblende, augite, garnet, epidote, zircon, tourmaline, rutile, magnetite and other iron ores, and abundant flakes of muscovite. Kyanite and staurolite are notably rare. Hornblende is extraordinarily abundant, both the common green variety and greenish blue arfvedsonite. Tourmaline is also abundant and shows many shades of colour, brown, pink, blue and green: the grains are as usual very well rounded. In comparison with other local sands, pyroxenes, epidote and zoisite are common.

The most striking features of this sand are the abundance of hornblende and muscovite, and the strange shapes of many of the grains, which can best be described as sharply angular chips. It is evident that in the case of most of the constituents there has been little rolling, either by water or wind action; only the grains derived from some older formation, such as tourmaline and kyanite, are conspicuously rounded. The significance of the

* Rastall and Romanes, 'The Boulders of the Cambridge Drift,' *Quart. Jour. Geol. Soc.* Vol. LXV. 1909, p. 254.

presence of much muscovite in this sand, and its absence in other localities, will be discussed in the concluding section. It is at any rate clear that this high-level deposit differs greatly from the low-level sands next to be described.

II. *Sands and Gravels of the Ancient River System.*

(*a*) Pit on Newmarket Road, half a mile beyond Barnwell Junction, 46 ft. above O.D.

In this pit, which is now being worked for gravel, are seen seams of very white sand, interbedded with fine flint gravel. On examination with a pocket-lens, the sand is seen to be very rich in minute fragments of Chalk, flint chips and grains of glauconite. It also contains many small white prismatic objects, which, as Dr Bonney suggests, are probably prisms from the disintegrated shells of *Inoceramus*, together with minute spines of Echinoids.

After washing, the sand effervesced very strongly with dilute acid, and a second washing removed a large amount of muddy material; hence the grains are evidently partly cemented by calcareous matter.

In the first separation in bromoform, many brown grains came down: the majority of these proved to be grains of light minerals with a coating of iron oxide (limonite). After prolonged treatment with acid this coating was removed, and many opaque grains were left, of various colours, white, pink, green and brown. These grains were easily separated from the true heavy residue by shaking with water in a dish. They are very well rounded and uniform in size, and certainly come from the Lower Greensand.

In the heavy residue the most abundant mineral is garnet, either colourless, pink or brownish red, for the most part in very angular chips of varying size. Green hornblende is also common, while blue-green arfvedsonite and pale green augite also occur. Other minerals noted are tourmaline, kyanite, staurolite, epidote, zircon and rutile, besides opaque iron ores.

(*b*) Pit behind the Travellers' Rest Inn, Huntingdon Road, 1 mile from Cambridge, 83 ft. above O.D.

This large pit is opened in gravel of very variable coarseness, with abundant seams of brownish sand. This gravel is notable in that it yields an unusual number of large boulders, up to one foot in diameter: the great majority of these are of sandstone, probably Carboniferous, but other rocks are fairly common. From this locality a large number of far-travelled erratics have been

recorded, including scratched blocks of Carboniferous limestone, Millstone Grit, basalt, pink granite and rhomb-porphyry*.

The sand is fairly clean, of a light brown colour, and effervescing very freely with acid. In this case a double separation was found necessary, and the light residue from the second separation was found to consist largely of glauconite, which had been coated with iron oxide.

Among the heavy minerals of this specimen, the following were identified: pink garnet, tourmaline, both brown and blue, staurolite, kyanite, epidote, hornblende, augite and hypersthene, together with black grains which are presumably magnetite. Zircon is very rare. The most notable features are the extreme angularity of most of the garnets, and the comparatively large size of the heavy mineral grains.

(*c*) Furze Hill, Hildersham, 200 ft. above O.D.

This specimen was obtained from the actual bed in which a Palæolithic implement was found *in situ* by Dr Marr†. It is a yellowish brown rather coarse sand with many small flints. After cleaning in the usual way the sample was found to consist chiefly of grains of clean white quartz, many of which are notably rounded. Coloured minerals are not abundant, consisting chiefly of grains of iron oxide. After separating in bromoform the heavy residue was examined microscopically, and was found to contain rounded grains of iron oxide together with brown and blue tourmaline, staurolite, zircon, rutile, garnet, and a very little kyanite. Most of the heavy grains are very much rounded, and they show much variation in size.

This sand is very unlike the specimen collected at about the same level on the Gog-Magog Golf-Links, and has evidently been laid down in rapidly moving water, with much more rolling of the grains. This fact helps to confirm the idea put forward by Dr Marr†, that these gravels and sands belong to the ancient river-system rather than to the plateau gravels.

III. *Gravels of the River Cam.*

Highest or Barnwell Terrace.

(*a*) Gravel Pit close to L. and N. W. Railway bridge between Trumpington and Shelford, 60 ft. above O.D.

This pit shows chiefly a moderately fine flint gravel of the usual type, with occasional seams of sand. From the most

* Rastall and Romanes, 'The Boulders of the Cambridge Drift,' *Quart. Jour. Geol. Soc.* Vol. LXV. 1909, p. 254.

† Marr, 'On a Palæolithic Implement found *in situ* in the Cambridgeshire Gravels,' *Geol. Mag.* 1909, pp. 534—537.

important of the latter the sample was collected. In the heavy residue the dominant mineral is pink garnet, and opaque grains of magnetite and other iron ores are also abundant. The grains are very variable in size, many of them, especially the garnet, being rather large.

The following is a list of the minerals identified: garnet, tourmaline, staurolite, kyanite, hornblende, augite, hypersthene, zircon, rutile, epidote and magnetite.

Garnet, both pink and colourless, is very abundant; some grains are rounded, most are subangular, while a few are very sharply angular. The grains of brown and pink tourmaline are generally very well rounded and appear to be derived, while the staurolite and kyanite grains exactly resemble those of the Lower Greensand, being almost certainly derived from that formation.

(*b*) Swan's Gravel Pit, Milton Road, 45 ft. above O.D.

This well-known pit, which is situated near the first milestone on the Cambridge and Ely road, is excavated chiefly in rather fine flinty gravel which contains a good many far-travelled rocks. The specimen was collected from a thick seam of rather ferruginous sand which forms the north-western corner of the pit. After cleaning in the usual way it was found to contain abundant dark grains together with glauconite. The proportion of grains sinking in bromoform was unusually large and most of them were so thickly coated with iron oxide as to require prolonged treatment with acid. It is noticeable in this and other cases that the iron oxide is often regularly deposited in concentric coats round some mineral nucleus, and when these shells are only partially destroyed a regular oolitic structure giving a black cross in polarised light can often be seen.

The final residue consisted of abundant rather large grains, most of which were fairly well rounded, sharply angular crystals, other than garnet, being uncommon. The list of minerals recognised is as follows: garnet, tourmaline, kyanite, staurolite, epidote, hornblende, augite, hypersthene, rutile and zircon: possibly also brookite and anatase. The garnets vary much in colour, being most commonly pink or brownish pink, sometimes colourless. The tourmaline grains are very round, and include brown, pink and blue varieties. There are many rounded grains of yellow-green epidote, which in everything but colour have a strong resemblance to monazite.

This sample is specially notable for the large size and rounded form of the kyanite grains. Generally speaking, the grains of heavy minerals are conspicuously more rounded than those of the sands previously described.

Second or Chesterton Terrace.

Pit near Pike and Eel, Chesterton, 24 ft. above O.D.

This pit is opened up in fine flint gravel with seams of rather coarse sand and in it have been found a good many far-travelled rocks, including rhomb-porphyry. To the naked eye the sand when dry is white and chalky in appearance. It shows many grains of Chalk and flint together with glauconite, fragments of the thin shells of fresh-water mollusca, prisms of *Inoceramus* and Echinoid spines. With dilute hydrochloric acid it effervesces strongly, and in the cleaned sample dark grains are fairly abundant.

The sand yields an abundant heavy residue in which nearly all the grains are covered with brown limonite. After treatment with strong acid many of these are found to be glauconite or a pale brown isotropic material of uncertain character, perhaps consisting of colloid silica. The principal minerals identified are garnet, hornblende, rutile, zircon, kyanite, staurolite and tourmaline. The garnets are mostly subangular and rounded, only a few being angular. The grains of tourmaline, which are chiefly brown, are well rounded. The minerals in general do not present any special features of interest: they are on the whole more rounded than in the older gravels.

IV. *Gravels of the Great Ouse Basin.*

Gravel Pit half a mile S.E. of Fenstanton, 36 ft. above O.D.

This pit is situated on the north side of the Cambridge and Huntingdon road, close to the county boundary. It shows a ferruginous gravel of the usual type, with brown 'pipes' and seams of sand.

The sand is brown in colour, with very little muddy matter. It was treated at once with dilute acid, giving only slight effervescence, and yielding a remarkably white sample consisting of clean bright colourless quartz and rounded grains of white flint with much glauconite. In the heavy residue pink garnet and bright grains of magnetite were conspicuous before mounting, while a few grains of glauconite sank in the bromoform.

The heavy grains are of unusually large size and garnet is predominant, being often extremely angular in shape; other transparent minerals are present only in small quantity. The minerals recognized are as follows: garnet, generally very angular, tourmaline, kyanite in fine large crystals, staurolite, hornblende, epidote and rutile; zircon is rare.

The exact stratigraphical position of this gravel is uncertain: it is situated near the margin of a low plateau, which slopes down somewhat suddenly within a short distance to the alluvial flats of the Great Ouse. In general appearance it strongly resembles the gravels of the Cam, and it is included here for the sake of comparison.

V. *Surface Deposits.*

Blown Sand, Lakenheath Warren.

This specimen, which was collected by Dr Marr as an example of the superficial wind-transported deposits of the Brandon district, consists of a very clean sand, remarkably free from muddy material, but containing a good deal of black fibrous vegetable matter. The effervescence with acid was very slight and the sample required very little preparation. It gave a fairly large dark residue in bromoform, and as most of this was obviously ferruginous it was at once treated with acid and allowed to stand for some time, washed and re-separated. The final product was small in amount and the constituent grains were of very small size as compared with those of the sands presumably laid down in water. The only mineral occurring in fairly large grains is garnet, which is also rather angular; all the other constituents are very round (except staurolite, which appears to remain angular under all circumstances).

The minerals identified are garnet, tourmaline (sometimes blue), kyanite, staurolite, hornblende, augite, hypersthene, epidote and rutile. Zircon is much more abundant than in any other specimen here described.

Surface Deposit, Fowlmere, near Thetford.

This is a sandy deposit, of a curious grey colour when fresh; the peculiar colour is probably due to the absence of the usual iron oxide. When washed and treated with dilute acid it is found to consist principally of clean white quartz sand, with a few reddish grains.

The minerals of the heavy residue are much as usual, namely, magnetite, garnet, tourmaline, hornblende, augite, epidote, zoisite, kyanite, staurolite and zircon. The staurolite is as usual angular, but the other minerals are well rounded, and this is specially notable in the garnet, which shows signs of more attrition than in any other sample examined. This may safely be attributed to wind action.

The particles show a wide variation in size, but the majority are small.

General Conclusions.

The most abundant constituent in all the sands here examined is, as might be expected, quartz. This is normally perfectly clear and colourless, but sometimes shows a reddish tinge. Next in abundance is flint, in white opaque grains, often well rounded. In many samples Chalk is abundant, while prisms of *Inoceramus* and Echinoid spines derived from the Chalk are often common. Glauconite is generally present, although not conspicuous till the sands are cleaned, since the grains of this mineral are frequently covered by a skin of iron oxide, which must be removed by acid before they become readily visible. The heavy residues, of a density higher than 2·95, include a large proportion of opaque grains of magnetite and other oxides of iron. Among the transparent constituents of the heavy residues the following are generally present: garnet, tourmaline, kyanite, staurolite, hornblende, augite, hypersthene, epidote, zoisite, zircon, rutile. Of these the most abundant is, in most cases, garnet, which occurs in angular, subangular and rounded fragments often of considerable size; tourmaline varies much in colour, and blue varieties are common; other colours noted are brown, pink and green. Staurolite is common, always in angular fragments, while the numerous crystals of kyanite are notably more angular in the older gravels. Hornblende is locally very abundant and includes the blue variety (arfvedsonite) which is so characteristic of Norwegian soda-rocks. Zircon is much less common than in most sands, *e.g.* the Bagshot Sands described by Dick.

The minerals of these sands may be divided into two groups, as follows:

A. Glauconite, tourmaline, kyanite and staurolite: these are almost certainly derived from the Neocomian. As is well known, staurolite nearly always remains angular, but the tourmaline and kyanite are somewhat more rounded than in that formation. The tourmaline especially occurs in extraordinarily well-rounded forms. The blue variety, which is common, is almost certainly derived, viâ the Lower Greensand, from the ancient Armorican land to the south-west. Kyanite is a very common and characteristic mineral both in the Carstone and in the Sandringham Sands, and staurolite is also present in both of these. The abundance of glauconite in the Neocomian hardly needs mention.

B. Garnet, hornblende, augite, hypersthene, epidote: these minerals are almost unknown in the Neocomian, and have certainly been derived at first hand from the disintegration of far-travelled rocks of Scotch and Scandinavian origin during the

glacial period. They are much more angular than the first group, especially in the older sands, and show little evidence of derivation from pre-existing sandy deposits: in particular the angularity of the chips of garnet is often very marked, and to use a simple word they look very *new*, having undergone little rolling or attrition: well-rounded garnets are very rare, except in the superficial and wind-borne deposits of the Breckland.

The most striking feature of the sands here examined is the almost complete absence of muscovite, this mineral being one of the commonest constituents of sands of all ages, with a noteworthy exception in the case of deposits of one class. It is stated by Retgers* and Thoulet† that mica and other minerals with very perfect cleavage are almost completely absent from wind-blown sands, and this observation may have some significance in this case. The only deposit here described containing muscovite is the high-level or plateau gravel on the summit of the Gog-Magog Hills. This is undoubtedly much older than the gravels and sands of the Cam system, and it is generally regarded as being of glacial age. The great abundance and variety of far-travelled erratics in it certainly lend support to this conclusion, and the sand itself is of a very different character to the river and surface deposits seen at lower levels.

On the retreat of the ice the whole country must have been covered by vast spreads of sand and gravel, the relics of which are still to be seen in the Breckland of western Suffolk and south-west Norfolk. It is generally believed that the Glacial period on the continent was succeeded by a time of warm and dry climate, the Steppe period, and somewhat similar conditions must have prevailed in England. It is possible that during this time the sands were to a certain extent worked over by the wind, removing the muscovite and other light and flaky minerals. The chief difficulty in the way of this view is the extreme angularity of the garnets and some other minerals in the earlier river-gravels.

The general conclusions arrived at from a study of the mineral composition of the sands here examined may be stated as follows: the materials have been derived from two sources, partly from the Neocomian sands of Cambridgeshire and the neighbourhood of the Wash, and partly from far distant sources by ice-transit; that is, from the solid matter transported on and in the ice from Norway, Scotland and the north of England. The mineral grains obtained from the former source are almost exactly like those characteristic

* Retgers, 'Über die chemische und mineralogische Zusammensetzung der Dünensande Hollands,' *Neues Jahrb. für Min.* 1895, p. 22.

† Thoulet, 'Étude minéralogique d'un sable du Sahara,' *Bull. Soc. Min. France*, Vol. IV. 1881, p. 262.

of the Neocomian sands of Norfolk, the Carstone and the Sandringham sands, only differing from them in their slightly more rounded form. The far-travelled grains on the other hand are largely such as might be obtained from igneous and metamorphic rocks, then undergoing disintegration for the first time: they are angular in the earlier deposits, subangular in the later, and only in the obviously wind-borne surface deposits are all alike reduced to a small size and a fairly uniform degree of roundness. Thus during the deposition of the Cam gravels water action was predominant; at a later time wind asserted its power as the principal agent of distribution of the latest superficial accumulations of the drier parts of East Anglia.

Note on the Röntgen radiation from cathode particles traversing a gas. By R. WHIDDINGTON, M.A., St John's College.

[*Read* 11 November 1912.]

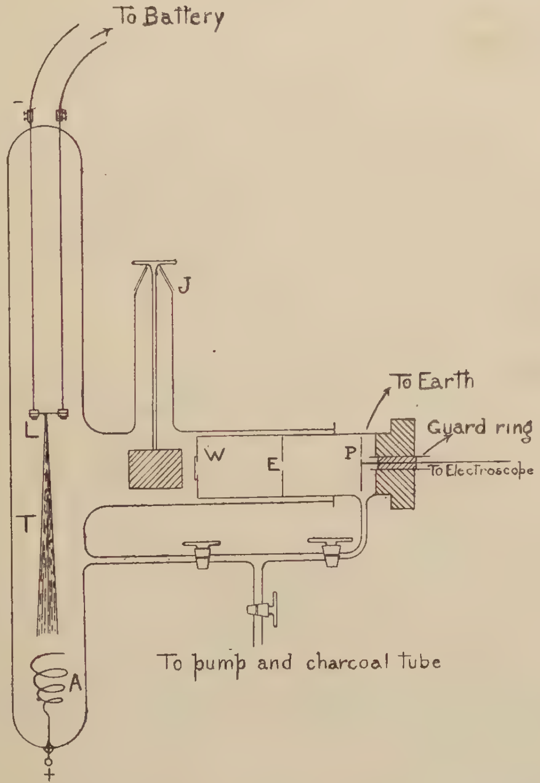
IT is now a commonplace that Röntgen rays are emitted from a target struck by cathode particles, and very many researches have been carried out with the object of determining how the nature of the target and the velocity of the incident particles determine the quantity and the quality of the emitted Röntgen rays. The experiments shortly to be described bring forward evidence to show that when very slow cathode particles traverse a gas at low pressure, Röntgen rays are emitted all along their path*.

The form of apparatus used is indicated in the figure. A hot lime cathode L projects a fine beam of cathode particles down the axis of the evacuated glass tube T, the anode being a spiral of aluminium wire A. An insulated aluminium plate P is enclosed in an earthed brass tube E, provided with a small opening at W. This opening is closed by a thin blown-glass window, so thin as to show interference colours of a low order. P is connected to the leaf of a Wilson tilted electroscope. The arrangement of taps shown in the diagram enables connection between T and E to be severed after a moderate vacuum has been produced so that a slightly higher vacuum may prevail in E than in T. This is desirable since we wish to observe the potential to which P will rise, and it is necessary under such circumstances to eliminate any gas leak due to ionisation.

In the earlier experiments it was found that when the cathode stream from L was passing down the tube there was an enormously rapid deflection of the gold leaf in such a direction as to show that the plate P was charging up positively. This effect was finally traced to defects in the window W, small cracks being present which apparently allowed a diversion of the main current into the brass cylinder, P functioning as a subsidiary anode and becoming positively charged. When the window, however, was apparently perfect there was still an effect in the same direction, the plate P charging up positively as before but at a very much diminished rate. A strong magnetic field applied across W had little or no effect upon the rate of charging of P. Moreover, when the narrow pencil of rays from L was curled up into a close spiral in front of the window, but not touching it, P charged

* No rays can be detected unless the applied potential exceeds 90 volts.

up much more rapidly, an effect easily explained if P is regarded as emitting negative electricity as the result of a radiation proceeding from the cathode stream itself. A test which conclusively showed that the effect was really due to a radiation from the cathode stream was devised. A solid obstacle was placed in front of W, the effect at once disappeared; on removal of the obstacle, P again commenced to charge up. In the actual experiment the obstacle was a small brass plate, which could be rotated from the



outside by means of the ground glass joint J. In this way P could be given an uninterrupted view of the cathode stream or the view could be cut off.

There is thus definite evidence of a radiation proceeding from the path of the cathode stream. That this radiation does not consist of charged particles of small mass is shown by the fact that it is not cut off by a transverse magnetic field. Strong evidence in favour of supposing it to be a very soft Röntgen

radiation is afforded by the observation that the velocity of the negative particles ejected from P is not very different from the velocity of those streaming out from the lime cathode*. This experiment was carried out as follows. An electrostatic voltmeter connected between L and A gives the velocity (expressed in volts) of the cathode particles shot out from L. The potential to which P charges up is a measure of the velocity of the particles ejected from P. In practice it was found necessary to keep the gold leaf in as sensitive a position as possible, and a convenient method of ensuring this was to keep the potential difference between the electroscope plate and the leaf very nearly equal to V, the voltage giving instability. Thus the leaf was charged to a positive potential v, and the plate was kept at such a potential p (positive or negative according as v was greater or less than V) as would satisfy the relation $v - p = V$. The voltage across the tube was then varied until the leaf did not drift from its zero position when insulated. We can then say that cathode particles of velocity not greater than v are being ejected from P. The following table gives a few of the results obtained in this way.

Velocity of lime cathode rays (expressed in Volts)	Velocity of particles from P (expressed in Volts)
128	100
145	117
187	152
218	178

It is to be noticed that the ratio $\dfrac{\text{velocity of lime cathode rays}}{\text{velocity of particles from } P}$ is nearly constant*.

* In this connection see *Proc. Roy. Soc.* A. Vol. LXXXVI. 1912.

The Gravels of East Anglia. By Professor HUGHES.

[*Read* 25 November 1912.]

IN introducing the subject of the gravels of East Anglia the author pointed out that too much importance must not be attached to the absolute height and level of the river terraces, firstly because of the rise of the valley from its mouth to its source and secondly on account of the earth movements which have affected the area. He showed that there had been considerable depressions in the valley of the Cam since the deposition of some of the existing river silt.

Only a small proportion of the flints of which the gravels were chiefly composed were likely to have been derived directly from the Chalk and very few from the London Tertiaries. They were probably produced on the Miocene land surface over which the Crag sea advanced rapidly sweeping up the old surface soils and forming the first deposits of angular flints from which so much of our stained gravel has been derived.

The subsequent depression of this area, while adjoining mountain regions were uplifted, would account for the material of the Norfolk cliffs which might be referred to the action of an ice-laden sea on the land. Shore ice and pack ice early impinged upon the sinking and afterwards the rising land, mixing up and contorting the material upon it, and creeping up the long slopes and over the wide plains with similar results.

He traced these glacially formed or glacially modified beds from the coast to the hills inland, pointing out that every character which was seen in isolated sections inland could be seen along the coast in continuous sections. They differed from the river gravels in their tumultuous arrangement and in their great variety of composition.

This group, of glacial origin, were well represented by the Whittlesford beds.

The loam of the cutting near Chesterford Station, and the sands, gravels, loam and marl of Hildersham, Gog Magogs and Hare Park were assigned to the same series, and connected with the coast by the deposits of Roslyn Pit, Sedgeford and Hunstanton. As the land rose the agents of subaerial denudation began their work at once and the rain-wash and river-terraces thus formed,

being often fossiliferous, were more easily grouped in chronological succession.

Of the newest age were the gravels of Jesus College, Little Downham, &c. In an earlier stage he placed the gravels of Barnwell, Stow, the Botanic Garden, Trumpington, Shelford, and the mammoth gravel of Chesterford, and to a still earlier stage he referred the Girton and Observatory gravels and those of the Newmarket Road.

The Barrington beds must be considered by themselves, being very different in composition, situation and fossil contents.

On the Properties of a Liquid connected with its Surface Tension. By R. D. KLEEMAN, D.Sc. (Adelaide), B.A., Emmanuel College.

[*Received* 18 December 1912.]

Calculation of the Absolute Mass of the Hydrogen Atom.

If no transition layer were formed on a liquid surface the equation*

$$\lambda_a = \frac{U m_a^{\frac{1}{3}} \rho_1^{\frac{2}{3}}}{6 \times \cdot 876} \quad \text{......................(1)}$$

could be used to calculate the absolute molecular weight m_a of a molecule where U denotes the energy expended against molecular attraction in separating the molecules of a gram of substance an infinite distance from one another, ρ_1 the density of the substance, and λ_a the surface tension that would exist if no transition layer were formed. At low temperatures when the density of a liquid is large in comparison with that of its saturated vapour $U = L$ the internal heat of evaporation, if the internal energy of a molecule is practically independent of the vicinity of other molecules, which is very likely the case. But although a transition layer is formed the equation may be used to obtain an approximate value of the quantity in question. It was found that if values of m_a are calculated by equation (1) for different temperatures of a liquid, substituting for λ_a in the equation the values λ found in practice, and plotted against the corresponding temperatures, the points obtained lie approximately on a straight line. Now the effect of the transition layer on the surface tension decreases with decrease of temperature†, and at the absolute zero is probably negligible. According to a formula given by the writer‡ the surface tension at the absolute zero is about sixteen times that at room temperature, and the effect of the transition layer on the surface tension therefore small according to the paper first quoted. An approximate value of the absolute molecular weight is therefore given by the intercept of the straight line on the m_a axis.

* *Phil. Mag.* Dec. 1912, p. 883.
† *Loc. cit.*
‡ *Ibid.* Jan. 1911, pp. 99—101.

TABLE I.

Ether			Carbon tetrachloride		
T	λ	L	T	λ	L
313	14·05	75·36	363	17·60	40·62
363	8·63	63·31	423	11·21	34·42
$m_a = 1{\cdot}34 \times 10^{-24}$ grm.			$m_a = 1{\cdot}67 \times 10^{-24}$ grm.		

Methyl formate			Benzene		
T	λ	L	T	λ	L
303	23·09	107·5	353	20·28	85·62
363	14·29	85·1	413	13·45	74·09
$m_a = 1{\cdot}62 \times 10^{-24}$ grm.			$m_a = 1{\cdot}62 \times 10^{-24}$ grm.		

Table I contains the values of m_a for the hydrogen atom calculated in the way described from the values of λ for four liquids corresponding to the temperatures given in the table*. It will be seen that the values on the whole agree well with one another, the only serious deviation being shown by that obtained from ether. The mean value is

$$1{\cdot}56 \times 10^{-24} \text{ grm.},$$

which is very nearly equal to $1{\cdot}61 \times 10^{-24}$ grm., the value deduced by Rutherford from experiments on the α particle.

Relations connecting Surface Tension with other Quantities.

A number of relations have been found connecting the surface tension of a liquid with its internal heat of evaporation, coefficient of compression, coefficient of expansion with rise of temperature, and other quantities. They should have as foun-

* The values of L were obtained from a paper by Mills, in the *Journ. of Phy. Chem.* vol. VIII. p. 405 (1904).

dation the relations that can be obtained from the fundamental equations the writer deduced from the law of molecular attraction. Some of these relations will be pointed out here.

From the equations *

$$\lambda = \kappa'' \left(\frac{\rho_1}{m}\right)^2 (\Sigma c_a)^2 \text{ and } L = \frac{B^2}{m}\left(\frac{\rho_1}{m}\right)^{\frac{4}{3}} (\Sigma c_a)^2$$

we have

$$\lambda = D_1 \frac{L\rho_1^{\frac{2}{3}}}{m^{\frac{5}{6}}} \quad\text{.........................(2)},$$

where m denotes the molecular weight relative to hydrogen, and D_1 is a quantity which is the same for all substances at corresponding states.

The equation †

$$p = M^2 \left(\frac{\rho_1}{m}\right)^{\frac{7}{3}} (\Sigma c_a)^2$$

and one of the foregoing equations give

$$\lambda = D_2 p \left(\frac{m}{\rho_1}\right)^{\frac{1}{3}} = D_3 p_c \left(\frac{m}{\rho_c}\right)^{\frac{1}{3}} \quad\text{..................(3)},$$

where D_2 and D_3 are corresponding quantities, p denotes the pressure of the saturated vapour, and the suffix c indicates that the quantity to which it is attached refers to the critical point.

The coefficient of compression β is given by

$$\beta = \frac{1}{v}\frac{dv}{dp} = \frac{a_1}{p},$$

where a_1 is a corresponding quantity, and hence equation (2) may be written

$$\lambda\beta = D_4 \left(\frac{m}{\rho_1}\right)^{\frac{1}{3}} \quad\text{.........................(4)},$$

where D_4 is a corresponding quantity.

The coefficient of expansion α is given by

$$\alpha = \frac{1}{v}\frac{dv}{dp} = \frac{a_2}{T_c}.$$

By means of this equation and the equation

$$p_c = \frac{RT_c\rho_c}{3.7m}$$

equation (3) may be written

$$\alpha\lambda = D_5 \left(\frac{\rho_1}{m}\right)^{\frac{2}{3}} \quad\text{.........................(5)},$$

where D_5 is a corresponding quantity.

* *Phil. Mag.* Oct. 1909, pp. 491—510.
† *Ibid.* Dec. 1909, p. 903.

The different values of each of these corresponding quantities for a number of liquids will probably not differ much from one another for a constant temperature, if it is low, *i.e.* that of the room, since the differences between the values of a corresponding quantity for a constant temperature decrease with decrease of temperature.

A case of special interest is the following. From one of the foregoing equations by differentiation and elimination we obtain $\frac{d\lambda}{dT} = D_6 \frac{\lambda}{T_c}$, where D_6 is a corresponding quantity. This equation may be written $\frac{d\lambda}{dT} = D_4 \lambda\alpha$, by means of the equation giving the coefficient of expansion, where D_7 is a corresponding quantity. Now the surface tension is approximately a linear function of the temperature, and is therefore approximately given by an equation of the form $\lambda = \lambda_0 (1 + \delta_1 T)$, where δ_1 is approximately a constant. The former of the last two equations may therefore be written

$$\frac{\delta_1}{\alpha} = g_1 \quad \text{..............................(6)},$$

where g_1 is a corresponding quantity. Since δ_1 is approximately a constant and α does not vary rapidly with the temperature, g_1 is approximately constant over a considerable range of temperatures. This is the empirical relation connecting δ_1 and α proposed by Cantor*, who showed that $g_1 = 2{\cdot}3$ approximately. Table II

TABLE II.

Name of substance	λ	δ_1	α	g_1
Argon	11·7	0·013	0·00491	2·6
Carbon monoxide ...	13·5	0·013	0·00454	2·9
Cadmium	830	0·00042	0·000170	2·5
Tin......................	610	0·00027	0·000114	2·4
Lead	510	0·00029	0·000129	2·2
Benzene..............	29·4	0·0035	0·00139	2·5
Ethyl alcohol	34·3[6]	0·0027	0·00124	2·2
Phenol	40·6	0·0029	0·00089	3·3
Aniline	45·0	0·0025	0·00092	2·7
Nitro benzene	48·2	0·0028	0·00089	3·1
Carbon disulphide	54·6[6]	0·0029	0·00121	2·0

contains the values of g_1 for a number of substances selected from a table given by Freundlich in his book on *Kapillarchemie*.

* *Zeitschr. f. phys. Chemie*, XXXIX. 129 (1902).

The Influence of the Curvature of Surface of a Liquid on its Surface Tension in Connexion with the Radius of the Sphere of Action of a Molecule.

Since the surface tension of a liquid is due to the existence of molecular forces we should expect that it should depend on the curvature of the liquid surface. But the effect of curvature of surface on the surface tension becomes appreciable only when the radius of curvature becomes comparable with the radius of the sphere of action of a molecule. The latter quantity may be defined as the distance that two molecules in a liquid must be separated in order that the energy expended in overcoming their attraction on one another during the process is approximately equal to that expended in separating them an infinite distance from one another.

Consider a sphere of liquid in the centre of which is a tiny spherical air bubble whose diameter is equal to the radius of the sphere of action of a molecule. Suppose the air bubble increased slightly in size by forcing an additional quantity of air into it. Now it will be easily seen that during the increase of the liquid surface in contact with the bubble the molecules in and near the surface do not get out of each other's range of molecular action for an increase of surface ds to such an extent as if the surface were plane. The amount of work done per unit increase of surface is therefore less than in the case of a plane surface, and the surface tension therefore decreases with increase of the curvature of the surface when it is concave with respect to a point outside the liquid.

In a similar way it can be shown that if the diameter of a sphere of liquid is equal to the radius of the sphere of action of a molecule, less energy would be expended in producing an increase of surface area ds than if the surface were plane. The surface tension will therefore also decrease with increase of curvature of the surface when it is convex. It will be observed that these results should also hold if a liquid is incompressible and no transition layer is formed.

Properties of Plane Liquid Films.

It can be easily shown that the average external work done in evaporating by a reversible process a film of liquid of constant mass, and expanding the vapour till its density is equal to a given density, increases with a decrease of the thickness of the film. Thus let W_1 denote the external work done in one case. Next let the process of evaporation be carried out in a different way. Thus let the area of the film first be considerably increased by stretching it out, and let w_s denote the work done against the

surface tension. Let the film now be evaporated and the vapour expanded or compressed at constant temperature till its density is the same as that of the vapour at the end of the previous process, and let w_v denote the external work done. Now according to thermodynamics the external work done at constant temperature during a reversible process between given limits is independent of the nature of the path of the process. Hence $W_1 = w_v - w_s$, and therefore w_v is greater than W.

It follows therefore that if a film of liquid is gradually stretched out a thickness would ultimately be reached at which the pressure of the saturated vapour begins to increase with decrease of thickness of film. It can be easily shown that this thickness is equal to twice the radius of the sphere of action of a molecule. For if a film has a thickness equal to or less than this quantity it requires the expenditure of less energy to move a molecule to infinity than in the case of a much thicker film. Consequently a larger proportion of molecules will possess sufficient velocity to be able to overcome the molecular attraction and escape from the liquid surface in the former case than in the latter, producing a corresponding difference in the pressures of the saturated vapours. Also the attraction of one half of the film on the other half per cm.^{2} of surface would in such a case be less than the attraction of a large mass of liquid on a cm.^{2} of its surface transition layer. This should have the effect of making the transition layer for the film less abrupt than for a much thicker film. Further, the density of the liquid mid-way between the surfaces of such a film should be less than that in the centre of a large mass of liquid.

An inferior limit of the radius of the sphere of action of a molecule can be obtained. When the surface of a liquid is increased at constant temperature an amount of heat is absorbed equal to $T\frac{d\lambda}{dT}$ per unit increase of surface. Therefore if a c.c. of liquid be stretched out till it consists of a film containing one layer of molecules only the amount of heat absorbed should be practically equal to the internal heat of evaporation $L\rho_1$ of the c.c. of liquid, or

$$\int 2T\frac{d\lambda}{dT}\cdot dA = L\rho_1.$$

It is evident therefore that the surface tension should undergo a change during the process before the area of the film is equal to A, where

$$A2T\frac{d\lambda}{dT} = L\rho_1.$$

If Δ denote the thickness of the film we have $\Delta A = 1$.

Table III contains values of Δ at different temperatures for two liquids. The inferior limit of the radius of the sphere of action of a molecule obtained, which is equal to $\frac{\Delta}{2}$, is of the same order of magnitude as the usually accepted radius of a molecule.

TABLE III.

T	$T\frac{d\lambda}{dT}$	L	Δ	n_c
Ether				
313	35·06	75·36	$3{\cdot}2 \times 10^{-8}$ cm.	1·6
333	36·60	70·79	3·7 ,,	1·7
353	37·43	65·85	4·2 ,,	1·8
373	37·87	60·33	4·9 ,,	1·8
Carbon tetrachloride				
363	40·3	40·62	$3{\cdot}2 \times 10^{-8}$ cm.	1·6
383	41·39	38·64	3·6 ,,	1·7
403	42·33	36·58	4·0 ,,	1·7
423	42·69	24·42	4·5 ,,	1·9

The table also contains the values of the number of layers of molecules n_c in a film of thickness Δ, which is given by

$$n_c = \Delta \left(\frac{\rho_1}{m}\right)^{\frac{1}{3}} + 1.$$

The values are all less than 2, and a film therefore consists of a single layer of molecules when stretched out till its area is equal to A. Now the surface tension must obviously decrease before this stage is reached. Since n_c is calculated on the supposition that $T\frac{d\lambda}{dT}$ is independent of the thickness of film, it follows therefore that the ratio

$$T\frac{d\lambda}{dT}\Big/\lambda$$

on the average increases with a decrease of the thickness of film when it is less than that of twice the radius of the sphere of action of a molecule.

The Polymerization of Molecules in a Substance.

The equations*

$$T_c = H_2^2 \left(\frac{\rho_c}{m}\right)^{\frac{4}{3}} (\Sigma \sqrt{m_1})^2 \quad \dots\dots\dots\dots\dots\dots (7)$$

and

$$p_c = M^2 \left(\frac{\rho_c}{m}\right)^{\frac{7}{6}} (\Sigma \sqrt{m_1})^2 \quad \dots\dots\dots\dots\dots\dots (8)$$

and the relations† connecting the internal heat of evaporation, surface tension, and other quantities with one another, obtained by giving the function (ϕ) in the law of molecular attraction different forms, may be used to investigate whether a substance is polymerized. The quantities T_c, p_c, ρ_c, denote the critical temperature, pressure, and density respectively of a substance of molecular weight m, m_1 denotes the atomic weight of an atom in the molecule and H_2 and M are constants. If a relation does not fit the facts, it signifies that the molecular weight does not correspond to that indicated by the chemical formula, but to some multiple of it. Provided the substance does not consist of a mixture of molecules polymerized to different extents the actual molecular weight can be determined by finding the factor of the chemical molecular weight which gives an agreement with the facts. Oetvos' surface tension equation has been used in this way. By means of it Ramsay and Shields found that water, acetic acid, and the alcohols are more or less polymerized. This may be verified by applying to these substances the latent heat and other relations quoted.

A few additional important cases of polymerization will be pointed out here. By means of Dieterici's equation ‡

$$L = \frac{KRT}{m} \log \frac{\rho_1}{\rho_2},$$

giving the internal heat of evaporation L in terms of the densities ρ_1 and ρ_2 of the liquid and vapour respectively, it can be shown that NH_3 is considerably polymerized in the liquid state. The constant K instead of being equal to 1·75, the value it has for liquids that are not polymerized, varies from 2·6 to 3·9 when the temperature varies from $-10°$ to $40°$ C. It will also be found that the critical quantities of O_2 do not fit in with equation (8), and O_2 is therefore to a certain extent polymerized. Mercury is usually supposed not to be polymerized. It will, however, be found that if the internal heat of evaporation is calculated by means of Clapeyron's thermodynamical equation, and substituted

* *Phil. Mag.* May 1910, pp. 783—809.
† *Ibid.* Jan. 1911, pp. 83—102.
‡ *Ibid.* Oct. 1910, pp. 688, 689.

in the equation $L = E_1(\rho_1^2 - \rho_2^2)$, the value of E_1 is not constant as it should be*, but increases rapidly with the temperature, showing that mercury is in a polymerized state.

A liquid may consist of molecules whose molecular weight is that indicated by the chemical formula, or of molecules polymerized in equal degrees, or of a mixture of molecules polymerized in different degrees. The formulae discussed do not distinguish between the last two cases. It will therefore be of importance to develop one that does.

In a previous paper† it was shown that

$$\frac{\lambda}{\rho_1^2} = \frac{32{\cdot}96}{m^2\rho_c^2}(\Sigma\sqrt{m_1})^2,$$

when the density of the liquid is large in comparison with that of its surrounding vapour, where λ denotes the surface tension. Since $\frac{m}{\rho_c}$ according to Traube is approximately proportional to $\Sigma\sqrt{m_1}$ we may write

$$\frac{m}{\rho_c} = 10\,\Sigma\sqrt{m_1}.$$

Hence the foregoing equation becomes

$$\frac{\lambda}{\rho_1^4} = 3296\left(\frac{\Sigma\sqrt{m_1}}{m}\right)^4 \quad\text{.....................(9).}$$

This equation holds independently of the extent of the polymerization of a substance, provided all the molecules are polymerized to the same extent, in which case m and $\Sigma\sqrt{m_1}$ have the same polymerization factor. Let us apply this equation to the four liquids mentioned in Table IV, which are known not to be polymerized.

TABLE IV.

Name of liquid	$\frac{\lambda}{\rho_1^4}$	$3296\left(\frac{\Sigma\sqrt{m_1}}{m}\right)^4$
Methyl formate	27·53	32·6
Carbon tetrachloride ...	3·99	3·25
Benzene	46·5	45·66
Ether	62·92	66·11

The table contains the values of the right- and left-hand sides of

* *Phil. Mag.* Oct. 1910, p. 687.
† *Ibid.* Jan. 1911, pp. 99—101.

the equation (9) for these liquids, which, it will be seen, agree fairly well with one another, as we should expect. The values of the left-hand side are the mean values given in a table in a previous paper*.

Since equation (9) holds independently of the temperature and the values of the critical quantities of the substances under investigation it can be easily applied to the facts. We may, for example, use it to investigate the polymerization of fused metals and salts. The results of such an investigation are given in

TABLE V.

Symbol of substance	λ	ρ_1	$\frac{\lambda}{\rho_1{}^4}$	$3296\left(\frac{\Sigma\sqrt{m_1}}{m}\right)^4$
Pd	1339	11·4	·07929	·2935
Pt	1658	21·32	·008024	·08670
Hg	435·6	13·55	·01293	·08241
S	42	1·811	3·904	3·22
Se	70·4	4·26	·2138	·5282
Ag	782·4	9·51	·09563	·2825
Bi	381·9	10·04	·03759	·07652
Zn	812·2	6·48	·4606	·7802
Sn	587·1	7·02	·2418	·2374
Sb	244·5	6·41	·1447	·2289
Pb	448	10·37	·03877	·07691
Cd	693·5	7·975	·1713	·2628
Fe	950	6·88	·4239	1·051
Au	612·2	19·23	·004476	·08492
K	363·9	·8298	767·4	2·167
Cu	581	8·217	·1275	·8307
Na	519·7	·9287	699·2	6·232
P	41·15	1·7555	4·340	3·431
KBr	48·4	1·991	3·079	·003787
K_2CO_3	160·2	1·90	12·28	·01654
KCl	69·3	1·45	15·67	·01592
LiCl	63·4	1·375	17·74	·04725
$LiCO_3$	152·5	1·765	15·71	·05360
NaBr	49	2·212	2·046	·005526
$NaCO_3$	179	1·9445	12·52	·02847
NaCl	66·5	1·500	13·13	·03251
Na_2SO_4	182	2·065	10·02	·0115
AgBr	121·4	6·479	·0689	·0009854

* *Loc. cit.*

Table V. It will be seen that in most cases the value of $\frac{\lambda}{\rho_1^{\frac{1}{4}}}$ differs very considerably from that of

$$3296 \left(\frac{\Sigma \sqrt{m_1}}{m}\right)^{\frac{1}{4}},$$

indicating that the fused substances consist of a mixture of molecules polymerized to different extents. We would, of course, expect that the molecular weight of these substances is not that indicated by their chemical formula since fused metals and salts do not possess any appreciable vapour pressure. But this investigation gives additional information. It will be seen that in the case of each of the four substances S, S_n, Cd, and P, each molecule is polymerized to the same extent.

PROCEEDINGS

OF THE

Cambridge Philosophical Society.

The Minerals of some Sands and Gravels near Newmarket. By R. H. Rastall, M.A., Christ's College.

[*Read* 10 February 1913.]

In a recent communication to this Society* a brief account was given of the composition and characters of certain sands and gravels of Pleistocene and Recent age found in the neighbourhood of Cambridge. A few specimens have since been collected from deposits of the same general age in the Newmarket district, and the present paper embodies the results of an examination of these by methods similar to those previously employed, with slight modifications suggested by experience.

The deposits here described are not connected with any streams now existing and must have been formed by rivers belonging to an ancient drainage system which has now totally disappeared, having undergone complete modification by capture or other processes of a like nature. For the present purpose however it is unnecessary to discuss in detail the origin of these deposits; the main object is to place on record their characters and composition for the guidance of future workers. It is hoped that a systematic investigation of the lithological characters of the gravels and sands of this region may eventually succeed in establishing the existence of definite types of deposit which may be correlated with the physical conditions prevailing at different periods in the past.

The deposits here described are referred to very briefly in the publication of the Geological Survey† dealing with this district, and no detailed descriptions seem to exist. When mentioned at

* Rastall, "The Mineral Composition of some Cambridgeshire Sands and Gravels," *Proc. Camb. Phil. Soc.*, vol. XVII. 1913, pp. 132—143.

† "The Geology of parts of Cambridgeshire and Suffolk," *Mem. Geol. Surv.* 1891, p. 72.

all they are generally assumed to be similar to the "Gravels of the Ancient River System" of Cambridge, and the field evidence, such as it is, goes to support this view. In the neighbourhood of Exning and Snailwell the gravels and sands form a ridge, no doubt once continuous, but now cut through in places by existing rivers. This ridge is no doubt analogous to the Quy-Cambridge-Girton ridge, which is similarly cut through by the Cam. I have been unable to find in the scanty literature any mention of the loam or brick-earth here described, although brick-earth is said to exist in some abundance at Bury St Edmunds.

It is unnecessary to describe in detail any of the methods adopted in collecting, preparing and separating the material. These processes were all carried out by the methods mentioned in the paper just cited. The only addition here made was the adoption of a new optical arrangement to facilitate the examination of the separated minerals, which may be briefly described.

The heavy residues from all the sands dealt with contain a considerable proportion of opaque grains, all of which look very similar when examined by transmitted light in the usual way. Hence it became highly desirable to devise some method by which grains of different colours could be readily distinguished. It was important also to make such an arrangement that it was possible to revert to the normal method of examination without disturbance of the microscope or of the slide, enabling any individual grain to be examined by each method alternatively. After some trials it was found that the best results were obtained by strong oblique illumination, and that various colours of background were required for objects of different characters. Thus highly-coloured transparent or semi-transparent minerals are made most conspicuous by a black background, while for completely opaque and metallic grains a partially illuminated background gives the best results. The method finally adopted was as follows: a powerful electric lamp, of 16 or preferably of 32 candle-power, was placed to one side of and as close as possible to the revolving stage of an ordinary Swift petrological microscope, the light being raised very slightly above the plane of the stage. The illumination of the field of view was thus very nearly horizontal. The plane mirror was arranged to reflect a beam of light up the tube in the usual way for examination by transmitted light, with or without crossed nicols. To obtain a dark background all that is necessary is to insert a card of the required shade, black, grey or white, into the space between the lower nicol and the stage. It is convenient to have a single card, half black and half white, or with a grey strip in addition; this card can be moved about as desired for different grains, and it can also have a hole cut in it to be brought to the centre when a solid background is not desired. This card in no

way interferes with the working of the nicols, the revolving stage, or other parts of the microscope and can be left in position permanently. This method of examination is especially useful for the detection of glauconite, magnetite, haematite or any metallic minerals, and it might perhaps be of service in the search for small traces of gold or platinum in sands and gravels and also in the examination of soils for agricultural purposes. If a strong source of artificial light is not available, sunlight acts as well, or even better in the case of minerals of some shades of colour, but unfortunately sunlight is not always obtainable when required. The special advantage claimed for this arrangement depends mainly on the extremely oblique illumination, which reveals clearly pittings and inequalities on the surfaces of the grains, thus bringing out the characteristic appearance. Among transparent minerals it can be usefully employed for the detection of thin flakes of mica, which always lie flat in properly mounted slides. Since the light strikes them edgeways, owing to their thinness they are practically invisible against the dark background, though conspicuous when viewed by transmitted light. This complete disappearance of a mineral may be taken as a proof of its occurrence in very thin flakes. By this means also the colours of very minute grains are made easily visible. This simple apparatus was employed to a considerable extent in the examination of the specimens described in the following pages.

(1) *Pit at Newmarket Station.*

Numerous shallow excavations have recently been opened on the slope of the hill immediately adjoining the north side of Newmarket station, behind the Coronation Hotel, and from these numerous mammalian bones have been obtained. Among the specimens preserved in the Sedgwick Museum, Cambridge, Mr C. E. Gray has identified the following: *Elephas, Hippopotamus, Rhinoceros, Bos, Bison, Ursus* and *Cervus*. The general character of the deposit is best described as brown, rather loamy sands and gravels with many fairly large flints: pebbles of other rocks are extraordinarily rare, only one or two fragments of sandstone being observed.

After washing, the heavy constituents of the sand were concentrated by panning and separated directly in bromoform; a large and very ferruginous residue was obtained, necessitating prolonged digestion in strong hydrochloric acid. It then appeared that a considerable proportion of the grains that sank in bromoform consisted of glauconite of various shades of brown, green and bluish green, coated over with a skin of brown iron oxide. These were so abundant that a second separation was necessary.

The final residue was nevertheless large in amount and contained a considerable variety of minerals. Black grains of magnetite are very abundant, and the chief transparent minerals found are zircon, rutile, garnet, tourmaline, staurolite, hornblende, augite and probably spinel. Zircons are very common and rather large: sometimes idiomorphic, but more commonly well rounded; unusually so for such a hard mineral. Rutile is also very plentiful in prismatic crystals and in grains varying in colour from yellow to deep red. It is possible that some of the deep red grains here assigned to rutile may really be cassiterite: this would be difficult to prove. Pale pink and brown garnets are common in well rounded grains rather larger than the average size. There are also a few opaque white, yellow and brown grains of indeterminable nature, and a very little opaque red iron oxide, probably haematite. Kyanite is very rare. When examined on a dark background the most striking feature of the sample is the abundance of magnetite and rutile, and its general appearance is very characteristic.

(2) *Dane Hill, Kennett.*

At this locality, which is about four miles north-east of Newmarket, there are curious patches of marl or loam resting on gravel. The marl is very pale grey in colour, weathering pale yellowish brown. It is very fine in texture and soft, disintegrating readily when placed in water. By repeated washings much fine mud can be removed, leaving a residue of pale brownish sand, with small chips of flint and some white mica. It is difficult to get rid of the last residue of the mud by washing, and this is important, since separation of heavy minerals is prevented by the presence of mud. However addition of a little acid facilitates the process, and produces a copious effervescence, thus justifying the designation of marl employed above. Larger elements which fail to pass the first sieve are small in amount, consisting of pinkish quartz and grains of iron oxide, together with chips of pale blue flint.

The portion sinking in bromoform also for the most part consists of iron oxide in rounded and curiously polished grains. After further treatment with acid these are removed, and all that remains is a very few rounded grains of garnet, together with still fewer crystals of hornblende, zircon and rutile. These grains are not by any means exceptionally small: in fact they are considerably larger than in the sands from Newmarket station and from the cemetery pit at Exning. This fact probably has some bearing on the origin of the marls, as will be explained more fully in the case of the Exning deposits.

(3) *Cemetery Pit, Exning.*

A large pit adjoining the cemetery at Exning now shows a good exposure of sands and gravels, apparently lying in a pocket in the Chalk. It is possible that sands and gravels of two different ages are here seen in juxtaposition, since a good deal of variation is to be observed in different parts of the pit. The eastern end is excavated in brown, highly ferruginous, sand with wisps of chalky clay and many blue flints. The north wall of the same pit consists of a lighter coloured sand, with much more Chalk and at the top the usual pipes filled with brown loamy material. All over the pit large flints are numerous, but judging from a careful examination of the piles of stones riddled out from the gravel, pebbles of distant origin are very rare; only a few blocks of sandstone were seen, ranging up to 6 inches in diameter: no pebbles of igneous rocks could be found.

Since some doubt exists as to the identity of the sands seen in various parts of the pit, samples were collected from different points and treated separately by the usual methods.

(*a*) Eastern side of pit. This is a pale brown, clean, bright sand, of rather fine grain, with few stones: when washed it was found to be remarkably free from mud. The heavy portion was partially concentrated by panning, and being obviously ferruginous it was at once treated with acid, yielding a remarkably white sand with much glauconite. A fairly abundant heavy residue sank in bromoform.

This residue consists of rather small grains which are as a rule distinctly rounded: the chief minerals present are iron oxides, brown tourmaline, abundant zircons, rutile, epidote, staurolite, hornblende in rounded grains and ragged fragments, together with kyanite and garnet, the last-named mineral being fairly abundant and in grains distinctly larger than the rest, suggesting a somewhat different origin. The zircons are notably abundant and more rounded than in samples from the neighbourhood of Cambridge.

(*b*) Northern side of pit. This specimen consists of a rather light coloured fine grained quartz sand with fairly abundant dark grains and many very small chips of flint. A small sample when tested with acid gave a strong effervescence. The heavy constituents were partially concentrated by panning and then separated in bromoform without previous treatment with acid. Grains of iron oxide were found to be so abundant as to necessitate treatment with acid and a second separation in bromoform. This process brought to light a considerable amount of green glauconite and some pale brown grains of doubtful character. The heavy residue after the second separation was abundant and consisted of unusually small grains. Its general mineral composition also was

very similar to that from Newmarket station, and unlike the sands near Cambridge. The chief minerals identified are zircon, rutile, garnet, hornblende, augite, epidote and staurolite. Tourmaline is notably scarce and kyanite was not found.

The grains of garnet, though larger than most of the other minerals, are still small and generally a good deal rounded, while the crystals of epidote, of a bright yellowish green, are conspicuously round. The hornblende includes not only the common green variety, but also a blue form, referable to arfvedsonite. The most notable feature of the specimen is certainly the extraordinary abundance of zircon and the extreme rounding of many of the crystals of this very hard and resistant mineral, indicating prolonged rolling, or derivation in a rolled condition from some older rock. The entire absence of kyanite and rarity of tourmaline are also notable and difficult to explain, since both these minerals are so abundant in the sands of the same general age near Cambridge and both occur with fair frequency in the sample of sand taken from another part of the same pit. This fact goes to confirm the idea that sands of two different origins and ages are exposed in this pit.

(4) *Pit near Exning Village.*

This pit is situated a short distance from the village of Exning, on the road to Snailwell, and lies on fairly high ground. It shows a good exposure of material of peculiar character, which is best described as brown and grey loam or marl, or the somewhat vague designation of *brick-earth* might well be applied to it. Although the material is of very fine texture it contains a few large flints, those of an elongated form often lying with their long axes vertical. There are also well rounded boulders of sand up to a foot in diameter embedded in the marl. The occurrence of the vertical flints and of the boulders of sand suggest deposition from floating ice in comparatively still water. It is difficult to account for the sand boulders unless they were frozen when deposited in their present position, as they are well rounded and with sharp outlines clearly demarcated from the fine marl.

Two large samples were taken from this pit, one of a brown loamy material and the other of a fine greyish-yellow sandy marl. The treatment of these samples presented considerable difficulties, owing to the large amount of fine mud. However by long-continued washing in a large porcelain dish the mud was eventually got rid of, and the sandy residue could be treated in the usual way. Naturally this residue was of unusually fine grain, and did not submit to the operation of panning so readily as the coarser sands. In the case of the fine marl in particular the amount of heavy minerals ultimately obtained was very small.

(a) Brown loam. In this specimen all the grains are rather small and even in size; a few garnets only being somewhat larger than the average. Among the opaque constituents magnetite is moderately abundant and there are a few grains of red iron oxide. Among the transparent constituents zircon and rutile are very abundant, both in sharply angular and well rounded crystals. Hornblende, epidote and staurolite were also noted, while flakes of muscovite are abundant. Tourmaline is scarce, while kyanite is very rare indeed. Garnet occurs in angular chips and also in rounded grains.

The notable features here are the abundance of zircon and rutile and the rarity of kyanite and tourmaline, together with the presence of abundant muscovite.

(b) Grey marl. In this specimen the grains are all very small indeed, and of uniform size. Magnetite is not very abundant and there are a few pale yellow and white opaque grains. Minute flakes of muscovite occur in great profusion, the other notable minerals being zircon and rutile: the crystals of both are for the most part sharply angular, though a few are rounded. Other minerals are in such minute quantity as scarcely to be worth mention. Allowing for the difference of size of grains this specimen agrees very closely with the last.

General Conclusions.

Although the specimens here described are few in number, and show a certain amount of variation among themselves, they all possess certain characters in common, with the exception of the Dane Hill marl, which is quite exceptional. In all the others the most notable heavy minerals are zircon and rutile, while kyanite, staurolite and tourmaline, so abundant in the gravels near Cambridge, are notably rare or even completely absent. In the loam and marl of the Snailwell road pit at Exning muscovite is also very abundant.

From the general mineralogical composition of the sands and gravels here examined it may be inferred that the greater part of the material has been derived from distant sources by glacial transport, minerals obtained at second hand from the Neocomian series being quite subordinate in amount, and notably in less proportion than in the gravels near Cambridge. Not enough evidence is yet available to enable any explanation of the causes of this difference to be offered, and more work is required on this subject.

As for the actual manner of formation of these deposits, the coarser types appear to be due to rapidly moving water, either the waters of ancient rivers, or possibly in some cases fluvio-glacial, while as before stated the fine marls have possibly been deposited in still or slowly moving water in which ice was floating.

Observations on Polyporus squamosus, *Huds.* Preliminary Communication. By S. REGINALD PRICE, B.A., Clare College (Frank Smart Student in Botany in the University).

[*Read* 10 February 1913.]

Polyporus squamosus, the "Saddle-back Fungus" or "Dryad's Saddle," is well known as a parasite upon living trees and a powerful timber destroyer. Perhaps the most important work which has been done on the biology of this species is detailed in the papers published by Prof. A. H. R. Buller* and by Mr F. T. Brooks†.

There were certain features, however, with regard to its biology which merited further investigation, and the object of the work here summarised was to attempt to throw further light on the general life-history of the species.

The fungus has never before been grown on wood in artificial culture. Buller obtained some cultures on gelatine media, but did not get infection of wood to take place.

By making use of the property of the species of forming a spore cloud, as described by Brooks and Buller in the papers mentioned above‡, it was possible to collect the spores in large quantities and in an uncontaminated condition. For the culture, the usual methods, as described by Marshall Ward§ for *Stereum hirsutum* and Brooks|| for *Stereum purpureum*, were employed. The spores were placed on the surfaces of sterilised wood blocks contained in wide plugged test-tubes, the wood being kept moist by a pad of cotton-wool saturated with water.

Two months or more elapsed before any sign of mycelium made its appearance, after which it gradually spread over the surface. Generally the mycelium in older cultures formed a white felt-like mass of very fine hyphae covering the surface of the block. Later this became brown in colour and numerous "oidia" were found to be produced. Drops of a brown watery liquid also exuded from the mycelium at about this stage.

Spore cultures were made on elm wood; further cultures were made by mycelial transfer in many cases. The mycelium grew on blocks of the wood of the elm, lime, sycamore, horse-chestnut, and very feebly on that of *Pinus*.

* Buller, A. H. R. "The Biology of *Polyporus squamosus* Huds., a timber destroying Fungus," *Journ. Econ. Biol.*, 1906, vol. I. p. 101.

† Brooks, F. T. "Notes on *Polyporus squamosus*," *New Phyt.* VIII. 1909, p. 348.

‡ *Vide* also Buller, *Researches on Fungi*.

§ Ward, H. Marshall. "On the Biology of *Stereum hirsutum*," *Phil. Trans. Roy. Soc.* 1898.

|| Brooks, F. T. "Silver-leaf Disease," *Journ. Agric. Sci.*, vol. IV. p. 138.

Ultimately peculiar fructifications, which were, however, always sterile, and showed no signs of basidia formation, were produced in many cases, so far as present observations go always on elm wood. They took the form of rather elongated stipes with no indication of a pileus. On the whole they resembled the natural fructifications, obtained by Buller, from logs filled with the mycelium, which were kept in darkness*. Some were unbranched, others freely branched and tree-like.

Cultures kept in darkness usually developed a more abundant mycelium than those in light, but never produced any fructifications.

The mycelium penetrated the wood comparatively slowly, and except for the outer layer the blocks remained quite hard even after twelve months' action of the mycelium. This is very probably a cultural effect as the decay is certainly much more rapid in nature.

Attempts were made to grow the mycelium on a decoction of elm wood extract solidified with gelatine and also on agar containing the same decoction. The growth was slow and feeble on both these media.

The spores germinated in hanging drops of solutions rich in nitrogenous substances, as Buller found†. To follow out the manner of the germination of the spores on wood, and the penetration of the hyphae, numerous spores were placed on small sterile blocks of wood. The germination was slow, but the penetration of the hyphae could be traced after four to six weeks.

Inoculations were also made on living elm trees, using both mycelium and spores, on new and old wound surfaces. The mycelium again penetrated slowly as determined by cutting sections in the region of inoculation. Infection seemed to take place more readily in autumn and winter than in spring and summer, while old wound surfaces and especially dead twigs were easily attacked.

This last point is probably definitely connected with the slow germination of the spores and the relative activity of the region on which they are placed.

The work was suggested to me by Mr F. T. Brooks, to whom I wish to record my best thanks for his constant interest and advice.

* Buller, A. H. R. *Researches on Fungi*, p. 59 *sqq.*

† Buller, "Biology of *P. squamosus*," *loc. cit.*, p. 116.

Note on the respiratory movements of Torpedo ocellata. By
G. R. MINES, M.A., Fellow of Sidney Sussex College. (From the Stazione Zoologica, Naples.)

[*Read* 10 February 1913.]

(Plates III and IV.)

THE researches of Bethe, Rynberk, Baglioni and others have shown that in a great variety of fishes, both bony and cartilaginous, the rhythm of the respiratory movements is very closely dependent on peripheral stimuli. It has been remarked that the sequence of regular rhythmic movements which are ordinarily called the respiratory movements, is liable to be interrupted by movements of a different type, in which the cavity of the pharynx is opened more widely than usual and then forcibly contracted, driving the water out through channels which ordinarily conduct an inflowing stream. These movements can be elicited with great ease by the introduction of any foreign solid object into the mouth cavity, and also by the introduction of bubbles of air: they occur when a fish is removed from water, and are called variously gasping movements, *Auspeireflexe* or, in the case of elasmobranchs, *Spritzreflexe*.

Baglioni* remarks that these spouting movements may be seen occasionally if one watches a fish kept in a tank under conditions as nearly as possible normal. He attributes their occurrence to the entry into the mouth cavity of some foreign particle with the inspired water, such as a grain of sand or a shred of mucus: such objects are often to be seen in the jet of water shot out by the Auspeireflex. Baglioni thus regards the sporadic appearance of these movements as being conditioned by the chance occurrence of some slight external stimulus. He considers them equivalent to a cough or a sneeze.

While it is undoubtedly true that spouting movements can be elicited by very small external stimuli, I have made some observations which seem to me to point to a tendency on the part of the nerve cells concerned in the reflex to discharge at rhythmic intervals which are not correlated with rhythmic external stimuli. The development of rhythms of long period in nerve cells is a matter of interest from several points of view, and I have therefore thought it worth while to give a brief account of these observations, which were made incidentally in the course of another enquiry as yet unfinished.

* *Z. f. allg. Physiol.* VII. p. 177, 1907.

Methods. In the study of a series of rhythmic movements three points may receive attention: these are the frequency, the amplitude and the form of the movements. The first may be determined with sufficient accuracy by simple observation, but for the study of the second and third of these factors a graphic record is required. For the accurate study of the form of the movements the recording surface must move fairly rapidly, that is to say at the rate of some centimetres per second. From such a record all the information required as to frequency and amplitude could be obtained, but only as the result of examining and measuring many metres of tracing, even in an experiment of moderate duration. As a matter of fact, in the case of respiratory movements, the factors of greatest interest are the frequency and amplitude of the movements; it is rare to find changes in form unaccompanied by any change either in frequency or in amplitude. For the investigation of frequency and amplitude the most usual practice is to take a tracing on a drum travelling at such a rate that the movements are just distinctly separated. Such a record shows at once the amplitude, and on counting and comparing with a time record it gives the frequency. Anyone who has had the task of counting up the number of movements per minute from such a tracing knows that it is exceedingly tiresome. The tracings moreover are very cumbersome if the experiments are prolonged, and mere inspection gives no indication of any but relatively large changes in frequency. Following on the work of Marey, Noguès* has elaborated an apparatus known as an odograph for the investigation of such series of rhythmic movements. The frequency of a rhythm studied by this apparatus is indicated by the inclination of a continuous line to the horizontal axis. If the rhythmic movements continue at a uniform rate, the line ascends regularly: if they accelerate, the line goes up more steeply, if they retard, less steeply. In the course of an experiment, it may be of several hours' duration, the frequency recorder traces a single oblique line.

The method which I have used differs from that of Noguès in that the counting mechanism records each minute by a separate vertical line the number of contractions which have taken place in a predetermined interval of time. It is arranged as follows. The lever shown in Fig. 1 is used to record the amplitude of the movements, in the usual way. The recording lever is pivoted at *A*. At *B* a second lever is pivoted, the screws supporting it being insulated from the brass holder. This second lever consists of a wire bent to the form shown. On its axis there is pressed a piece of hair-spring which introduces sufficient resistance to

* *Trav. Inst. Marey*, II. p. 31, 1910.

prevent this little lever from falling by its own weight, and at the same time brings it into electrical connexion with the insulated binding screw *C*. The extremity of the second lever is bent back on itself so as to form a pair of jaws which pass on either side of the first lever, about 15 mm. from its axis. To the upper jaw is fastened a small piece of platinum wire, which can make contact with another piece of platinum wire fastened to the main lever. The under jaw is prevented from making contact with the first lever by a thin piece of mica. The amount of play between the recording lever and the jaws of the second lever is made as small as possible consistent with the making and breaking of contact with each up and down movement of the levers. This arrangement provides for a rubbing contact between the platinum pieces. *D* is a stop.

During the ascent of the recording lever, electrical contact is maintained between the binding screw *C* and the brass support; during the descent of the lever, the contact is broken. This holds good for a wide range of positions of the recording lever, and the arrangement will work even when the amplitude of the excursions is very small: it is limited, of course, by the width of the gap between the jaws of the second lever. This lever is connected to a source of current of about 4 volts and to the coils of a light relay. To obviate any sparking at the contacts a shunt may be introduced. The intermittent movements of the relay actuate another circuit which controls the apparatus shown in Fig. 2.

This consists of a toothed wheel (a small circular saw was used) which is moved on, one tooth at a time, by a ratchet actuated by an electric magnet. A difficulty in the construction of an electric counter is that the movements of the armature of the magnet are apt to be so abrupt that the wheel may be shot on more than one step at a time. This difficulty is overcome by the use of a glycerine brake, which is seen below the toothed wheel. The ratchet actuating the wheel and that preventing it from slipping back, are placed near together on the periphery of the wheel. A second electro-magnet *E* is so arranged, that when it is energised both these ratchets are pulled away from the toothed wheel, so that it is free to move. On the same axis as the toothed wheel is a small grooved pulley, on which is wound a piece of silk thread. The other end of the thread is attached to a suitably weighted lever. The wheel is furnished with a stop so that when the magnet *E* is energised, the wheel and the lever return to a definite zero position. The magnet *E* is excited for a period of eight seconds once a minute, the circuit being made by a wire connected to the second-hand of an ordinary clock dipping into a pool of mercury contained in a cavity in a paraffin block on which the clock stands.

The lever connected with the wheel is arranged to write under that recording the amplitude of the movements. It gives a record showing the number of movements which have occurred during a period of 52 seconds in each minute and at the same time provides a time tracing in minutes by which the upper tracing may be read.

The general arrangement of the apparatus is shown in Fig. 3. Records are taken on a drum moving about 10 cms. per hour, and thus quite a short tracing shows at a glance the frequency and amplitude of the movements over periods of several hours. Two sources of instrumental error must be noted, though they do not affect any of the conclusions to be drawn from the experiments to be described. In the first place variations in frequency within the interval of 52 seconds would escape notice in the record if they chanced to include an acceleration and a retardation which exactly balanced one another. In the second place, when the frequency of the movements is quite regular, there is liable to be a variation of one in the record from minute to minute, since the number of movements will not as a rule be an exact sub-multiple of the number of seconds during which they are counted.

Examples of the records obtained are shown in Figs. 4 and 5. The method is obviously suitable for the study of the heart beat or for counting drops.

Experiments were made on moderate sized specimens of *Torpedo ocellata.* The fish was held lying with the ventral surface uppermost by means of von Uexküll's device. A gentle stream of sea-water was led into the mouth by a rubber tube. The fish was placed in a small tank, provided with an overflow pipe, and as a rule was covered with water. The recording lever was connected by a thread to one of the gill clefts.

The temperature of the water varied in different experiments from 18° to 22° C.

Observations. After the irregularity caused by the manipulation involved in fixing the fish in position, the respiratory movements became regular, and often continued so with little change in amplitude or frequency for several hours. During this period it was very usually found that the fish made occasional spouting movements.

The graphic records showed that these spouting movements tended to recur at fairly regular intervals.

Fig. 4 is an example taken from the close of a period of two hours during which the behaviour had been practically the same the whole time. It is to be noted that the spouting movements do not occur at exactly equal intervals, yet sufficiently nearly so to suggest a slightly-distorted rhythm. The period of this rhythm was sometimes as great as 5 minutes—in other cases the movements recurred several times in a minute.

The frequency of the spouting movements was greatly influenced by changes in the mechanical conditions. The effect of an increased flow of water through the pharynx always caused an increase in the frequency of the ordinary respiratory movements with a decrease in the frequency of the spouting movements. These points are illustrated by Fig. 5 (*a*) and (*b*). The tracings are from different experiments.

In (*a*) the flow of water through the pharynx was reduced during the times indicated by the white horizontal lines. In (*b*) the horizontal line indicates an *increase* in the rate of flow.

The frequency scale at the side applies to both tracings.

I have not found any definite relationship between the chemical composition of the water and the frequency of the rhythm. A considerable increase in the carbon dioxide tension, changing the hydrogen ion concentration of the sea-water from about $10^{-8 \cdot 3}$ to 10^{-6} caused violent movements and upset the rhythm altogether. In one case increase in the oxygen tension of the water was accompanied by cessation of the spouting movements which returned when the oxygen tension was restored to its original value. This result was not confirmed on repetition with other specimens. Further work on the subject is needed.

The object of the present communication is to point out that these spouting movements do often occur in the resting fish, separated by intervals so nearly alike as to make it improbable that the occurrence of each movement is the result of a fortuitous external stimulus. External stimuli will readily affect the reflex mechanism, but it appears that when external conditions are kept as uniform as possible the periodicity of the spouting movements is determined by the central nervous system.

I wish to express my warmest thanks to Doctor Dohrn and his Staff for the kindness with which they aided my work during my tenure of the University Table at the Stazione Zoologica in the autumn of 1912.

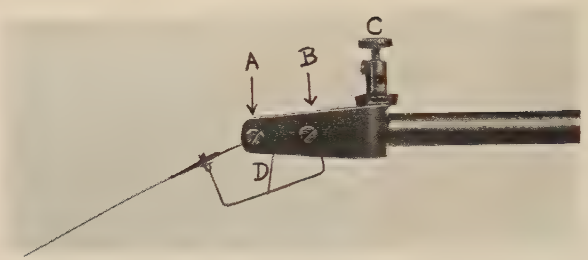


FIG. 1.

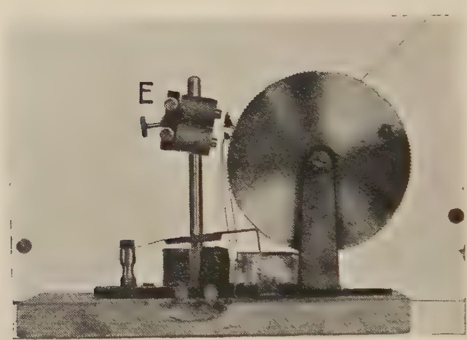


FIG. 2.

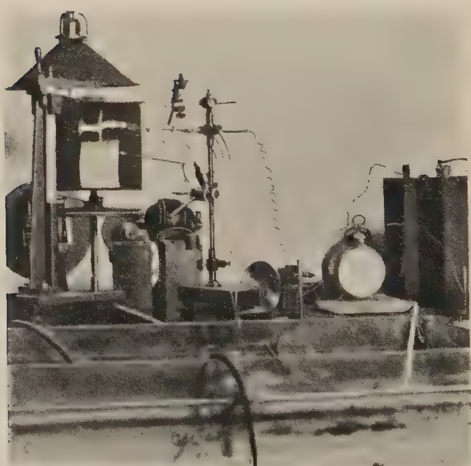
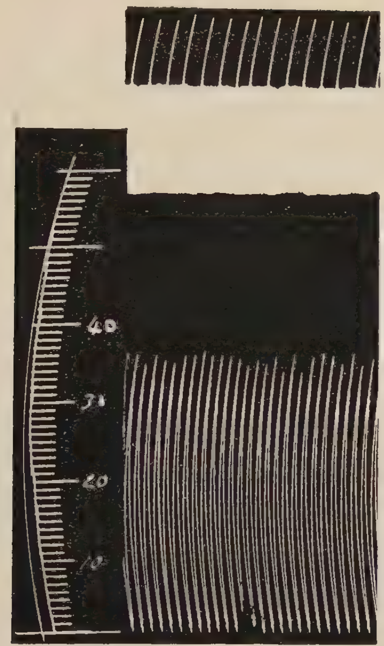


FIG. 3.



Amplitude

Frequency

Fig. 4.

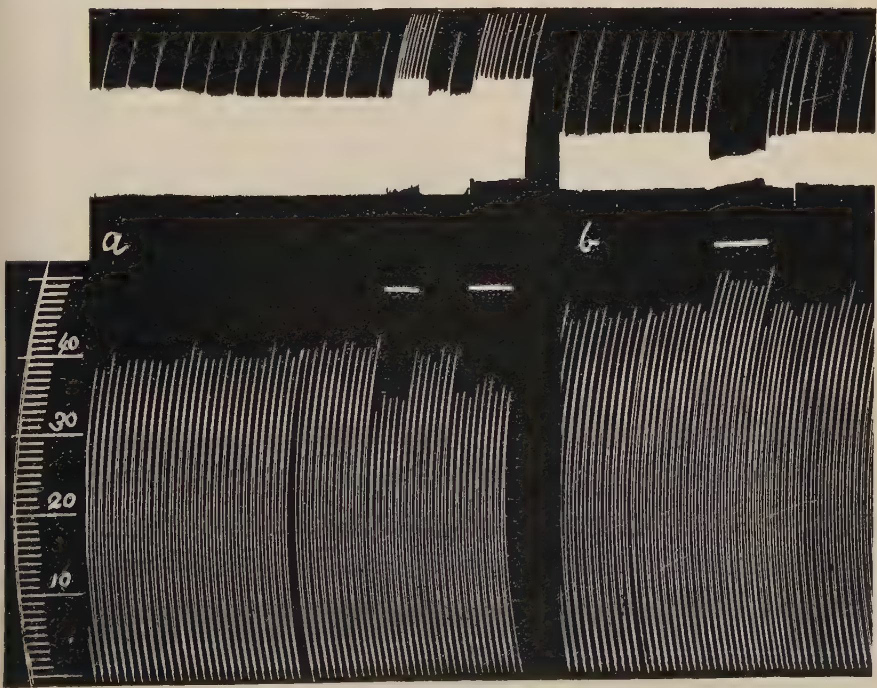


Fig. 5.

The Atomic Constants and the Properties of Substances.

By R. D. KLEEMAN, B.A., Emmanuel College, D.Sc. (Adelaide).

[*Read* 27 January 1913.]

THE quantity $\frac{m}{\rho}$ of a substance, where ρ denotes its density and m the molecular weight of a molecule relative to that of the hydrogen atom, is the volume of a gram molecule of the substance, and accordingly proportional to the molecular volume of a molecule. It has long been known that it is approximately an additive quantity of the atom for substances at their boiling points at atmospheric pressure. It is evident, however, from the investigations of the law of corresponding states in connexion with the law of molecular attraction by the writer, that the molecular volumes obtained for corresponding states are likely to be of greater value and importance. Table I contains the values of $\frac{m}{\rho}$ for a number

TABLE I.

Values of c_v.

$$H = 1, \quad C = 2{\cdot}034, \quad O = 2{\cdot}298, \quad F = 2{\cdot}098, \quad Cl = 4{\cdot}105,$$
$$Br = 5{\cdot}805, \quad I = 7{\cdot}734, \quad Sn = 8{\cdot}59.$$

Name of substance	$\frac{m}{\rho_c}$	$14{\cdot}06\Sigma c_v$	Name of substance	$\frac{m}{\rho_c}$	$14{\cdot}06\Sigma c_v$
Di-isopropyl	356·7	368·4	Stannic Chloride	351·6	351·6
Di-isobutyl	482·2	482·2	Benzene	256·2	256·2
Pentane	310·3	311·7	Ether	284·1	287·3
Isopentane	307·7	311·7	Propyl formate	288·5	291·5
Hexane	367·5	368·4	Propyl acetate	344·9	348·2
Heptane	427·6	425·1	Carbon dioxide	94·8	93·2
Octane	490·4	482·2	Ethyl butyrate	420·4	404·9
Fluor benzene	271·4	271·4	Methyl acetate	231·3	234·8
Bromo benzene	323·5	235·5	Carbon tetrachloride	276·2	259·4
Iodo benzene	350·6	350·6	Methyl formate	172·0	178·0
Chloro benzene	307·8	299·6	Hydrochloric acid	61·4	73·7
Hexamethylene	307·5	340·3			

of substances at the critical point—the most important corresponding point—and the values of the molecular volumes calculated from the expression $K\Sigma_{c_v}$, where c_v denotes the apparent volume of an atom relative to that of a hydrogen atom. The values of c_v are given at the top of Table I. Those of the H and C were obtained from the two substances di-isobutyl and benzene, while the values for the other atoms are the mean values obtained from the remaining substances. K has been put equal to 14·06. A fair agreement between calculation and experiment is obtained.

There is no obvious simple reason why the apparent molecular volume of a molecule should be an additive property of its atoms, since an increase in the volume of a substance is not attended by an equal increase in the actual space occupied by the molecules. The molecular volume in the case of a liquid or vapour, it may be pointed out here, is really the outcome of the equilibrium between the intrinsic pressure due to the attraction between the molecules, and the pressure exerted by the molecules due to their motion of translation. We might therefore expect that the constant c_v and the atomic attraction constant c_a should be connected with one another. Thus the writer has shown that the attraction constant of an atom is proportional to $\sqrt{m_1}$, where m_1 denotes its atomic weight. Traube has shown that the atomic volume of an atom at the absolute zero, which according to the law of corresponding states is proportional to that at the critical point, is proportional to $\sqrt{m_1}$. But the values of the constants c_a and c_v (the average values determined from the facts) agree on the whole better with the facts than the constant $\sqrt{m_1}$, as might be expected.

The factors of $\sqrt{m_1}$ that have to be introduced in both cases to obtain a better agreement with the facts are of interest. In the case of the constant c_a we may accordingly write $c_a = \gamma_1 \sqrt{m_1}$. The values of γ_1 for a number of atoms corresponding to the values of c_a deduced from the internal heat of evaporation* are given in Table II, the value for the carbon atom being put equal to unity. It is very probable that γ_1 depends on the chemical constitution of the molecule in which the atom occurs, and should therefore strictly not be given the same value for each substance. Probably it will be found when more extensive data are available that it has the same value for all substances belonging to the same chemical group. Further investigations of the properties of the attraction constants in the law of molecular attraction along these lines will probably lead to interesting results.

In the case of the constant c_v we may similarly write $c_v = \gamma_2 \sqrt{m_1}$. The values of γ_2 for a number of atoms are given in Table II, the value for the carbon atom as before being put equal

* *Phil. Mag.*, Oct. 1909, p. 507.

to unity. An inspection of the table will show that the greatest deviation from the square root law in the case of both the quantities c_a and c_v is shown by the hydrogen atom. The deviations are in opposite directions. This adds another anomaly to the large number for which hydrogen is already famous.

TABLE II.

Atomic symbol	γ_1	γ_2	Atomic symbol	γ_1	γ_2
H	·653	1·70	Cl	·920	1·171
C	1·000	1·00	Br	·778	1·105
O	·974	·977	Sn	·882	1·346
F	·863	·819	I	·897	1·168

The writer has deduced two fundamental relations from the law of molecular attraction and the laws of thermodynamics*. These equations may now be written

$$p = M_1^2 \frac{(\Sigma_{c_a})^2}{(\Sigma_{c_v})^{\frac{7}{3}}} \quad \text{.........................(1)},$$

$$T = M_2^2 \frac{(\Sigma_{c_a})^2}{(\Sigma_{c_v})^{\frac{7}{3}}} \quad \text{.........................(2)},$$

where p denotes the pressure of the substance at the temperature T, and M_1^2 and M_2^2 are quantities which have the same values for all substances at corresponding states. Thus if the chemical formula of a substance be known the critical constants can approximately be calculated by means of the atomic constants c_a and c_v. The values of T_c for a few substances were calculated in this way by means of equation (2) and are contained in Table III, putting for M_2^2 its mean value 17·69. The agreement between calculation and experiment is not quite so good as obtained in Table I, due probably to the fact that higher powers of Σ_{c_a} and Σ_{c_v} are involved which increases the effect of the errors in their values on the value of T_c. But still the formulae should be useful in obtaining approximately the values of the critical quantities.

They may be of use in chemical investigations, especially when the properties of a new compound are being investigated whose chemical formula can only be conjectured. Thus if the critical constants of a hypothetical substance be calculated, its pressure,

* *Phil. Mag.*, Oct. 1909, p. 509, and Dec. 1909, p. 903.

TABLE III.

Values of c_a.

H = 1, C = 5·30, O = 5·94, F = 5·76, Cl = 8·40, Br = 10·65, Sn = 14·68, I = 15·49.

Name of substance	Σc_a	Σc_v	T_c (Exp.)	T_c (Cal.)
Chloro benzene	45·2	21·31	633	620·7
Octane	60·4	34·27	569·2	588·4
Ether	37·2	20·43	467·4	442·7
Carbon dioxide	17·2	6·63	304·3	426·5
Methane	9·3	6·03	191	141·6
Hydrogen	2	2	38·5	28·5
Ethylene	14·6	8·07	282	290·7
Carbon tetrachloride	38·9	18·45	556·1	557·2
Pentane	38·5	22·17	470·3	427·4
Heptane	53·1	30·23	539·0	537·8
Di-phenyl methane	73·6	34·41	768·6	869·2

and density of liquid and saturated vapour, can be obtained for any temperature from those of a known substance by means of the law of corresponding states. By comparing these quantities with those found by experiment useful information for the guidance of further experiments is obtained. As an example of the application of these principles let us calculate the critical and other quantities of the substance O_5, which Ladenburg believes is produced in a vacuum tube through which an electric charge passes, supposing that it exists as a pure substance, that is, not as a mixture of substances whose formulae are of the type O_n. The critical density, temperature, and pressure in atmospheres are found to be ·495, 603, and 83·3 respectively. At a temperature of 16° C. (room temperature) the substance would have a vapour pressure of 30·7 mm. of mercury, and the temperature of its boiling point would be 134° C.

Useful information may also be obtained by means of the foregoing equations about the purity of a substance, that is, whether or no it consists of a mixture of two or more substances, a special case of which is partial polymerization of the molecules. For example, the values of c_a and c_v for an atom of copper, deduced by interpolation from the values of c_a and c_v for a number of atoms given in Tables I and III are 9·81 and 5·18 respectively. The critical temperature of liquid copper should therefore be 190° C.

Since, however, it is undoubtedly enormously higher, it follows that copper in the solid and molten state consists in practice of molecules having the formula Cn_n, where n is probably quite large. The same conclusions can be arrived at in respect to all the other metals, and a large number of chemical compounds such as the various salts etc.

It will be apparent from equation (2) that the value of T_c increases with that of Σ_{c_a}. Therefore the partial polymerization of a liquid should have the effect of raising its critical temperature. An important case in point is water, whose critical temperature would be 159·5° C. instead of 631° C., if the chemical formula for each molecule were H_2O. If each molecule were polymerized to the same extent its chemical formula would be 7·8 (H_2O). It appears, however, from surface tension considerations that water consists of molecules polymerized to different extents. The molecular weight of some of the molecules must thus be greater than that according to the above formula. It may be pointed out here that the determination of the extent of polymerization of a liquid from Oetvos' surface tension equation cannot lead to very accurate results since it is tacitly assumed that the critical temperature is not influenced by polymerization.

Another interesting case in this connexion is the molecular weight of liquid mercury. If we take c_a and c_v equal to $\sqrt{m_1}$, which we have seen is approximately the case, we obtain 604·9° C. for the critical temperature. But it is undoubtedly much higher, from which it follows that mercury must be partially polymerized. This we have shown to be the case by a different method in a previous paper*.

* *Proc. Camb. Phil. Soc.*, vol. XVII. p. 157.

Note on the effect of heating paraformaldehyde with a trace of sulphuric acid. By J. G. M. DUNLOP, M.A., Gonville and Caius College.

[*Read* 24 February 1913.]

PRATESI (*Gaz.* XIV. 139) described a polymeric variety of formaldehyde, soluble in alcohol and other solvents, and to which he gave the name α trioxymethylene. This he prepared by heating paraformaldehyde (trioxymethylene) with a trace of concentrated sulphuric acid for some hours in a sealed tube at 115° C. The α trioxymethylene was described as condensing in the cooler portion of the tube in long needles.

The present author, having occasion to require a strong solution of formaldehyde in alcohol (in which paraformaldehyde is insoluble), began to prepare α trioxymethylene in this way.

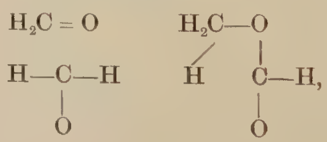
For convenience it was thought desirable to use a tube about 60 cm. long, bent in the middle into a right angle, so that the end containing the paraformaldehyde and sulphuric acid could be heated in a tube furnace, the α trioxymethylene being condensed in the other limb, which was immersed in a beaker of water.

The distillate was found to consist of a very mobile liquid, which when distilled gave two fractions, one distilling at 33° C. and the other at about 100° C.

The more volatile fraction had an ethereal odour, and did not react with sodium bisulphite. On investigation it was found to be methyl formate, and this was confirmed by comparison with a specimen of this ester, the boiling point and density being identical. On hydrolysis with moist lead oxide it yielded lead formate and methyl alcohol.

The yield of methyl formate is very variable, and depends on the amount of sulphuric acid and also on the temperature. With about six drops of acid to ten grams of trioxymethylene, a yield of about one to two grams of ester appears to be usual. In an experiment in which about five grams of acid to ten grams of trioxymethylene were taken, great charring took place, and practically no ester was formed.

The reaction appears to take place by an intermolecular exchange of linkages thus:



i.e. H—$COOCH_3$.

It may also take place by the hydrolysis of two molecules of formaldehyde to methyl alcohol and formic acid, (analogous to the Cannizzaro reaction in the case of aromatic aldehydes), and subsequent esterification in the presence of the sulphuric acid. In this case however it would be reasonable to expect that by using a larger proportion of acid, the yield of ester would be increased, which does not accord with the experiment described above.

The higher boiling fraction was found to consist chiefly of a liquid boiling at 95—96° C. and which appears to be a polymer of formaldehyde. It is still under investigation. Experiments with other condensing agents are also being carried on.

The Oxidation of Ferrous Salts. By F. R. ENNOS, B.A., St John's College. (Communicated by Mr C. T. Heycock.)

[*Read* 24 February 1913.]

THE rate of oxidation of ferrous salts in aqueous solution and in absence of free acid was studied by bubbling air or oxygen through the solutions at a constant rate of about one litre in three hours, portions of the solution being removed at definite intervals and titrated with potassium permanganate or bichromate.

At the ordinary temperature ferrous sulphate and chloride are oxidised exceedingly slowly. In the case of the former the initial rate of oxidation in an $\frac{N}{10}$ solution at 25° C. is of the order ·03 per cent. per hour, and this rate is doubled roughly for a rise of 10° C. A temperature of 60° C. was finally employed in comparing different ferrous salts and it was found that the oxidation of the chloride is about one-tenth and of the acetate ten times as fast as that of the sulphate. For the sulphate the reaction appears to be of the second order as regards the ferrous salt and also proportional to the partial pressure of the oxygen, results agreeing with those obtained by McBain by a different method (*Jour. Phys. Chem.* 1901). It has not yet been possible to find the order of reaction for the chloride or the acetate.

The influence of temperature, dilution and nature of the acid radicle indicates that the oxidation depends on the non-ionised part of the ferrous salt molecule. It remains to be seen whether there is any quantitative relationship between the two.

A Simple Method of determining the Viscosity of Air. By
G. F. C. SEARLE, Sc.D., F.R.S., University Lecturer in Experimental Physics, Fellow of Peterhouse.

[*Read* 11 November 1912.]

§ 1. *Introduction.* The determination of the viscosity of water by the flow through a capillary tube has, for many years, been one of the experiments done in my practical class at the Cavendish Laboratory. The determination of the viscosity of air was introduced in October 1912 for the benefit of those students who had already determined the viscosity of water in Mr T. G. Bedford's class at the Cavendish Laboratory or in other laboratories. The apparatus can be constructed at small cost and without the aid of a highly-skilled mechanic—advantages which will appeal to many teachers. No attempt has been made to introduce refinements into the apparatus.

Air is pumped into a large vessel and is allowed to escape through a capillary tube. The pressure of the air at the beginning and end of a measured interval of time is determined, and it is assumed that the temperature of the air in the vessel and in the capillary tube is always equal to that of the surrounding atmosphere.

In the determination of the viscosity of water the difference of pressure forcing the liquid through the tube is kept constant and the density of the liquid may be treated as uniform. But in the present experiment the density of the air is not uniform along the flow tube at any given time, and the difference of pressure driving the air through the tube is not constant, but diminishes as the time increases. The theory must therefore take account of these two facts. For the convenience of those who may wish to repeat the experiment, I give the details of the necessary calculations.

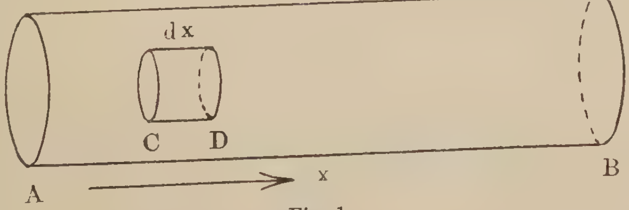


Fig. 1.

§ 2. *Calculation of velocity.* Consider a tube AB (Fig. 1) of radius a cm. and length l cm., through which air is passing in the

direction from A to B. Let CD be a length dx of a coaxal geometrical cylinder of radius r cm., the end C being at a distance x cm. from the end A of the tube.

Let the velocity of the air in the positive direction at any point defined by r and x be v cm. per sec. Then the velocity gradient at the curved surface of the cylinder CD is dv/dr sec.$^{-1}$. Hence, if the viscosity of the air be η dynes per sq. cm. per unit velocity gradient or η grm. cm.$^{-1}$ sec.$^{-1}$, the force due to viscous action on the curved surface of CD in the positive direction is

$$\eta \frac{dv}{dr} \cdot 2\pi r\, dx \text{ dynes.}$$

If the pressure at any point on the end C of the cylinder be p dynes per sq. cm., the pressure at any point on the end D is $p + (dp/dx)\, dx$, and thus the resultant in the positive direction of the forces due to the pressures is

$$-\pi r^2 \frac{dp}{dx}\, dx \text{ dynes.}$$

When the velocity of the air at any point in the tube is very small compared with the velocity of sound in air, the rate at which momentum enters the cylinder CD by the end C by convection differs from the rate at which it leaves the cylinder by the end D by an amount negligible compared with $\pi r^2 (dp/dx)\, dx$. Since the motion is steady, the momentum within the cylinder remains unchanged. Hence the resultant force vanishes and thus

$$\eta \frac{dv}{dr} \cdot 2\pi r\, dx - \pi r^2 \frac{dp}{dx}\, dx = 0$$

or

$$\frac{dv}{dr} = \frac{r}{2\eta} \frac{dp}{dx}. \qquad \text{.........................(1)}$$

Since $v = 0$ when $r = a$, because there is no slipping at the wall of the tube, the solution of this equation is

$$v = -\frac{1}{4\eta}(a^2 - r^2)\frac{dp}{dx}. \qquad \text{....................(2)}$$

If the flow of air across the plane defined by x be U c.c. per sec.,

$$U = \int_0^a 2\pi r v\, dr = -\frac{\pi}{2\eta}\frac{dp}{dx}\int_0^a (a^2 - r^2)\, r\, dr$$

$$= -\frac{\pi a^4}{8\eta}\frac{dp}{dx}. \qquad \text{....................................(3)}$$

The mass of air crossing this plane per second is σU grammes, where σ grm. per c.c. is the density of the air under the pressure p at the temperature θ prevailing at the plane x; this temperature

s assumed to be equal to θ_0, the temperature of the surrounding atmosphere.

§ 3. *Calculation of pressure.* When the motion is steady, the mass of air crossing each section of the flow tube per second is the same, and hence

$$\sigma U = \sigma_0 U_0, \qquad (4)$$

where σ_0 and U_0 are the density and volume of the same mass of air at temperature θ_0 and at the atmospheric pressure P_0. Thus

$$U = \frac{\sigma_0}{\sigma} U_0 = \frac{P_0}{p} U_0. \qquad (5)$$

Inserting this value of U in (3), we find the following differential equation for p, viz.

$$\frac{P_0 U_0}{a^4} = \frac{pU}{a^4} = -\frac{\pi}{8\eta} p \frac{dp}{dx}. \qquad (6)$$

In practice the radius of the tube will not be quite constant at different parts of the tube, but it will generally vary so slowly as we pass along the tube that the conditions of flow for any infinitesimal length dx may be treated as if they were those which would exist there if the whole tube had the same radius as the element dx. On these assumptions, equation (6) holds good for all values of x. Integrating it with respect to x from $x = 0$ to $x = l$, and remembering that η is independent of the pressure, we have

$$P_0 U_0 \int_0^l \frac{dx}{a^4} = -\frac{\pi}{8\eta} \left[\tfrac{1}{2} p^2\right]_{x=0}^{x=l}. \qquad (7)$$

If the pressure in the vessel, i.e. where $x = 0$, be P, then

$$P_0 U_0 \int_0^l \frac{dx}{a^4} = \frac{\pi}{16\eta} (P^2 - P_0^2), \qquad (8)$$

since the pressure at $x = l$ is the atmospheric pressure P_0. With moderately good tubes the integral on the left differs very little from l/a_0^4, where πa_0^2 is the value of the cross-section deduced from the formula

$$\pi a_0^2 l \rho = M, \qquad (9)$$

and M grammes is the mass of mercury, of density ρ grm. per c.c., which fills the tube. The necessary calibration correction is investigated in § 6. Neglecting it for the present, the formula becomes

$$U_0 = \frac{\pi a_0^4}{16\eta l} \cdot \frac{P^2 - P_0^2}{P_0}, \qquad (10)$$

an equation due to O. E. Meyer.

§ 4. *Formula for viscosity.* In the experiment the mass of the air which passes through the tube is deduced from the fall of pressure of the air in the vessel during a time t seconds.

Let the volume of the vessel up to the end $x = 0$ of the flow tube be S c.c. and let the mass of air in the vessel at any time t be M grms. Then if, as is assumed, the temperature be θ_0, we have

$$M = S\sigma = S\sigma_0 P/P_0 . \qquad \text{.....................(11)}$$

The rate at which mass escapes from the vessel is $\sigma_0 U_0$ grms. per sec. and this is equal to $-dM/dt$. Hence, by (11),

$$U_0 = -\frac{1}{\sigma_0}\frac{dM}{dt} = -\frac{S}{P_0}\frac{dP}{dt}.$$

Using this value of U_0 in (10), we obtain the following differential equation for P, viz.

$$-\frac{S}{P_0}\frac{dP}{dt} = \frac{\pi a_0^4}{16\eta l}\frac{P^2 - P_0^2}{P_0}.$$

Putting $1/(P^2 - P_0^2)$ into partial fractions, we have

$$\frac{1}{2P_0}\left\{\frac{1}{P+P_0} - \frac{1}{P-P_0}\right\}\frac{dP}{dt} = \frac{\pi a_0^4}{16\eta l S}. \qquad \text{..........(12)}$$

Integrating from $t = 0$ to $t = t$, we have

$$\frac{1}{2P_0}\left[\log_e \frac{P+P_0}{P-P_0}\right]_{t=0}^{t=t} = \frac{\pi a_0^4 t}{16\eta l S}.$$

If P_1 and P_2 be the pressures in the vessel at $t = 0$ and at $t = t$, then

$$\frac{\pi a_0^4 P_0 t}{8\eta l S} = \lambda, \qquad \text{.........................(13)}$$

where

$$\lambda = \log_e \left\{\frac{P_1 - P_0}{P_2 - P_0}\cdot\frac{P_2 + P_0}{P_1 + P_0}\right\} \qquad \text{................(14)}$$

Hence

$$\eta = \frac{\pi a_0^4 P_0}{8lS}\cdot\frac{t}{\lambda}. \qquad \text{........................(15)}$$

From the last expression η can be determined. The value of P_0 on the right side of (15) must be expressed in dynes per sq. cm. Thus, if the barometric height be h_0 cm. and the density of mercury at the temperature of the barometer be ρ grms. per c.c.,

$$P_0 = g\rho h_0.$$

Since only *ratios* are involved in the formula (14) for λ, the pressures in that formula may be expressed in cm. of mercury.

In the experiment it is the differences $P_1 - P_0$ and $P_2 - P_0$

which are observed by means of a gauge. The quantities $P_1 + P_0$ and $P_2 + P_0$ are obtained as follows:—

$$P_1 + P_0 = (P_1 - P_0) + 2P_0, \quad P_2 + P_0 = (P_2 - P_0) + 2P_0.$$

§ 5. *Experimental details.* A "tin" can C (Fig. 2) of about 10 litres capacity (price 10*d.*) is used to contain the air. Through a well-fitting rubber bung a tube passes, and into this tube are

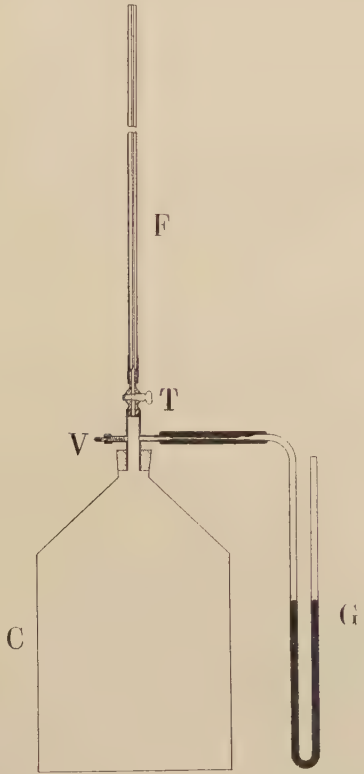


Fig. 2.

soldered a cycle tyre valve V (price 4*d.*), an ordinary gas-fitter's tap T (price 8*d.*), and a side tube. The side tube is connected with the mercury gauge G by a stout rubber tube. The flow tube F is connected with the gas tap by a rubber joint. The various joints must, of course, be air-tight.

Air is pumped into the can by means of an ordinary cycle pump attached to the valve V, until the pressure is raised to

20 to 25 cm. of mercury above the atmospheric pressure. The apparatus is then allowed to stand for some minutes in order that the rise of temperature due to the compression of the air may die away. A screen should be placed between the can and the observer to prevent the transfer of heat from his body to the can.

When the gauge readings have become steady, they are recorded. The tap is then opened for a time t seconds—one minute or more—and is then closed, and the new gauge readings are taken and recorded*. This process is repeated for various initial pressures. If the barometric pressure and the temperature remain constant, we see, by (15), that the value of t/λ should have the same value for each of the experiments with a given tube.

The radius of the flow tube is found from the mass of mercury required to fill the tube. It is here assumed that the bore of the tube is uniform; the method of obtaining the small calibration correction is explained in § 6.

The volume of the can is best found from the mass of water required to fill it; small corrections are required for the various tubes connected to the can.

§ 6. *Calibration correction.* We shall now investigate the correction required when the flow tube is not of uniform radius. By (8) we see that we have to replace l/a_0^4 in (10) by

$$\int_0^l \frac{dx}{a^4},$$

and thus formula (15) becomes

$$\eta = \frac{\pi P_0}{8S}\left[\int_0^l \frac{dx}{a^4}\right]^{-1} \cdot \frac{t}{\lambda}. \quad \text{.....................(16)}$$

The value of $\int dx/a^4$ is found as follows. Suppose that $n-1$ marks are made on the tube, dividing it into n parts, each l/n cm. in length. Let a thread of mercury of mass m grms., whose length is approximately l/n cm., be introduced into the tube. Let the thread be moved along the tube so that its centre approximately coincides with the centre of each of the n parts in turn, and let $q_1, q_2, \ldots$ be its length in the n positions. Then, if the tube be treated as uniform over each of the n parts, we may put

$$\pi\rho a_1^2 = m/q_1, \quad \pi\rho a_2^2 = m/q_2, \quad \&c.,$$

where ρ is the density of mercury.

* After the tap has been closed, the pressure in the vessel sometimes rises gradually by a few tenths of a millimetre. This indicates that, although the thermal conduction from the walls of the vessel has not kept the temperature of the expanding air quite constant, yet the fall of temperature has been very slight.

If the mass of mercury filling the whole tube be M grms.,

$$M = \pi\rho \int_0^l a^2 dx.$$

Taking l/n as an element of length instead of dx and replacing integration by summation, we have

$$M = \pi\rho \int_0^l a^2 dx = \pi\rho\Sigma a^2 . \frac{l}{n} = m\Sigma \frac{l/n}{q} = \frac{ml}{n} \Sigma \frac{1}{q}.$$

Hence $$\frac{1}{m} = \frac{l}{nM} \Sigma \frac{1}{q}. \qquad (17)$$

In a similar way we find

$$\int_0^l \frac{dx}{a^4} = \Sigma \frac{1}{a^4} . \frac{l}{n} = \frac{\pi^2\rho^2}{m^2} \Sigma q^2 . \frac{l}{n} = \frac{\pi^2\rho^2 l}{m^2 n} \Sigma q^2.$$

Hence, by (17),

$$\int_0^l \frac{dx}{a^4} = \frac{\pi^2\rho^2 l^3}{M^2} . \frac{1}{n^3} \left(\Sigma \frac{1}{q}\right)^2 . \Sigma q^2.$$

If a_0^2 be the mean value of the square of the radius deduced from the mass of mercury filling the whole tube,

$$a_0^2 = M/\pi\rho l.$$

Thus $$\int_0^l \frac{dx}{a^4} = \frac{l}{a_0^4} . \frac{1}{n^3} \left(\Sigma \frac{1}{q}\right)^2 . \Sigma q^2.$$

Hence the factor F by which l/a_0^4 must be multiplied to correct for the inequality of the tube is

$$F = \frac{1}{n^3} \left(\Sigma \frac{1}{q}\right)^2 . \Sigma q^2. \qquad (18)$$

This factor is only very slightly greater than unity when the tube is nearly uniform.

We may exhibit the correcting factor in another form. Let q_0 be the mean value of the n quantities q_1, q_2, ... and let

$$q_1 = q_0 + d_1, \quad q_2 = q_0 + d_2, \quad \&c.$$

Then $$\Sigma d = d_1 + d_2 + \ldots = 0.$$

Hence $$\Sigma q^2 = \Sigma (q_0^2 + 2q_0 d + d^2) = nq_0^2 + \Sigma d^2.$$

Also, since with a nearly uniform tube d_1, d_2, ... are small compared with q_0,

$$\Sigma \frac{1}{q} = \Sigma \frac{1}{q_0 + d} = \Sigma \left(\frac{1}{q_0} - \frac{d}{q_0^2} + \frac{d^2}{q_0^3} - \ldots\right) = \frac{n}{q_0} + \frac{1}{q_0^3} \Sigma d^2 - \ldots.$$

Hence, going as far as Σd^3, we find for the correcting factor

$$F=\frac{1}{n^3}\left(\Sigma\frac{1}{q}\right)^2\Sigma q^2=\left(1+\frac{2\Sigma d^2}{nq_0^2}+\ldots\right)\left(1+\frac{\Sigma d^2}{nq_0^2}\right)=1+\frac{3\Sigma d^2}{nq_0^2}. \quad (19)$$

Since
$$\Sigma\frac{1}{q}=\Sigma\frac{1}{q_0+d}=\Sigma\frac{q_0-d}{q_0^2-d^2},$$

and since q_0+d and q_0-d are both positive,

$$\Sigma\frac{1}{q}>\frac{1}{q_0^2}\Sigma(q_0-d)>\frac{n}{q_0}.$$

Hence
$$F>\frac{1}{n^3}\left(\frac{n}{q_0}\right)^2(nq_0^2+\Sigma d^2)>1+\frac{\Sigma d^2}{nq_0^2},$$

and thus F always exceeds unity.

To illustrate the practical application of the calibration correction, I give the data for the flow tube No. I used in the measurements described in § 7. The following table gives the length q of a mercury thread in 11 equally spaced positions along the tube; the length of the tube was 64·82 cm.

Length of thread q	q^2	$\frac{1}{q}$	$d=q-q_0$	d^2
cm.	cm.^{2}	cm.$^{-1}$	cm.	cm.^{2}
6·32	39·9424	0·158228	−0·15	0·0225
6·40	40·9600	0·156250	−0·07	0·0049
6·40	40·9600	0·156250	−0·07	0·0049
6·38	40·7044	0·156740	−0·09	0·0081
6·40	40·9600	0·156250	−0·07	0·0049
6·43	41·3449	0·155521	−0·04	0·0016
6·49	42·1201	0·154083	+0·02	0·0004
6·53	42·6409	0·153139	+0·06	0·0036
6·60	43·5600	0·151515	+0·13	0·0169
6·60	43·5600	0·151515	+0·13	0·0169
6·58	43·2964	0·151976	+0·11	0·0121
$q_0=6·47$	$\Sigma q^2=460·0491$	$\Sigma\frac{1}{q}=1·701467$		$\Sigma d^2=0·0968$

Hence, since $n=11$, we find, by (18),

$$F=\frac{1}{n^3}\Sigma q^2.\left(\Sigma\frac{1}{q}\right)^2=\frac{460·0491\times(1·701467)^2}{11^3}=1·00063.$$

The need of tables giving q^2 and q^{-1} to several significant figures may be avoided by finding F by formula (19). Thus

$$F=1+\frac{3\Sigma d^2}{nq_0^2}=1+\frac{3\times 0·0968}{11\times 6·47^2}=1·00063.$$

§ 7. *Practical Example.* The following observations were made with two flow tubes.

Volume of can and tubes (found by mass of water) $=S=9646$ c.c.

Barometric height $=h_0=76{\cdot}36$ cm.

Barometric pressure $=P_0=g\rho h_0=1{\cdot}016\times10^6$ dyne cm.$^{-2}$.

Density of mercury $=\rho=13{\cdot}56$ grm. cm.$^{-3}$.

Length of flow tube I $=l=64{\cdot}82$ cm.

Mass of mercury filling flow tube I $=M=3{\cdot}671$ grm.

Hence $\qquad a_0^2=M/\pi\rho l=0{\cdot}001329$ cm.^{2}, $\quad a_0^4=1{\cdot}767\times10^{-6}$ cm.^{4}.

By § 6,

$$F=\frac{1}{n^3}\Sigma q^2\,.\left(\Sigma\frac{1}{q}\right)^2=1{\cdot}00063.$$

If

$$\eta=Kt/\lambda,$$

then

$$K=\frac{\pi P_0a_0^4}{8Sl}\left\{\frac{1}{n^3}\Sigma q^2\,.\left(\Sigma\frac{1}{q}\right)^2\right\}^{-1}=\frac{\pi P_0a_0^4}{8Sl}\cdot\frac{1}{F}.$$

Thus, for tube I,

$$K=\frac{\pi\times1{\cdot}016\times10^6\times1{\cdot}767\times10^{-6}}{8\times9646\times64{\cdot}82\times1{\cdot}00063}=1{\cdot}127\times10^{-6}\text{ dyne cm.}^{-2}.$$

The results obtained with tube I are shown in the following table. The table also includes results found with tube II. The data for tube II were

$$l=77{\cdot}56\text{ cm.},\quad a_0^4=1{\cdot}386\times10^{-6}\text{ cm.}^4,\quad F=\frac{1}{n^3}\Sigma q^2\,.\left(\Sigma\frac{1}{q}\right)^2=1{\cdot}00467.$$

Hence, for tube II, $\qquad K=7{\cdot}356\times10^{-7}$ dyne cm.$^{-2}$.

Tube	Time	Gauge readings		P_1-P_0	P_2-P_0	λ	$\eta=\frac{Kt}{\lambda}$
	secs.	cm.	cm.	cm.	cm.		
I	0	36·26	13·80	22·46		0·710	$1{\cdot}90\times10^{-4}$
	120	30·20	19·93		10·27		
I	0	33·81	16·28	17·53		0·727	$1{\cdot}86\times10^{-4}$
	120	29·08	21·08		8·00		
I	0	31·01	19·12	11·89		0·749	$1{\cdot}80\times10^{-4}$
	120	27·80	22·40		5·40		
II	0	36·39	13·66	22·73		0·742	$1{\cdot}78\times10^{-4}$
	180	30·10	20·06		10·04		
II	0	30·15	20·00	10·15		0·751	$1{\cdot}76\times10^{-4}$
	180	27·40	22·77		4·63		
II	0	32·49	17·65	14·84		0·785	$1{\cdot}69\times10^{-4}$
	180	28·30	21·88		6·42		

In the case of tube I the tap was opened for 120 seconds, so that $t = 120$ secs. For tube II the tap was opened for 180 seconds. The temperature was 13·8° C. throughout.

The mean value for the viscosity of air at 13·8° C. is

$$\eta = 1\cdot 80 \times 10^{-4} \text{ grm. cm.}^{-1} \text{ sec.}^{-1}.$$

R. A. Millikan (*Physical Review*, No. 1, 1913, p. 79) gives

$$\eta = 1\cdot 824 \times 10^{-4} \{1 + \cdot 0027\,(t - 23)\}$$

for the viscosity of air at t° C. This gives $\eta = 1\cdot 779 \times 10^{-4}$ for the viscosity at 13·8° C.

The Swarming of Odontosyllis. By F. A. POTTS, M.A., Trinity Hall.

[*Read* 10 February 1913.]

THROUGHOUT the months of June and July in 1911 I had frequent occasion to take and examine dredgings from the sea bottom outside the harbour of Nanaimo, British Columbia. The bottom deposits there are of a peculiar character, consisting largely of the debris of dead Hexactinellid sponges. The long interwoven spicules form a matted mass which furnishes a secure retreat for many species of Polychaet worms. One of the most abundant and interesting of these is a species of Syllid which proved to be *Odontosyllis phosphorea* described by Moore* in 1909. During the whole period the worms of this species contained reproductive products. The females were of a bright red colour due to the eggs, while the males showed the natural yellow colour and transverse markings of the species. Both sexes were very irritable under handling and broke up entirely when attempts were made to fix them with sublimate solutions or alcohol, and at the same time an intense phosphorescence was produced.

No appearance of the mature worms at the surface was noticed during these months, though a close lookout for phenomena of this kind was maintained. On August 15th, however, Professor McMurrich of Toronto informed me that he had observed just before sunset hundreds of small worms swimming on the surface of the sea in Departure Bay, a shallow inlet adjacent to the grounds described above. Examination of a number of these which had been brought into the laboratory showed me that they belonged to the species found so abundantly amongst the sponge debris.

The next night Professor McMurrich and myself followed the phenomenon as closely as possible. Sunset was about seven o'clock, and half an hour or more before isolated individuals began to appear. At first these were mostly males, but as the numbers increased so did the proportion of females to males until there was approximate equality. On their appearance from the depths the individuals of both sexes swam round and round in circles with swift undulatory movements. A short time after, the movements became slower, finally ceasing, and during spasmodic flexures of the whole body the eggs and spermatozoa were discharged. Then the spent individuals sank slowly beneath the surface. No approximation of the two sexes was observed to take place, but my impression was that each individual sought the surface without fixed plan or

* J. Percy Moore, *Proc. Acad. Nat. Sci.*, Philadelphia, vol. LXI. 1909, p. 327.

direction, rid itself of its contents as quickly as might be, and then lost no time in descending to its accustomed habitat. The phenomenon continued till the light had nearly faded, but by then there were only very occasional individuals to be seen, so that probably the whole period of swarming is less than an hour.

On the following day I left Nanaimo, but I was afterwards told that, though a few individuals were seen on the surface on that night and the next, there were nothing like the numbers of the two preceding nights.

The accompanying map shows the distribution of *O. phosphorea* in the neighbourhood of Nanaimo. In explanation of the small area assigned to the swarming worms, it must be noticed that the short time did not allow a complete investigation of their distribution, but doubtless future observers will be able to map out the area more completely. One fact is quite clear; that the worms, in seeking the surface, migrate inshore in considerable numbers, for I never, in the course of many dredgings, found a single example of *Odontosyllis* in Departure Bay itself.

Recently Professor McMurrich has been kind enough to send me very interesting information which establishes the periodic appearance of the swarms of *Odontosyllis.* The weather was very unfavourable in 1912 and there had been a great deal of rain just before August 18th. On that night, however, the weather was fine, and on going out at 7.30 p.m. Professor McMurrich found *Odontosyllis* swarming at the surface as in the year before. Dr Fraser again observed the phenomenon on the two or three evenings following.

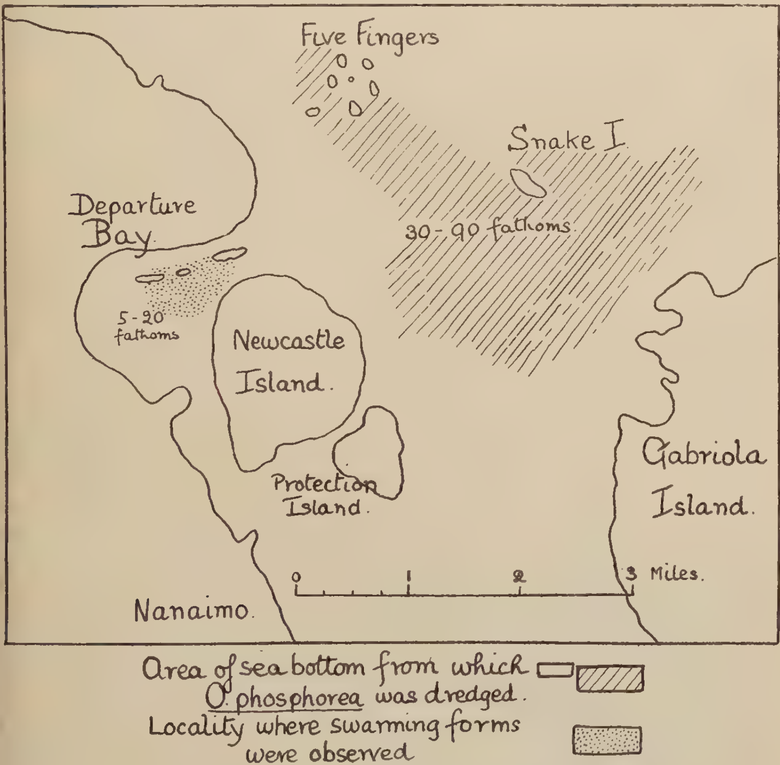
It seems certain, then, that the swarming of *Odontosyllis phosphorea* takes place at Nanaimo at approximately the same date every year. In 1912 it occurred three days later than in 1911, but this may possibly have been due to the unfavourable weather. In 1911 swarming took place during the last quarter of the moon and in 1912 at the beginning of the first quarter. In both cases the tide was full or just falling. The close proximity of the Dominion Government Biological Station will allow of a close watch being kept on the circumstances of the appearance from year to year, and I trust that the scanty data which I give here will soon be supplemented. In particular I hope that enquiry will be made as to whether swarming takes place at any other period of the year*.

For some time I supposed that the phenomenon witnessed at Nanaimo had not been previously described in *Odontosyllis*, and it

* In the original description by Moore (*loc. cit.*) he states that the type specimens were labelled "Phosphorescent annelids caught at surface; Avalon Bay, Catalina Island, evening, April 11, 1904." This fixes a date for the swarming of this species in California.

was with great interest that I discovered and read a paper by Galloway and Welch* on very similar phenomena in an Atlantic species *Odontosyllis enopla*. In the summer of 1904 this species was observed to appear in the surface waters of Harrington Sound, Bermudas, in considerable numbers and with striking regularity. The following dates of appearance are chronicled:

July 3—7 reaching a maximum on the 4th,
July 29—31 „ „ „ „ 30th,
August 23 (no further details),



Distribution of O. phosphorea.

so that there appears to be an interval of approximately 26 days. On the two first occasions when they were carefully observed they appeared each day within fifteen minutes of the same time just at twilight. The daily period of swarming lasted from 20—30 minutes. "Only a few appeared at first each evening. The numbers gradually

* *Trans. Amer. Micr. Soc.*, vol. XXX. 1911, pp. 13–39.

increased to a maximum when scores might be seen at once. The display waned somewhat more suddenly than it waxed.

"The males and females differ considerably in size, the females often being twice as long as the males....Both sexes are distinctly phosphorescent—the female with strong and more continuous glow and the male with sharper intermittent flashes.

"In mating the females, which are clearly swimming at the surface of the water before they begin to be phosphorescent, show first as a dim glow. Quite suddenly she becomes acutely phosphorescent, particularly in the posterior three-fourths of the body, though all the segments seem to be luminous in some degree. At this phase she swims rapidly through the water in small luminous circles two or more inches in diameter. Around this smaller vivid circle is a halo of phosphorescence growing dimmer peripherally. This halo of phosphorescence is possibly caused by the escaping eggs together with whatever body fluids accompany them. At any rate the phosphorescent effect closely accompanies ovulation and the eggs continue mildly phosphorescent for a while.

"If the male does not appear this illumination ceases after 10 to 20 seconds. In the absence of the male the process may be repeated as often as four or five times by one female at intervals of 10—30 seconds. The later intervals are longer than the earlier. Usually, however, the males are sufficiently abundant to make this repetition unnecessary; and the unmated females are rare, if they are out in the open water. One can sometimes locate the drifting female between displays by the persistence of the luminosity of the eggs; but the male is unable to find her in this way.

"The male first appears as a delicate glint of light possibly as much as 10 or 15 feet from the luminous female. They do not swim at the surface as do the females, but come obliquely up from the deeper water. They dart directly for the centre of the luminous circle, and they locate the female with remarkable precision when she is in the active stage of phosphorescence. If, however, she ceases to be actively phosphorescent before he covers the distance he is uncertain and apparently ceases swimming, as he certainly ceases being luminous, until she becomes phosphorescent again. When her position becomes defined he quickly approaches her, and they rotate together in somewhat wider circles, scattering eggs and sperm in the water. The period is somewhat longer on the average than when the female is rotating alone; but it, too, is of short duration.

"So far as could be observed the phosphorescent display is not repeated by either individual after mating. Very shortly the worms cease to be luminous and are lost."

This is, I submit, a very remarkable account. In its general

aspects the swarming in this species resembles that in *O. phosphorea*, *i.e.* in the appearance of great numbers of individuals at the surface of the sea at definite times of the year and at a definite period of the day in the neighbourhood of sunset. But the divergencies in the habits of the two species are very interesting. While *O. phosphorea* first reaches the surface well before sunset, *O. enopla* is not seen till dusk has fallen. In this latter species, moreover, the phosphorescence, which is characteristic of most species of this genus, is developed to a most extraordinary extent and is adapted to serve as a means of sexual recognition. This is almost without parallel in the animal kingdom. It is, however, stated that the function of the phosphorescent organs in the Fireflies is to attract the other sex. In the case of the common glow-worm it is generally allowed that the male finds the female by means of her phosphorescence. Mr J. C. F. Fryer has told me of a Lampyrid beetle in Ceylon, the female of which actually remains in deep holes, but that she emits a most powerful light from organs on the underside of the abdomen, which is the better displayed by the flexion of that part of the body. As soon as the male approaches, however, the light dies down as in *Odontosyllis.* The account of *O. enopla*, which I have quoted above at length, shows, however, that we have among much lower animals almost as complex a phenomenon in which the production of phosphorescence is equally essential for the meeting of the sexes. It is possible, however, to make one criticism of the interpretation which is given above. This is that an arrangement to secure close approximation between the male and female of a marine worm would seem in general to be unnecessary for the successful propagation of the species. There is none such in *O. phosphorea*, and in the case of our two commonest British species of *Odontosyllis*, *O. ctenostoma* and *gibba*, the heterosyllids seem to occur, not in swarms, but as scattered individuals, and probably discharge their eggs or spermatozoa when no other member of the species is near. In the only reference respecting the pelagic occurrence of these species that I am able to find Gravely* says: "The brown *Odontosyllis* (*i.e. O. gibba*) is frequently seen in the adult condition—occasionally accompanied by *O. ctenostoma* and sexual specimens of *Autolytus* and *Myrianida*—swimming at the surface of the sea at the mouth of Port Erin Bay and further out towards the Calf on calm evenings during July." This passage, I think, clearly points to an irregular and sporadic occurrence.

If there are these considerable differences in the reproductive habits of the different species there must, I think, be equivalent physiological differences in the reproductive cells of the species. One would expect, from the elaborate devices practised by *O. enopla* to ensure fertilisation, that the independent life of the eggs and

* *Q. J. M. S.*, vol. LIII. p. 600.

spermatozoa of that species was very brief and that fertilisation must take place very shortly after dehiscence, while probably in *O. gibba* and *O. ctenostoma* the genital products can survive for a much longer time in sea-water.

In the literature of swarming amongst Annelids a majority of the observations refer to the genus *Nereis* and show a great diversity of reproductive habits. Hempelmann*, who has investigated the case of *N. dumerilii* very thoroughly, found that at Naples heteronereid forms occurred at the surface of the sea from 1st October, 1908—15th May, 1909. From 15th May, 1909—15th August, 1909 there were no heteronereids. The mature worms appear at the surface indifferently in the day or the night and usually occur not as swarms but scattered individuals which discharge their eggs or spermatozoa when no other heteronereid is near. But on one occasion at least, on May 2nd, 1908, there was seen in the Bay of Naples a great swarm of *N. dumerilii* and *N. coccinea*.

Similarly attention has been lavished on the heteronereids which are seen off the east coast of England. Sorby† was accustomed to observe the phenomena in the summer throughout a long series of years. On several occasions he saw immense numbers of heteronereids on the surface. The date, time and place of occasion of these are indicated in the following table:—

23 May, 1885.	*N. dumerilii.*	In the evening at the mouth of the Colne.
16 July, 1898.	„ „	At 5 o'clock in the morning at the mouth of the Stour and Orwell, the sea being covered with millions of worms.
11 May, 1882.	*N. longissima.*	In the evening near Sheerness.
24 May, 1889.	„ „	At the mouth of the Orwell.
9 September, 1889.	„ „	In the evening at Queenborough.

In twenty years Sorby only saw five such great swarms of nereids. He does not state that he ever observed isolated heteronereids on the surface of the sea, and for this reason the record of these English occurrences is lamentably incomplete. But his observations go to establish several facts which Hempelmann's more thorough, but less extended, investigations give no clue to, viz.:—

* "Zur Naturgeschichte von *Nereis dumerilii*," *Zoologica*, Bd. xxv. Heft 62, 1911, pp. 92 ff.

† Sorby, *Journ. Linn. Soc.*, London, vol. xxix. 1906.

(1) that these great swarms of nereids are only seen rarely;

(2) that they occur at almost any time of the day (at early morning, at midday* or in the evening) and nearly any period of the summer;

(3) that the date of swarming has no definite relation to the full moon.

On the whole I think we are justified in stating that in *N. dumerilii* and probably other species the swarming habit is not fixed. Hempelmann has noticed a slight correlation between the appearance of the heteronereids and the phases of the moon, but this is by no means marked. There is no doubt from Hempelmann's observations that the ascent of the sexually mature worms is due to a combination of causes which act throughout a long period and whose efficacy fluctuates considerably.

Enough has been said to indicate the irregularity in period and time of the swarming of *Nereis dumerilii* and other associated forms. There is, however, at least one good case in the genus where an absolute periodicity has been established. I refer to *N.* (*Ceratocephale*) *osawai* of Japan†, the heteronereids of which regularly issue forth four times in the year in the months October and November, in 3—4 day periods. Their date of appearance is absolutely fixed for the days following the new moon. Their presence on the surface is limited to from one to two hours in the evening, but the time of appearance is by no means so definitely fixed as in the case of *Odontosyllis phosphorea* and *enopla* (sunset). They appear in fact well after sunset and often after moonset, so that the immediate stimulus would appear to be independent of the action of light.

Other cases might be quoted, but I think my main contention, the diversity of swarming habits in *Nereis*, is sufficiently proved.

The phenomenon of swarming, at least in its final form, does appear to be of a definitely adaptive nature. The object is the fertilisation of the maximum number of eggs, and this is gained by the simultaneous emission of eggs and spermatozoa from a crowded mass of male and female individuals. Galloway and Welch found in *O. enopla* that of eggs collected in connection with the swarming worms from 45—80 °/$_{\circ}$ were already fertilised. In the case of the Atlantic Palolo worms, which turn the clear blue waters of the Tortugas into a thick milky mass with their eggs and spermatozoa, it is difficult to imagine how any of the eggs escape fertilisation. Yet Professor Mayer tells me that the

* Mr William Brockett in the month of June, 1910, collected a rather large number of heteronereids off Mersea Island on the Essex coast between midday and two o'clock in the afternoon.

† Izuka, "Observations on the Japanese Palolo *Ceratocephale osawai*," *Journ. Coll. Sc.*, Tokio, T. XXXVI. 1903.

eggs laid by females on the borders of the swarm are certainly not developed.

How, then, does it come about that a species like *N. dumerilii*, the heteronereids of which become sexually mature at any time within a widely extended period, and in which the swarming habit is very indefinitely developed, is able to maintain its great numbers and wide distribution. Not only is there no arrangement to ensure the simultaneous swarming of the sexes, but it has frequently been observed in this and related species that large swarms consist of a single sex*. Under these circumstances there must be an enormous waste.

There is, however, one obvious explanation. *N. dumerilii* is a polymorphic species, and according to the results of von Wistinghausen and Hempelmann all individuals have more than one period of sexual maturity. When they first reach a certain length the female lays eggs, within the tube she normally inhabits, to the number of 1000 or more. These are generally fertilised in the most economical manner by the male, who creeps into the tube and spreads his sperm over the eggs. It would seem probable that this is the method most responsible for the maintenance of the species, the production of sexually ripe heteronereids at a later period of life being a subsidiary (possibly incipient or degenerate) phenomenon.

These comments on swarming in *Odontosyllis* and *Nereis* are only intended to illustrate the diversity in the habit existing among related forms. I have not attempted to discuss the thorny question of the part played by external stimuli or the possibility of an inherited rhythm in the organism.

Note. Since writing the above I have had access to a paper by Lillie and Just (*Biol. Bull.*, Feb. 1913) on the breeding habits of *Nereis limbata.* In this species swarming takes place quite regularly in four runs during the summer corresponding to the lunar cycles in the months June, July, August, September, occurring on many successive nights shortly after sunset and lasting little longer than an hour. This is, then, an advanced case of the swarming habit, but the great interest lies in the fact that the female produces a substance which acts on the male causing the emission of sperm. It will be of great interest to see whether the distribution of this phenomenon is at all general or whether it is a peculiar development of the swarming habit to ensure fertilisation, like the mating relations in *Odontosyllis enopla.*

* Cf. Sorby, Hempelmann, *loc. cit.*

Further applications of positive rays to the study of chemical problems. By Professor Sir J. J. Thomson.

[*Read* 27 January 1913.]

The author described the application of positive rays to the detection of the rare gases in the atmosphere. Sir James Dewar kindly supplied two samples of gases obtained from the residues of liquid air; one sample which had been treated so as to contain the heavier gases was found on analysis to contain Xenon, Krypton, Argon, there were no lines on the photograph unaccounted for, hence we may conclude that there are no unknown heavy gases in the atmosphere in quantities comparable with the known gases. The other sample which had been heated so as to contain the lighter gases was found to contain helium and neon and in addition a new gas with the atomic weight 22, the relative brightness of the lines for this gas and for neon shows that the amount of the new gas is much smaller than that of neon.

The second part of the paper contains an investigation of a new gas of atomic weight 3 which this method of analysis had shown to be present in the tube under certain conditions. The gas had occurred sporadically in the tube from the time of the earliest experiments but its appearance could not be controlled. After a long investigation into the source of this gas, it was found that it always occurred in the gases given out by metals when bombarded by cathode rays, a trace of helium was also usually found on the first bombardment. The metals used were iron, nickel, zinc, copper, lead and platinum; the gas was also given off by calcium carbide. Various experiments were described which illustrated the stability of the gas.

The chemical and bacterial condition of the Cam above and below the sewage effluent outfall. By J. E. Purvis, M.A., and A. E. Rayner, M.A.

[*Read* 24 February 1913.]

The river was investigated at various points extending from 100 feet above the outfall and at 8 feet from the outfall: and at $\frac{1}{4}$ of a mile, $\frac{1}{2}$ a mile, $\frac{3}{4}$ of a mile, $1\frac{1}{2}$ miles, 2 miles, $2\frac{1}{2}$ miles, 3 miles and 4 miles below the outfall.

Chemically, the river purifies itself moderately well from the contaminating effluent; for at about $\frac{3}{4}$ of a mile below the effluent, the albuminoid ammonia and the oxygen absorbed figures were lower than at 100 feet above the effluent outfall.

Bacterially, the dangerous pollution, as indicated by *B. coli*, is well-marked at between 3 and 4 miles below the outfall. The potential danger of such contamination is in the direction of cattle quenching their thirst, of bathers, and of watercress.

On the optically active semicarbazone and benzoylphenylhydrazone of cyclo-*hexanone-4-carboxylic acid.* By W. H. MILLS, M.A., and Miss A. M. BAIN.

[*Read* 24 February 1913.]

THE semicarbazone of *cyclo*-hexanone-4-carboxylic acid can be obtained in an optically active form by crystallising its morphine salt from dilute alcohol, the highest value obtained for the molecular rotation in alkaline solution being $[M]_D$ 38·8°. The benzoylphenylhydrazone of the acid can similarly be obtained in an optically active form by crystallisation of its quinine salt from aqueous alcohol, the highest value found for the molecular rotation in alkaline solution being $[M]_D$ 238·6°.

These optically active compounds agree so closely in their behaviour with the optically active oxime of this acid previously described by the authors that there can be little doubt that the optical activity is due to similar causes in the three cases.

The observations accordingly lend great support to the view that stereoisomerism in the sense of the Hautzsch-Werner hypothesis exists in the case of semicarbazones and phenylhydrazones.

The ten Stereoisomeric Tetrahydroquinaldinomethylenecamphors. By Professor POPE and J. READ, M.A.

[*Read* 24 February 1913.]

THE two enantiomorphously related tetrahydroquinaldines condense readily with the two similarly related oxymethylenecamphors yielding products of the constitution, $C_{10}H_{12}N.CH:C\left\langle\begin{array}{l}C_8H_{14}\\ \,|\\ CO\end{array}\right.$. Since each component of the condensation can be obtained in a dextro- and a laevo-rotatory form, four simple optically active condensation products can be obtained; the configurations of these may be described by the following symbols, in which d- and l- represent the configurations of the tetrahydroquinaldine residue and D- and L- that of the oxymethylenecamphor nucleus.

(1) d—D. (2) l—L. (3) d—L. (4) l—D.

The two members of a pair of enantiomorphously related isomerides, (1) and (2), or (3) and (4), combine to form a double or racemic compound, so that the following externally compensated substances can also be prepared.

(5) [d—D, l—L], (6) [d—L, l—D].

Amongst substances such as these, which contain two dissimilar centres of asymmetry, six stereoisomerides of the above types are the only ones ordinarily obtainable but in the present instance four more can be prepared. These are the two pairs of partially racemic compounds of the configurations stated below.

(7) [d—D, d—L], (8) [l—L, l—D],
(9) [d—D, l—D], (10) [l—L, d—L].

No case has been previously recorded of the formation of partially racemic compounds in the manner just described and it would be anticipated that no resolution of externally compensated tetrahydroquinaldine into its optically active components would be possible with the aid of d- or l-oxymethylenecamphor. It is shown, however, that on treating externally compensated tetrahydroquinaldine with less than one-half an equivalent of d-oxymethylenecamphor a resolution can be effected because the l-base condenses more rapidly than the d-isomeride with d-oxymethylenecamphor; under these conditions the condensation yields about 80 per cent. of the partially racemic compound (9) and 20 per cent. of the simple optically active substance (4) from which l-tetrahydroquinaldine may be separated.

Experiments illustrating flare spots in photography. By G. F. C. SEARLE, Sc.D., F.R.S., University Lecturer in Experimental Physics, Fellow of Peterhouse.

[*Read* 24 February 1913.]

§ 1. *Introduction.* When light strikes a refracting surface, it is partly refracted and partly reflected. For glass of refractive index μ, the ratio of the intensity of the reflected light to that of the incident light is $(\mu-1)^2/(\mu+1)^2$ for normal incidence and this ratio increases as the angle of incidence increases. For glass of refractive index 1·5 the ratio is 1/25. If the light suffers a second reflexion, the ratio of the intensity of the twice reflected light to that of the original light is $1/(25)^2$ or 1/625. For $2n$ reflexions the ratio is $1/(25)^{2n}$, which diminishes very rapidly as n increases.

If a luminous point be placed on or near the axis of a lens, an image will be formed by rays which have passed through the lens without suffering reflexion. This is the ordinary image used in photography. A second and fainter image will be formed by rays which have been twice reflected and there are other images formed by rays which have been reflected 4, 6, ... times, but the latter images will be very faint unless the source of light is very powerful.

When there are n lenses, the rays which suffer their first reflexion at the last air-glass surface can be reflected a second time at any one of the $2n-1$ air-glass surfaces in front of the last surface. The rays which suffer their first reflexion at the last surface but one can be reflected a second time at any one of the $2n-2$ surfaces in front of that surface, and so on. Hence, if N be the total number of images formed by twice reflected rays

$$N = (2n-1) + (2n-2) + \ldots + 2 + 1 = n(2n-1).$$

Thus we have the results

Number of lenses	1	2	3	4	5
Number of images formed by twice reflected rays	1	6	15	28	45

In many photographic lens systems, two or more pieces of glass are cemented together with Canada balsam. The light

reflected at the cemented surfaces is inappreciable, and for the present purpose such a cemented component of a lens system is to be considered as a single lens. It is only the air-glass surfaces which count.

The image of any object formed by rays which have not suffered reflexion in their passage through the lens will be called the *primary* image of that object, and any image formed by rays which have suffered two reflexions will be called a *secondary* image (*Nebenbild*) of the object.

§ 2. *Ghosts in photography.* In photography the sensitive plate is adjusted so that the primary image of an object, i.e. the image formed by rays which have not been reflected in their passage through the lens system, is sharply focussed upon it. Thus, if S (Fig. 1)* be an object point on the axis of the lens

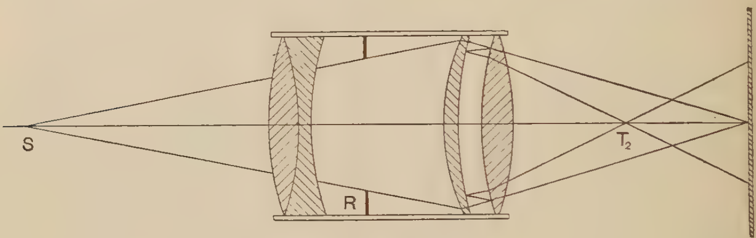


Fig. 1.

system, the plate is placed in the plane GH which is normal to the axis and passes through the conjugate primary image point T. If T_2 be a real or virtual secondary image of S formed by twice reflected rays, the cone of rays which meet in T_2 will be cut by the plane through T in a circle of diameter GH. If T_2 is so far from T that the diameter of this circle is greater than that of the sensitive plate, the intensity of illumination due to T_2 will be very small and the only effect will be a slight uniform fogging of the plate. The effect is so small that the number of air-glass surfaces has to be considerable before there is any serious fogging due to all the cones of twice reflected light whose sections by the plane GH have diameters greater than that of the plate.

If, however, T_2 be close to T, the illuminated patch GH will be small and the illumination may then be sufficient to cause trouble. This small patch is known as a "ghost."

If a secondary image T_2 coincides with T, any object in a plane normal to the axis and passing through S will have its corresponding secondary image in the plane GH. But the secondary

* Fig. 1 is merely diagrammatic and does not show accurately the paths of the rays.

image will not generally be of the same size as the primary image and may even be erect when the latter is inverted.

Thus when T_2 coincides with T, a faint secondary image of the objects to be photographed will be impressed upon the plate in addition to the primary image. Chapman Jones (*Science and Practice of Photography*, p. 248) states that he "has seen a portrait spoiled by an inverted image of the shirt front appearing over the model's head."

The position and magnitude of any secondary image can be determined when we know the positions of the corresponding cardinal points, viz. the principal (or unit) points and the principal foci. To each pair of surfaces which act as reflectors there corresponds a set of cardinal points. When the lens system is of considerable length, the cardinal points corresponding to any pair of surfaces may occupy all sorts of positions.

In the case of the secondary image described by Chapman Jones, the system behaved as one of negative focal length with its cardinal points so placed that it formed a real erect image of a distant object. A system of two thin convex lenses separated by a distance greater than the sum of their focal lengths behaves in the same manner with respect to a primary image. The system has a negative focal length and forms a real erect image of a distant object.

§ 3. *Flare spots in photography.* The rays which have been twice reflected may give rise to trouble in another way. The primary image of the stop R (Fig. 1) will be so far from the plate that its effects may be disregarded; the image will generally be virtual. But when the camera is directed towards any landscape or other scene, an infinite number of rays passes in all directions (within limits) through every point of the opening of the stop, and thus this opening will behave as if it were a self-luminous disk which, however, only emits light towards one side. One or more of the secondary images of this equivalent disk, formed by rays which have suffered the first reflexion at some surface between the stop and the plate and a second reflexion at any other surface of the lens system, may be accurately or nearly focussed on the plate and so give rise to a well defined bright spot in the centre of the plate. This spot is known as a "flare spot."

For a given background, the flare spot image of the stop aperture will be of the same intensity whatever the size of the stop, at least in those cases where the second reflexion takes place at a surface behind the stop. But the intensity of the primary image of the background will be proportional to the area of the stop. Hence, although the flare spot image of the stop may be

too faint to produce a noticeable effect during the short exposure employed with a large stop, it may be quite strong enough to produce a deleterious patch upon the plate during the long exposure required to obtain an image of the background with a small stop. There are many lenses which cannot be used with a small stop for this reason. A flare spot may be detected by watching the focussing screen as the opening of the stop is diminished, the camera being pointed towards the sky or other bright background.

If the edges of the stop are bright, they may reflect light forwards and this light after being reflected at one of the surfaces in front of the stop, may come more or less nearly to a focus on the plate. Since this light has suffered only one reflexion at an air-glass surface, it may produce a strong image or at least a considerable fogging of the plate. The same remark applies to the tops of screw threads, which may have been worn bright, and to the edges of the lenses themselves. The defect may be cured by properly blacking the surfaces with good optical black.

§ 4. *Experiment with spectacle lens.* A virtual flare spot is easily seen when a positive (convex) spectacle lens of focal length f is placed before the eye. If a bright object such as a gas flame or the bright sky is viewed through a small hole cut in a sheet of paper, a well defined secondary image of the hole will be seen when the distance between the lens and the hole is about $f/7$ (see § 7). The secondary image of the hole is virtual and at a considerable distance from the eye. By adjusting the position of the hole the secondary image may be seen clearly at the same time as the flame. This corresponds to the case where a secondary image of the stop of a camera is focussed on the plate at the same time as the primary image of distant objects.

If two pairs of spectacles are worn, six secondary images can be seen.

§ 5. *Thin lenses in contact.* The positions and magnitudes of the secondary images of any object formed by a system of any number of thick or thin lenses can be calculated by repeated applications of the formulae for reflexion and refraction at spherical surfaces. When, however, the system consists of a single thin lens or of two thin lenses in contact, the positions of the secondary images can be very readily calculated by the method of least time. The experimental tests of the formulae can be made with simple apparatus and form interesting exercises for students who have made a little progress in practical optics.

§ 6. *Focal length of thin lens.* The application of the principle of least time to find the focal length of a thin lens is well known,

but it will be convenient to give it here as the mathematical methods can be transferred, with little change, to the solution of the other problems considered.

Let AKB (Fig. 2) be a thin lens. Let S be a point source of

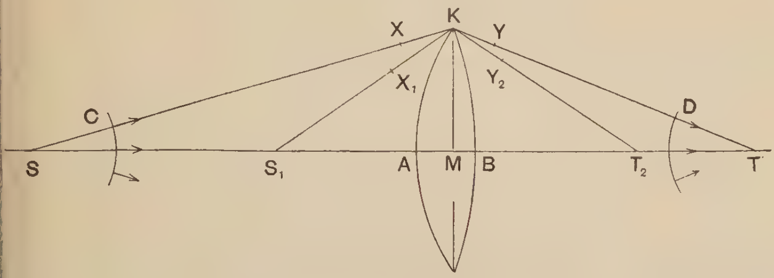


Fig. 2.

light on the axis of the lens and let T be its image. Let the radii of the faces AK, BK be a, b cm. and let the refractive index of the lens be μ. The radii are counted positive when the faces are convex, as in Fig. 2. Let the focal length be f cm.; we shall follow the rule of the practical opticians and count f positive when the lens is a converging one. Where convenient, capital letters will be used to denote "powers" or reciprocals of distances. Thus

$$\frac{1}{f} = F,$$

where F cm.$^{-1}$ or 100 F dioptres is the "power" of the lens.

When spherical aberration is negligible, any spherical wave front C, which expands from S as centre, becomes, after passing through the lens, a spherical wave front D, contracting to T as centre. Hence the time taken by light in passing from S to T is the same for every ray—an example of the principle of Least (or Stationary) Time.

Let M be the point in which the plane of the edge of the lens intersects the axis.

Let $\qquad SM = u, \; MT = v$ and $KM = h$.

The position of T is found by equating the optical length of the path from S to T for the ray which starts along SK to that for the ray which starts along SA. If X, Y be points on SK, TK such that $SX = SA$ and $TY = TB$, the optical length of the path from X to Y is equal to that of the path from A to B. Since the speed of light in glass is only μ^{-1} times its speed in air, the optical length of AB is μAB. Hence the optical equation is

$$XK + KY = \mu AB \quad \text{.....................}(1).$$

Since the distance of M from the centre of curvature of AK is $(a^2 - h^2)^{\frac{1}{2}}$, we have

$$AM = a\{1 - \sqrt{1 - h^2/a^2}\},$$

and similarly for BM. Expanding as far as h^2, we find

$$AM = \tfrac{1}{2}h^2/a, \quad BM = \tfrac{1}{2}h^2/b.$$

Hence $$AB = \tfrac{1}{2}h^2(1/a + 1/b) \quad \text{.....................(2).}$$

Now, as far as h^2,

$$SK = \sqrt{u^2 + h^2} = u + \tfrac{1}{2}h^2/u, \qquad TK = \sqrt{v^2 + h^2} = v + \tfrac{1}{2}h^2/v,$$

$$SX = SM - AM = u - \tfrac{1}{2}h^2/a, \quad TY = TM - BM = v - \tfrac{1}{2}h^2/b.$$

Hence $$XK = SK - SX = \tfrac{1}{2}h^2(1/u + 1/a) \quad \text{...........(3),}$$

$$KY = TK - TY = \tfrac{1}{2}h^2(1/v + 1/b) \quad \text{...........(4).}$$

Multiplying the optical equation (1) by $2/h^2$ and substituting from (2), (3) and (4), we have

$$(1/u + 1/a) + (1/v + 1/b) = \mu(1/a + 1/b),$$

or $$1/u + 1/v = (\mu - 1)(1/a + 1/b) \quad \text{..............(5).}$$

Hence, the focal length is given by

$$1/f = (\mu - 1)(1/a + 1/b) \quad \text{.................(6),}$$

and the "power" F by

$$F = (\mu - 1)(1/a + 1/b) \quad \text{.................(7).}$$

§ 7. *Secondary image with thin lens.* Some of the light from S (Fig. 2) which falls on BK will be reflected and will strike AK. Here it will be again reflected and then, on refraction at BK, it will pass out of the lens, forming a secondary image of S at T_2. Let $MT_2 = v_2$.

If Y_2 lie on T_2K and if $T_2Y_2 = T_2B$, the optical equation is

$$XK + KY_2 = 3\mu AB \quad \text{....................(8),}$$

since the light which moves along the axis has traversed the thickness of the lens three times. This equation only differs from the optical equation (1) corresponding to the primary image by having 3μ in place of μ and v_2 in place of v. Hence the method of § 6 gives at once

$$1/u + 1/v_2 = (3\mu - 1)(1/a + 1/b) \quad \text{..............(9).}$$

Hence, if f_2 be the focal length and F_2 the power of the lens for the secondary image,

$$\frac{1}{f_2} = (3\mu - 1)\left(\frac{1}{a} + \frac{1}{b}\right) = \frac{3\mu - 1}{\mu - 1} \cdot \frac{1}{f}, \quad \text{...........(10),}$$

and $$F_2 = \frac{3\mu - 1}{\mu - 1} F \quad \text{......................(11).}$$

The symmetry of (9) shows that if T_2 is the secondary image of S, then S is the secondary image of T_2. If $\mu = 1{\cdot}5$ we have $f_2 = \frac{1}{7}f$.

If we measure both f and f_2, we can find $\mu - 1$ from the equation

$$\mu - 1 = \frac{2f_2}{f - 3f_2} = \frac{2F}{F_2 - 3F} \quad \text{..............(12).}$$

The method can be applied at once to rays which have suffered 4, 6 ... reflexions. If F_{2n} be the power for rays which have been reflected $2n$ times,

$$F_{2n} = \frac{(2n+1)\,\mu - 1}{\mu - 1}\,F.$$

§ 8. *Images by once reflected rays.* Two images of S will be formed by rays which suffer a single reflexion. One image will be formed by reflexion at the surface AK. With this we are not here concerned except when that surface is concave; in that case, if S be placed at the centre of curvature of AK it will coincide with its own image.

A second image, S_1 (Fig. 2), will be formed by rays which have suffered one reflexion at BK and two refractions at AK. Let $S_1M = u_1$ and let u_1 be positive when the image, S_1, is real.

If X_1 lie on S_1K and if $S_1X_1 = S_1A$, the optical equation is

$$XK + KX_1 = 2\mu AB.$$

Multiplying this equation by $2/h^2$ and using the results of § 6, we have

$$(1/u + 1/a) + (1/u_1 + 1/a) = 2\mu\,(1/a + 1/b),$$

or $$1/u + 1/u_1 = 2\,(\mu - 1)\,(1/a + 1/b) + 2/b = 2/f + 2/b.$$

If S be adjusted so that the image S_1 coincides with S, and if p denote the common value of u and u_1 in this case,

$$P = 1/p = \tfrac{1}{2}\,(1/u + 1/u_1) = 1/f + 1/b \quad \text{...........(13).}$$

Similarly, if an object at a distance q from M on the other side of the lens coincides with its image formed by rays reflected once at AK and refracted twice at BK,

$$Q = 1/q = 1/f + 1/a \quad \text{...................(14).}$$

Adding (13) and (14), we have

$$1/p + 1/q = 2/f + (1/a + 1/b) = \{2 + 1/(\mu - 1)\}\,.\,1/f.$$

Thus, if p, q and f are known, we can find $\mu - 1$ from the equation

$$\mu - 1 = \frac{1/f}{1/p + 1/q - 2/f} = \frac{F}{P + Q - 2F} \quad \text{........(15).}$$

The value of $\mu - 1$ found by (15) from f, p and q can be compared

with that found by (12) from f and f_2. If we equate these two values of $\mu - 1$, we obtain

$$2(P+Q) = F + F_2 \qquad (16).$$

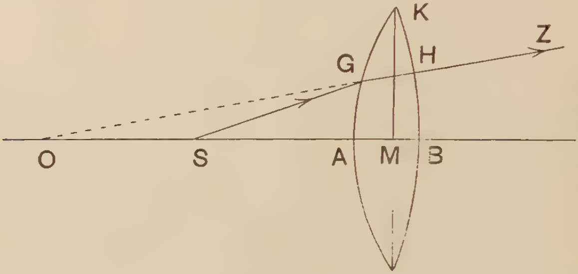


Fig. 3.

Equation (13) can also be obtained as follows:—Since S coincides with its own image, the ray SGH (Fig. 3) must be normal to the surface BK. Hence the emergent ray HZ is directed from O, the centre of curvature of BK, and thus O and S are conjugate points. When the lens is thin, we may put $MO = b$ and $MS = p$. Then

$$1/p - 1/b = 1/f,$$

which agrees with (13). The method of determining the radius b by making an object S coincide with its image by reflexion at BK is due to C. V. Boys.

If BK be so strongly concave that $1/f + 1/b$ is negative, p will be negative and then it will be impossible to make a real object coincide with its own image, and Boys's method cannot be applied. Since, however, the surface BK is concave, the radius b can be found directly by making an object on the B-side of the lens coincide with its image by reflexion at BK. If AK be smeared with vaseline, no image will be formed by reflexion at AK and thus confusion will be avoided.

§ 9. *Experimental details.* A spectacle lens (price 9*d.*) is suitable for the experiment; its primary focal length should be considerable—say, a metre,—so that its thickness may be neglected without much loss of accuracy. The primary focal length f is found by aid of a *very* distant object (500 metres) or of a *good* plane mirror, or by measuring the distances of an object and its image from the lens. An ordinary optical bench will not be long enough to allow the focal length to be conveniently found by the minimum distance method.

The secondary focal length f_2 is found on an optical bench (Fig. 4). One of the sliding carriages of the bench carries a tube

blackened internally, and the lens is attached to this tube*. A second carriage bears a plate pierced by a pinhole and furnished with cross-wires S. The hole is illuminated by a flame. The third carriage bears a pin T which may conveniently be adjustable in a plane perpendicular to the length of the bench, and T is adjusted to coincide with the secondary image of S. In making the adjustment, a lens of about 15 cm. focal length, held in a clip, is of much assistance.

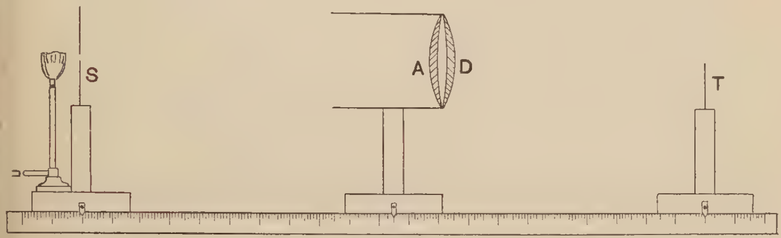


Fig. 4.

The secondary focal length is best found by the minimum distance method. The minimum distance between the cross-wires and the secondary image is $4f_2 + t$, where t is the distance between the principal points corresponding to the secondary image. The lens being "thin," t is negligible and hence the minimum distance may be taken as $4f_2$.

The measurement is very quickly made if S is placed at about $2f/7$ from the lens; the secondary image of the cross-wires will then be at a distance of about $2f/7$ from the lens on the other side. The minimum distance is then easily found.

The distance p is found by making a well illuminated pin coincide with its own image when the face AK of the lens is turned towards the pin. A black surface should be placed behind the lens. The distance q is found in a similar manner.

§ 10. *Practical example.* The following results were obtained in an experiment by J. L. Barritt. A double convex spectacle lens of a nominal power of one dioptre was used.

Primary focal length (found by plane mirror) $= f = 101{\cdot}85$ cm. Hence, power $= F = 1/f = 0{\cdot}009818$ cm.$^{-1} = 0{\cdot}9818$ dioptre.

Minimum distance between object and secondary image $= 4f_2 = 59{\cdot}62$ cm. Hence $f_2 = 14{\cdot}905$ cm. and $F_2 = 1/f_2 = 0{\cdot}06709$ cm.$^{-1}$. Thus, by (12), § 7,

$$\mu = 1 + \frac{2f_2}{f - 3f_2} = 1 + \frac{29{\cdot}81}{57{\cdot}135} = 1{\cdot}5218.$$

* Fig. 4 shows *two* lenses attached to the tube for the experiments described in § 12.

Distance from lens of pin and coincident image formed by light reflected at $BK = p = 51{\cdot}90$ cm. Hence $P = 1/p = 0{\cdot}019268$ cm.$^{-1}$.

Distance from lens of pin and coincident image formed by light reflected at $AK = q = 52{\cdot}38$ cm. Hence $Q = 1/q = 0{\cdot}019091$ cm.$^{-1}$. Hence, by (15), § 8,

$$\mu = 1 + \frac{F}{P + Q - 2F} = 1 + \frac{0{\cdot}009818}{0{\cdot}018723} = 1{\cdot}5244.$$

This value agrees closely with that obtained from the secondary image.

Also $2(P + Q) = 0{\cdot}07672$ cm.$^{-1}$ and $F + F_2 = 0{\cdot}07691$ cm.$^{-1}$. By (16), § 8, these two quantities should have the same value.

§ 11. *Secondary focal lengths for a system of two thin lenses in contact.* Let AKB, CLD (Fig. 5) be two thin lenses in contact. Let the radii of the faces AK, BK, CL, DL be a, b, c, d, the radii being counted positive when the surfaces are convex, as in Fig. 5. Let the planes KM, LN through the edges of the lenses cut the axis in M, N, and let $MK = NL = h$. Let S be a luminous object point and let T_2 be one of its secondary images formed by rays which have suffered two reflexions in their passage through the system. Let $SM = u$, $NT_2 = v_2$. Let the refractive index of the lens AKB be μ and let that of the lens CLD be μ'.

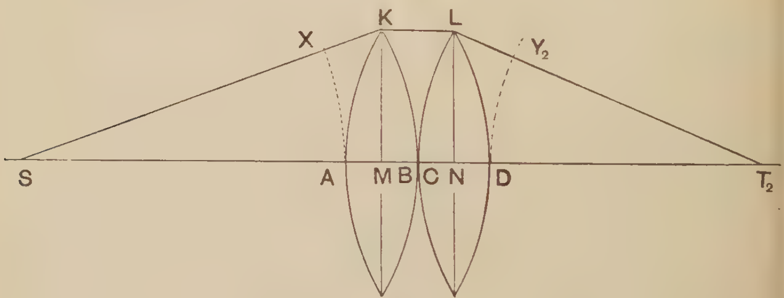


Fig. 5.

The position of T_2 is found by making the optical length of the path from S to T_2 for the ray which starts along SK equal to that for the ray which starts along SA.

Since the lenses are "thin" and since the angle KSA is "small", the ray between K and L is never inclined to the axis at more than a "small" angle, and consequently the optical equation will be, to the accuracy required, the same as if the path of the ray were actually $SKLT_2$.

If X, Y_2 be points on SK, T_2L such that $SX = SA$ and $T_2Y_2 = T_2D$, the optical length of the path from X to Y_2 is to be equal to that from A to D.

Now, by the methods of § 6, we have

$$SK = u + \tfrac{1}{2}h^2/u, \quad T_2L = v_2 + \tfrac{1}{2}h^2/v_2$$

$$SX = SM - AM = u - \tfrac{1}{2}h^2/a, \quad T_2Y_2 = T_2N - DN = v_2 - \tfrac{1}{2}h^2/d.$$

Hence

$$XK = SK - SX = \tfrac{1}{2}h^2(1/u + 1/a)$$

$$LY_2 = T_2L - T_2Y_2 = \tfrac{1}{2}h^2(1/v_2 + 1/d).$$

We have also

$$AB = \tfrac{1}{2}h^2(1/a + 1/b), \quad KL = \tfrac{1}{2}h^2(1/b + 1/c), \quad CD = \tfrac{1}{2}h^2(1/c + 1/d).$$

When we equate the optical length of the path *via* K to that *via* A in each of the six cases, we obtain the following six equations; the symbols [DC], [DB] ... indicate the two faces which have acted as reflectors*.

$$\begin{array}{lll}
[DC] & XK + KL + LY_2 & = \mu AB + 3\mu' CD \\
[DB] & XK + KL + LY_2 + 2KL & = \mu AB + 3\mu' CD \\
[DA] & XK + KL + LY_2 + 2KL & = 3\mu AB + 3\mu' CD \\
[CB] & XK + KL + LY_2 + 2KL & = \mu AB + \mu' CD \\
[CA] & XK + KL + LY_2 + 2KL & = 3\mu AB + \mu' CD \\
[BA] & XK + KL + LY_2 & = 3\mu AB + \mu' CD
\end{array}$$

Let the primary focal lengths of AKB, CLD be m, n cm. and let the corresponding powers be M, N cm.$^{-1}$. Then

$$(\mu - 1)(1/a + 1/b) = 1/m = M, \quad (\mu' - 1)(1/c + 1/d) = 1/n = N.$$

Let the secondary focal lengths of AKB, CLD be m_2, n_2 cm. and let the corresponding powers be M_2, N_2 cm.$^{-1}$. Then, by § 7,

$$(3\mu - 1)(1/a + 1/b) = 1/m_2 = M_2, \quad (3\mu' - 1)(1/c + 1/d) = 1/n_2 = N_2.$$

Let $$-2(1/b + 1/c) = 1/w = W.$$

It is worth noting that w is the *secondary* focal length of a "thin" lens of refractive index unity and radii b and c, the surfaces being concave in the case of Fig. 5. The primary focal length of such a lens is infinite. An optical system formed by air enclosed between two spherical soap films is a very close approximation to such a lens.

Let f_2 be one of the six secondary focal lengths and F_2 the corresponding secondary power of the system, so that

$$F_2 = 1/f_2 = 1/u + 1/v_2.$$

Then, when the six optical equations are multiplied by $2/h^2$, we obtain the following values for the six secondary powers:—

* A more general method, applicable to any number of thin lenses in contact, is given in § 14.

[DC]	$F_{DC} = 1/f_{DC} = M + N_2$
[DB]	$F_{DB} = 1/f_{DB} = M + N_2 + W$
[DA]	$F_{DA} = 1/f_{DA} = M_2 + N_2 + W$
[CB]	$F_{CB} = 1/f_{CB} = M + N + W$
[CA]	$F_{CA} = 1/f_{CA} = M_2 + N + W$
[BA]	$F_{BA} = 1/f_{BA} = M_2 + N.$

If the lenses are such that each of the six secondary powers of the system is positive, it will be possible to obtain six real secondary images of a real object.

A convenient system is obtained by using two meniscus lenses of positive powers, and placing them in contact so that their *concave* surfaces face each other. Then M, N, M_2, N_2 are positive and, since b and c are negative, W is positive. To avoid coincident images, M and N should be unequal. Meniscus lenses are used in spectacles, under the name of "periscopic" lenses.

§ 12. *Experimental details.* The six secondary images are readily observed if the system consists of two positive meniscus spectacle lenses (price 9*d.* each) placed in contact, with their concave surfaces facing each other. The following description refers to this case.

The primary and secondary focal lengths of each lens are found just as in § 9.

The radius of curvature of the concave surface of each lens is found by making a pin coincide with its image formed by reflexion at that surface. The *convex* surface is smeared with vaseline to stop reflexion there. The common distance of the pin and its image from the surface is equal to the radius. Since the surface is *concave*, a *square-ended* scale cannot be used to measure the distance; a proper appliance must be used.

The two lenses are then mounted on the tube as in Fig. 4. When S is sufficiently far from the lenses, the six real secondary images of S will be easily seen on looking through the system towards S, provided the eye be far enough from the lenses. If the images are very small, they may be increased in size by bringing the lenses nearer to S. If, however, the distance is made too small, some or all of the six secondary images will become virtual.

The six secondary focal lengths are best found by the minimum distance method. It is best to begin with the one of shortest focal length.

It is impossible to tell by the appearance of any particular secondary image which two surfaces have acted as reflectors for that image, as long as the lenses are *fixed* together. But if we

calculate the six secondary focal lengths from the quantities M, N, M_2, N_2 and W by § 11, we shall find that one of the observed values of the six secondary focal lengths will agree with any selected one of the calculated values, and so we can decide which two surfaces acted as reflectors in each case.

§ 13. *Practical example.* The following is a record of measurements made by G. F. C. Searle, using two meniscus spectacle lenses of nominal powers of one dioptre and of one-half dioptre. Barlow's tables of reciprocals were used; the powers are given to the nearest 0·00001 cm.$^{-1}$.

The primary power of each lens was found from the distances of an object and its image from the lens.

The primary power of AKB; $u = 767{\cdot}6$ cm., $v = 111{\cdot}36$ cm.

Hence

$$M = 1/u + 1/v = 0{\cdot}001303 + 0{\cdot}008980 = 0{\cdot}01028 \text{ cm.}^{-1} = 1{\cdot}028 \text{ dioptre.}$$

For primary power of CLD; $u = 633{\cdot}5$ cm., $v = 306{\cdot}7$ cm.

Hence

$$N = 1/u + 1/v = 0{\cdot}001579 + 0{\cdot}003261 = 0{\cdot}00484 \text{ cm.}^{-1} = 0{\cdot}484 \text{ dioptre.}$$

The secondary power of each lens was found on the optical bench by the minimum distance method.

For secondary power of AKB; minimum distance $= 4m_2 = 56{\cdot}48$ cm.

Hence $M_2 = 1/m_2 = 1/14{\cdot}12 = 0{\cdot}07082$ cm.$^{-1}$.

For secondary power of CLD; minimum distance $= 4n_2 = 120{\cdot}58$ cm.

Hence $N_2 = 1/n_2 = 1/30{\cdot}145 = 0{\cdot}03317$ cm.$^{-1}$.

Radius b of lens $AKB = -46{\cdot}44$ cm.

Radius c of lens $CLD = -35{\cdot}18$ cm.

The signs are *negative* since the surfaces are *concave*.

Hence $W = -2(1/b + 1/c) = 2(0{\cdot}021533 + 0{\cdot}028425) = 0{\cdot}09992$ cm.$^{-1}$.

By § 11, the six secondary powers of the system are given by the following equations:

$$\begin{aligned}
F_{DC} &= M + N_2 &&= 0{\cdot}01028 + 0{\cdot}03317 &&= 0{\cdot}04345 \text{ cm.}^{-1}.\\
F_{DB} &= M + N_2 + W &&= 0{\cdot}01028 + 0{\cdot}03317 + 0{\cdot}09992 &&= 0{\cdot}14337 \text{ cm.}^{-1}.\\
F_{DA} &= M_2 + N_2 + W &&= 0{\cdot}07082 + 0{\cdot}03317 + 0{\cdot}09992 &&= 0{\cdot}20391 \text{ cm.}^{-1}.\\
F_{CB} &= M + N + W &&= 0{\cdot}01028 + 0{\cdot}00484 + 0{\cdot}09992 &&= 0{\cdot}11504 \text{ cm.}^{-1}.\\
F_{CA} &= M_2 + N + W &&= 0{\cdot}07082 + 0{\cdot}00484 + 0{\cdot}09992 &&= 0{\cdot}17558 \text{ cm.}^{-1}.\\
F_{BA} &= M_2 + N &&= 0{\cdot}07082 + 0{\cdot}00484 &&= 0{\cdot}07566 \text{ cm.}^{-1}.
\end{aligned}$$

The following table gives the minimum distance between the cross-wires and each of the secondary images. The corresponding focal length, f_2, is one-quarter of this distance and the corresponding power F_2 is $1/f_2$. The power calculated from M, N, M_2, N_2, W is entered in

the last column but one and the last column shows which two surfaces acted as reflectors.

Minimum distance cm.	Focal length cm.	Observed power cm.$^{-1}$	Calculated power cm.$^{-1}$	Reflectors
19·46	4·865	0·20555	0·20391	*DA*
22·82	5·705	0·17528	0·17558	*CA*
27·91	6·978	0·14331	0·14337	*DB*
34·76	8·690	0·11507	0·11504	*CB*
53·22	13·305	0·07516	0·07566	*BA*
91·86	22·965	0·04354	0·04345	*DC*

It will be seen that there is fair agreement between the observed and the calculated values of the secondary powers.

§ 14. *Secondary focal lengths for a system of n thin lenses in contact.* The method of § 11 is easily extended to n thin lenses in contact. Let P (Fig. 6) be the object and Q a secondary image. Let the lenses be $A_1K_1B_1$, $A_2K_2B_2 \ldots A_nK_nB_n$. Let $\mu_1, \ldots \mu_n$ be the refractive indices and $F_1, \ldots F_n$ the powers of the lenses. Let the radii of the surfaces of the lenses be $a_1, b_1 \ldots a_n, b_n$, the radii being counted positive when the surfaces are convex as in Fig. 6. Let the distance of the edges $K_1 \ldots K_n$ from the axis be h. Let X, Y be points on PK_1, QK_n such that $PX = PA_1$, $QY = QB_n$. Let the planes of the edges of the first and last lenses cut the axis in M_1, M_n; let $PM_1 = u$, and let $QM_n = v$.

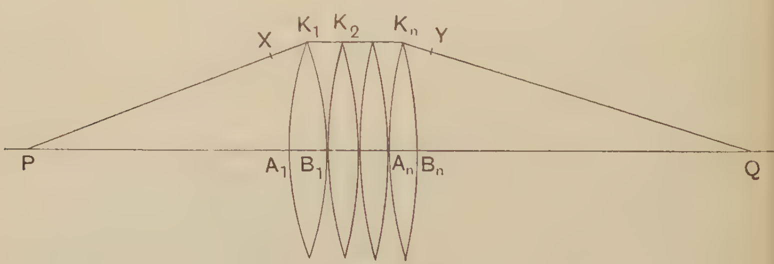


Fig. 6.

Since $F_1 = (\mu_1 - 1)(1/a_1 + 1/b_1)$ and since $A_1B_1 = \frac{1}{2}h^2(1/a_1 + 1/b_1)$, we have

$$A_1B_1 = \tfrac{1}{2}h^2F_1/(\mu_1 - 1),$$

and similarly for the other lenses.

If $W_{12} = -2(1/b_1 + 1/a_2)$, $W_{23} = -2(1/b_2 + 1/a_3)$, ..., we have

$$K_1K_2 = -\tfrac{1}{2}h^2 \cdot \tfrac{1}{2}W_{12}, \quad K_2K_3 = -\tfrac{1}{2}h^2 \cdot \tfrac{1}{2}W_{23}, \ldots.$$

Further, as in § 11,

$$XK_1 = \tfrac{1}{2}h^2(1/u + 1/a_1), \quad YK_n = \tfrac{1}{2}h^2(1/v + 1/b_n).$$

Now let Σ denote summation for *all* the quantities of any type and let $\mathbf{S}$ denote summation for only those quantities which refer to spaces through which a ray has passed *three* times.

The optical length of the path from X to Y is equal to that from A_1 to B_n (see § 11) and hence, if we omit the common factor $\frac{1}{2}h^2$, the general optical equation becomes

$$\frac{1}{u} + \frac{1}{a_1} + \frac{1}{v} + \frac{1}{b_n} - \Sigma\tfrac{1}{2}W - \mathbf{S}W = \Sigma\,\frac{\mu F}{\mu - 1} + \mathbf{S}\,\frac{2\mu F}{\mu - 1},$$

or

$$\frac{1}{u} + \frac{1}{v} + \Sigma\left(\frac{1}{a} + \frac{1}{b}\right) - \mathbf{S}W = \Sigma\,\frac{\mu F}{\mu - 1} + \mathbf{S}\,\frac{2\mu F}{\mu - 1}.$$

Since for any lens

$$\frac{1}{a} + \frac{1}{b} = \frac{F}{\mu - 1},$$

we have

$$\Sigma\,\frac{\mu F}{\mu - 1} - \Sigma\left(\frac{1}{a} + \frac{1}{b}\right) = \Sigma F,$$

and hence

$$\frac{1}{u} + \frac{1}{v} = \Sigma F + \mathbf{S}\,\frac{2\mu F}{\mu - 1} + \mathbf{S}W \quad \text{..............(17).}$$

For any lens

$$F + \frac{2\mu F}{\mu - 1} = \frac{3\mu - 1}{\mu - 1}F = \text{secondary power of lens.}$$

Further, by § 11, W_{12}, W_{23}, ... are the secondary powers of the successive air spaces, and hence the result (17) can be expressed in words as follows:

Secondary power of system = {*Sum of primary powers of lenses traversed once*} + {*Sum of secondary powers of lenses traversed three times*} + {*Sum of secondary powers of air spaces traversed three times*}.

The six secondary powers found in § 11 for a system of two lenses are particular cases of this general result.

If the lenses are so chosen that the primary power of each is positive and the value of every W is positive, *all* the secondary powers of the system will be positive.

When the lens used in § 10 is placed between the lenses used in § 13, the fifteen secondary images are easily seen.

PROCEEDINGS

OF THE

Cambridge Philosophical Society.

Sarcocystis colii, n. sp., a Sarcosporidian occurring in the Red-faced African Mouse Bird, Colius erythromelon.

By H. B. Fantham, D.Sc. Lond., B.A., Christ's College, Cambridge.

[*Read* 5 May 1913.]

(Plate V.)

Introduction.

The "red-faced" African mouse bird, *Colius erythromelon,* is of interest to the naturalist on account of its creeping habit. This small bird is a forest dweller, is not particularly shy, has a general greenish coloration in harmony with its surroundings and possesses buff feathers on the head, while the eye is surrounded by a scarlet ring of bare skin. A fine crest is on the head of the bird.

It has a new interest to the parasitologist, for I have in my possession a skinned specimen of the bird showing a heavy infection with a new Sporozoön, which I name *Sarcocystis colii,* since it presents several morphological features distinct from those found in the Sarcosporidia of other birds. Further, as Colies are considered by Kaffirs as good eating, it is possible that they are a source of sarcosporidiosis in man.

The Sarcosporidia are most frequently found in mammals (including man), but they have been noted as occurring in fowls, ravens and blackbirds, and the spores of the species (*S. rileyi*) occurring in the duck have been described in some detail by Crawley (1911). A few other avian hosts have been recorded by Stiles (1893), but details are not known.

Macroscopic Appearance of the Parasite. Its Distribution in the Host.

The *Colius* was sent to me for examination by Mr J. W. Cutmore, taxidermist to the Liverpool Museum, to whom my best thanks are due. The bird had been skinned recently and was quite fresh. Superficially it showed a number of elongate, white, opaque patches or streaks, distributed over the body surface, in the muscles (Pl. V). Teased in normal saline solution, one of these patches showed large numbers of perfect Sarcosporidian spores. The opaque bodies, then, were the large trophozoites of the Sarcosporidian, commonly known as Miescher's tubes or sarcocysts.

The parasites were seen superficially scattered over almost the entire body surface. Concentrations occurred along the cervical muscles near the oesophagus, the inter-coracoidal region, the insertion of the humerus and scapula, the muscles around the maxilla and near the preen glands of the tail. The areas around the insertion of the limbs were more heavily parasitised than their distal extremities, the dorsal surface containing more Miescher's tubes than the ventral. Parts bearing long quills, such as the muscles around the ulna, contained few or no parasites.

When the bird was opened the whole thickness of the breast muscles was seen to contain numbers of tubes, lying parallel to the long axis of the muscles. They had also penetrated the intercostal muscles and formed whitish streaks on the endothelial lining of the body-cavity. The bases of the abdominal muscles showed parasites on their peritoneal surfaces. The pericardium, connective tissue around the carotid arteries and jugular veins and the mesentery of the intestine also contained a few scattered trophozoites. The cardiac muscles contained a large number of tubes of smaller size than those in the pectoral muscles.

The most heavily parasitised region of the body was the large pectoral muscles. The right and left sides of the body seemed to contain about the same number of Miescher's tubes. The general dorsal surface (Pl. V) showed rather more trophozoites superficially than the ventral, but the enormously increased volume due to the pectoral muscles gave a preponderance of parasites on the ventral part of the body.

The effect of the parasite on the host is doubtful. While the bird showed no very obvious external signs of disease, yet there was no fat on the body nor around the viscera. The walls of the heart appeared somewhat thin, compared with those of an unparasitised bird. It is known that Sarcosporidia produce a toxin, sarcocystin, lethal to rabbits.

I am informed that the bird was not noticeably infected with ectoparasites, and so no indication as to the transmission of the parasites is available from that source.

Structure of the Parasite.

A brief account of the morphology of the parasite may now be given. When a fresh preparation of a teased Miescher's tube is examined the spores, sometimes known as Rainey's corpuscles, appear as elongate, sickle-shaped bodies with a clear centre and a distinct refractivity. The general cytoplasm seems homogeneous, though a vacuole may be present. One end of the organism is more pointed than the other. The spores examined by me exhibited very little power of progression.

Stained smears and sections show that the trophozoites present an elongate, tubular body with a definite envelope and a chambered or trabeculate structure. Various stages of the organism can be recognised. The large trophozoites contain numerous spores. Certain of them are solid owing to the spores practically filling their interior, but the oldest and longest of them show a hollow centre, as the spores there die and degenerate, leaving only the trabeculae. By the dehiscence of the tubes of the muscular system, invasion of the connective tissues is brought about.

Large Miescher's tubes are 2·5 mm. in length and have a breadth not exceeding 1 mm. They occur in the skeletal musculature. The cardiac muscle contains much smaller tubes, possibly the result of the less size of the cardiac muscles invaded, as well as the possibility of younger trophozoites being present there. The Miescher's tubes are easily seen in sections of infected tissue as they show great affinity for basic stains. Further details of the parasite as seen in sections will be given in a later publication.

The internal structure of the sickle-shaped spore, as seen in stained preparations, may be briefly outlined. Polymorphism occurs among the spores, at least two types being distinguishable: the one, narrow with more deeply staining contents, the other broader with paler contents. These spores are about $5\,\mu$ to $7\,\mu$ long, while their breadth varies from $1·5\,\mu$ to $2·5\,\mu$. The dimorphism may be due to growth and division. Occasionally "giant" spores are seen. The nucleus is not central but is near the blunter end. The structure of the nucleus varies. Sometimes it is vesicular with a karyosome, which may be central or excentric. At other times the chromatin is evenly distributed in granules throughout the nucleus. The differences in arrangement of the chromatin are due to cyclical development. Near the more pointed end of the spore a polar vesicle is often seen, and sometimes the remains

of a capsulogenous nucleus can be distinguished. A few well-stained specimens have shown portions of probable polar filaments. The commencement of protrusion of a filament has been followed. The polar capsule is seen as a vacuole in preparations stained *intra vitam.* Some spores show a curious sinuous line which might, perhaps, be compared with the sutural line of a Myxosporidian spore. Such a sinuous line is not to be confused with longitudinal division, which has been clearly followed in numerous, broad, bean-shaped spores. It should be mentioned that the spores show no marked metachromatic granules, nor have they the complicated structure described by Crawley for *S. rileyi* from the duck (see Fantham and Porter, 1912).

On account of the polymorphism and somewhat small size of the spores, the lack of marked metachromatic granules, and the presence of a definite polar vesicle, I propose the new specific name *colii* for this example of the genus *Sarcocystis.* Perhaps in the future, when our knowledge of the Sarcosporidia is more complete, it may be found advisable to place *S. colii* in an older species, but at present it certainly possesses several clear, distinguishing features.

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Crawley, H. (1911). "Observations on *Sarcocystis rileyi* (Stiles)." *Proc. Acad. Nat. Sci., Philadelphia*, Vol. LXIII, pp. 457—468, 1 plate.

Fantham, H. B. and Porter, Annie (1912). "The Structure and Homology of the Microsporidian Spore, as seen in *Nosema apis.*" *Proc. Camb. Philosoph. Soc.*, Vol. XVI, pp. 580—583.

Stiles, C. W. (1893). "On the Presence of Sarcosporidia in Birds." *Bulletin* 3, *Bureau of Animal Industry*, U.S. Dept. Agric.

EXPLANATION OF PLATE V.

Dorsal aspect of *Colius erythromelon*, showing the distribution of trophozoites of *Sarcocystis colii.* Approximately natural size.

SARCOSPORIDIA IN *COLIUS ERYTHROMELON.*

Observations on Hirneola auricula-judae, *Berk.* (*"Jew's ear"*). (Preliminary communication.) By M. J. Le Goc, B.A., Fitzwilliam Hall (University Frank Smart Prizeman, 1912). (Communicated by Mr F. T. Brooks.)

[*Read* 28 April 1913.]

Fries refers to the Jew's ear fungus as "antiquitus celebrata." This celebrity of the fungus was due to its supposed medicinal properties; for, on account of its fanciful resemblance to the *fauces*, it was frequently used as a cure for sore-throats in the days when the external form of a plant was thought to be a sufficient guarantee for its therapeutic quality. According to Berkeley, it was occasionally sold at Covent Garden for such a purpose as late as 1857.

The Jew's ear fungus received several different names: *Tremella auricula-judae**, *Exidia auricula-judae*†, *Exidia auriformis*, *Auricularia sambucina*‡, etc., until Fries and Berkeley fixed the name *Hirneola auricula-judae*§. *Hirneola* is derived from "hirnula," a small jug; "judae" refers to the host, the Elder tree, on which the legend represents Judas as having hanged himself; while the name "auris," ear, has been attached to it throughout its history on account of the ear-like form it often assumes. The popular name of "Jew's ear" is a corruption of Judas' ear.

The only person who has hitherto investigated the biology of this fungus is Brefeld, whose work was confined to a study of the germination of the spores and the structure of the fructifications‖. Mr F. T. Brooks suggested that I should attempt to establish pure cultures of this fungus and investigate some points of its biology which have hitherto been obscure.

As is well known, the Jew's ear fungus belongs to the Auriculariaceae, a group of the Basidiomycetes characterised by transversely-septate basidia arranged in a definite hymenium which becomes freely exposed during spore formation¶.

This fungus is of world-wide distribution and is of frequent occurrence in the neighbourhood of Cambridge. Most of the

* Lin. *spec.* (1625).

† Fr. *Syst. II.* p. 221; Berk. *Cryptogamic Botany* (1857), p. 355.

‡ Mart. *Erl.* p. 459.

§ Fr. *Fung. Nat.* p. 24; *Hym. europaei*, p. 695 (1874); Berk. *Outl. of Brit. Fung.* p. 289 (1860).

‖ Brefeld, *Untersuchungen aus dem Gesammtgebiete der Mykologie*, VII. Heft, pp. 70–76.

¶ Fuckel, *Symbolae*, p. 29; Winter, *Pilze*, p. 283.

material used has been collected from the vicinity of Byron's Pool. It is found abundantly on Elder bushes both living and dead, and also on dead branches and trunks of Elm trees in moist places. It has been suggested that the *Hirneola* growing on Elm is not perhaps identical with the *Hirneola* which lives on Elder: the hymenium is decidedly freer from folds, but as a matter of fact spores from *Hirneola* on Elder have germinated quite easily and are growing quite happily on blocks of Elm wood. It is usually stated that the fructifications reach a size varying from 4 cm. to 7 cm. in diameter*; but it is often much larger and a specimen collected in this locality measured 21·6 cm. by 12 cm.

The fructifications which are gelatinous under moist conditions shrivel to black, horny masses in a dry atmosphere. In this form the fungus remains alive for at least five months, because after this interval it revives again when moistened and produces an abundant supply of spores—as in the case of *Stereum purpureum* and some other fungi†.

If a fructification which has been moistened is suspended over a sterilized glass slide, spores are produced after an interval of some 10 hours and fall on the slide where they form a thick white deposit. They can then be transferred into tubes of sterilized water or directly picked up with a platinum needle and used for cultures.

Germination of spores on the fructification.

If the fruit body is kept moist for two or three days and the spores allowed to accumulate on its surface, it is often found that the spores germinate *in situ*. The mycelia reach a considerable length and in time form roundish protuberances projecting from the surface of the fructification. These projections prove to be webs of hyphae entangled together and enclosing a large number of ungerminated spores in a good state of preservation. The hyphae themselves soon undergo a process of disintegration; the glycogen contained in them breaks into small globules, regularly arranged in a chapelet which simulates a chain of "oidia." The ungerminated spores look healthy after an interval of more than three months and are capable of germination.

Germination of spores in liquids.

The spores can be cultivated in hanging drops in sealed cells and easily observed under the microscope. Under these

* Massee, *Diseases of Cultivated Plants*, p. 404.

† Brooks, F. T., "Silver-leaf Disease," *Journ. Agric. Sci.* Vol. IV. p. 143; Buller, A. H. R., *Researches on Fungi*, p. 106.

conditions germination has not been observed in distilled water, but in Elder-wood decoction it occurs occasionally. However, if a drop of the decoction is laid at the bottom of the same cell instead of being suspended from the cover glass, germination occurs regularly. The probable explanation of this anomaly is that the spores, being heavier than the liquid, fall to the bottom; thus in the case of the hanging drops the spores are only partially immersed, and osmosis is not carried out in the proper manner. In tubes or watch glasses the spores germinate very readily in Elder-wood decoction and occasionally in distilled water.

The hyphae grow rapidly, reach a considerable length, branch freely, and when the food and aeration have become deficient, they undergo disintegration ending in the formation of abundant drops of glycogen.

Here I must note some divergences with Brefeld's results. In the first place, Brefeld found that eight days immersion or more were required for the germination of spores*, while my observations show that two or three days are sufficient; after this interval large drops of glycogen develop in any ungerminated spores which at a later stage become ruptured and emit their contents. Secondly, Brefeld describes as of regular occurrence the formation of conidia on the germ tubes of the spores†. Such bodies have never been observed during the whole series of my experiments.

Pure cultures of the fungus.

(*a*) *Cultures in decoctions of Elder-wood + agar or gelatine.* The Elder-wood decoction has been solidified with 2 °/ₒ agar or 10 °/ₒ gelatine, after previous sterilization by the usual methods.

In the case of agar cultures the spores germinate quite easily and after some 15 days patches of mycelia become visible; these mycelia grow with some difficulty and not always with equal success. In my cultures they are still developing without any apparent attempt to produce fructifications.

The gelatine cultures are more interesting, showing an almost luxuriant growth. At first a woolly mass of mycelium appears on the surface: but the gelatine is soon liquefied and the fungus sinks into the medium, assuming a definite form and developing into bodies which imitate in shape and structure the fructifications of the Jew's ear fungus. At this stage a number of hyphae become very stout, irregular in form and branch freely in all directions. It has not yet been possible to determine the ultimate fate of these

* Brefeld, *loc. cit.* p. 72.
† Brefeld, *loc. cit.* pp. 73—76.

interesting structures. The production of a fruit body within a liquid is at least unusual in the case of fungi.

(*b*) *Cultures on wood.* Pure cultures of the fungus have been established on blocks of Elder, Lime and Elm wood, about 5 cm. in length and from 1 cm. to 4 cm. in diameter. These blocks of wood are enclosed in tubes or flasks containing a certain amount of water, and sterilized in the usual manner. Inoculation of the blocks of wood has been effected by transferring to them the spores of the fungus by means of a sterilized platinum needle. With few exceptions germination has always occurred. After about 20 days a white woolly growth appears on the spots infected and spreads all round forming a thick envelope. These woolly hyphae show at present signs of active growth, indicated by large drops of liquid which are oozed out as in the well-known example of *Pilobolus.* Also large, stout, irregular hyphae are being formed as in the case of the culture on gelatine. In some of the cultures on wood blocks fructifications have begun to appear, more especially on blocks exposed to a fair amount of light. The formation of fruit bodies on artificial cultures of one of the higher fungi is as interesting as it is rare.

The penetration of the hyphae inside the wood has been tested at intervals. It is very rapid, and after three months the hyphae have run all through the tissues. The path followed is along the vessels and tracheids, with penetration through the pits and more frequent branching in the medullary rays. For some time the fungus seems to be satisfied with the food found inside the cells, but from examination of material found in nature, it is evident that when hunger presses hard on the fungus, it encroaches on the cell walls: the xylem is then delignified, the hyphae bore their way locally through the walls which are gradually consumed and the ultimate result is that the whole tissue becomes spongy, crumbles when rubbed with the finger, and consists more of the hyphae than of the original tissue of the tree.

Inoculations of healthy Elder bushes with the fungus.

Inoculations of mycelia and spores have also been tried on living branches of Elder bushes. The spores have germinated, and the mycelia have penetrated the wood in the manner just described, but the process of penetration is slower than on dead wood and no further details are at present available.

I wish, as a pleasant duty, to express my hearty thanks to Mr F. T. Brooks for his kind guidance as well as for his repeated suggestions during these investigations.

Notes on additions to the Flora of Cambridgeshire. By A. H. EVANS, M.A., Clare College.

[*Read* 28 April 1913.]

THE publication last year in our *Proceedings* (Vol. XVI., Pt. iii.) of my "Short Flora of Cambridgeshire" has resulted, according to expectation, in a further revival of the interest now taken in Field Botany, and in considerable accessions to the county list. We are naturally much indebted to our senior botanists, several of whom are specializing in various divisions of the Cryptogams, but we feel that even more stress should be laid on the co-operation of the undergraduate members of the University, who have much time at their disposal in which to range the country, and keep submitting to us specimens of the plants with which they meet, so that it becomes possible to determine the exact forms, and settle whether they are new to Cambridgeshire or to any of its districts. The excursions under the guidance of Mr Moss come in the same category, while, extending as they do beyond the county limits, they enable us to institute comparisons between our Flora and that of the immediate neighbourhood, as well as to familiarize our students with rare plants that might not otherwise come under their notice. The extreme north of the county, however, is still in need of careful exploration.

Species which had been lost to sight have been rediscovered, and several that are new to our area met with, though, as will be seen from the subjoined list, new varieties are no less common than new species. It is most important to decide which of the varieties recognised on the Continent are also to be found in England, and we have been able to contribute to this desirable result, chiefly through the penetration of Mr Moss, who will incorporate the information in his forthcoming British Flora.

The genera *Arctium*, *Lycium* and *Bartsia*, as well as the aggregate *Polygonum aviculare*, have been entirely reconsidered in the light of recently published work, while several of the Latin names in the catalogue below have been corrected. As regards the Cryptogams we have been able to add somewhat to the lists furnished by our former coadjutors. Mr R. S. Adamson has sent a large number of notes on the Musci, in addition to remarks on the Phanerogams, and Mr Compton has given us the results of his year's work on the *Hepaticæ.* Mr F. T. Brooks is still studying the Fungi, but wishes to delay the publication of his results until he has examined further material.

The most striking occurrence of the year has been the discovery of a considerable quantity of *Prunella laciniata* in a fallow at Hardwick by an undergraduate (Mr A. W. Graveson of King's College), followed by the further discovery by Mr Moss that it was accompanied by a still greater quantity of the putative hybrid between it and *P. vulgaris.* This is new to the British Isles. The field furnished several other interesting plants, among others the radiate form of *Centaurea nigra*, found by the writer. The last-named was new to the county, as were *Epilobium roseum* from the town of Cambridge (Mr Compton), *Sparganium neglectum* from Waterbeach (Mr Moss), and *Anthoxanthum Puellii* from Gamlingay (Mr J. E. Little). The wonderfully productive greensand at the village just mentioned was also responsible for *Helianthemum Chamæcistus* subvar. *viridis*, and the rare *Arnoseris pusilla* was found there in abundance in the corner of a field, which had previously escaped investigation. Two uncommon forms of Orchids were met with at Dernford Fen by Mr Moss, *Orchis maculata* var. *O'Kellyi*, and *Habenaria* (*Gymnadenia*) *Wahlenbergi*; while the Babington Herbarium proved to contain *H.* (*G.*) *densiflora* from the same place. These two forms of *Habenaria* are new to the British Isles. *Juncus bufonius* var. *fasciculatus* was determined from Upware by Mr Adamson, who also found *Quercus sessiliflora* in Gamlingay Wood.

We have ascertained that a green as well as a glaucous form of *Stellaria Dilleniana* (= *S. glauca* or *palustris auct.*) grows in Wicken Fen, but it is too early to state definitely how many of the phases of this variable species the county really possesses. *Myosotis collina* var. *Mittenii* from Chippenham and the hybrid Willows *Salix alba* × *pentandra* and *S. cinerea* × *viminalis* must further be placed to the credit of Mr Moss.

Not one of our extinct plants has been rediscovered, but *Ranunculus parviflorus* has been met with, after the lapse of many years, at Hardwick and Caldecot; Mr Moss has found *Acorus Calamus* at Upware; Mr Shrubbs *Myosurus minimus* near Ickleton and *Muscari racemosum* near Hinxton; and the present writer a whole field of *Thlaspi arvense* near Pampisford, besides a quantity of *Pedicularis sylvatica* and new stations for *Claytonia perfoliata* and *Malva moschata* at Gamlingay. Both *Lycium chinense* and *L. barbarum* have been proved by Mr Moss to occur in the county; the same may be said of *Bartsia verna* and *B. serotina* and several of Professor Lindman's segregates of *Polygonum aviculare,* while various difficulties concerning the genus *Arctium* have been solved by the writer, who finds all the British forms in Cambridgeshire. Mr Adamson has named tentatively some of the Rosæ and Rubi, which will probably prove to be additions to our list.

ADDITIONS AND CORRECTIONS OF THE GENERAL LIST OF SPECIES.

I. ANGIOSPERMÆ.

Ranunculus trichophyllus var. *Godronii* Gren. 1.

R. auricomus 4.

R. parviflorus. Refound at Hardwick and Caldecot.

Erophila virescens 1. (The East Anglian plant seems to be rather *E. glabrescens* Jord. than *E. virescens.*)

Lepidium Draba 4.

Thlaspi arvense 4. (Also in profusion near Pampisford.)

Helianthemum Chamæcistus subvar. *viridis* Irvine 4 (Gamlingay, Moss).

Viola sylvestris var. *punctata* 3, 5.

Stellaria palustris must now stand as *S. Dilleniana.* We have in Wicken Fen both a green and a glaucous form.

Arenaria trinervia. Our form is var. *typica* Moss (var. nov. ined.).

Claytonia perfoliata 4 (Evans).

Malva moschata 4 (the typical plant).

[*Geranium lucidum.* Refound by Mr A. W. Graveson, at Linton.]

Ononis spinosa 4.

Rubus sylvaticus Wh. and N. 5 (*fide* Adamson).

R. macrophyllus Wh. and N. 4, 5 (*fide* Adamson).

Rosa canina var. *lutetiana* (Lem.) 1, 2, 3, 5 (*fide* Adamson).

„ „ *dumalis* (Bechst.) 1, 3, 5 (*fide* Adamson).

„ „ *dumetorum* (Thuill.) 1, 5 (*fide* Adamson).

„ „ *urbica* (Lem.) 3, 5 (*fide* Adamson).

Saxifraga granulata 4.

Epilobium roseum 1 (Cambridge, Compton).

Smyrnium Olusatrum 6.

Matricaria suaveolens 4.

Arctium majus must stand as *A. Lappa* L. (see Journ. of Bot. April 1913).

A. nemorosum (*auct. plur.*) must stand as *A. vulgare* subvar. *pycnocephalum* (Evans).

A. pubens Bab. must stand as *A. vulgare* (Evans).

Centaurea nigra forma *radiata* 5 (Evans).

Arnoseris minima. Found in quantity at Gamlingay by Mr J. E. Little of Hitchin.

Primula veris var. *suaveolens* 5 (Moss, Evans).

Vinca minor 4.

Blackstonia perfoliata 4.

Myosotis collina var. *Mittenii* (Baker) 6 (Moss).

Lycium barbarum L. and *L. chinense* (Mill), both occur, not uncommonly, in the county.

Bartsia verna 5.
Bartsia serotina 1, 5, etc. } These must replace *B. Odontites.*

Pedicularis sylvatica. In some quantity at Gamlingay in 1912 (Evans).

Prunella vulgaris × *laciniata* 5 (Hardwick, Moss).

P. laciniata 5 (Hardwick, A. W. Graveson).

Lamium hybridum 6.

Atriplex patula var. *erecta.* Babington's plant is merely a variety of *patula* and not the plant of Hudson or Smith.

Polygonum aviculare. This aggregate must now stand as follows for the county:

P. aviculare L. (= *P. heterophyllum* Lindman) 1, 5, etc.
P. rurivagum Boreau 3.
P. æquale Lindman 1, 5, etc.

P. maculatum 4.

Ulmus procera Salisb. must stand as *U. nitens* Mœnch., the former proves to be a synonym of *U. campestris* L.

Myrica gale. In quantity at a new station in Wicken Fen (Evans).

Alnus glutinosa Gaertn.

var. *typica* Moss (the common form of southern England).
var. *macrocarpa* (Loudon) 1 (Chippenham Fen, Moss).

Quercus sessiliflora 5 (Gamlingay Wood, Adamson).

Salix alba × *pentandra* 1 (Moss).

Salix cinerea × *viminalis* 1 (Moss).

Populus canadensis. Our common tree should stand as × *P. serotina.* It is always male, and always planted; × *P. canadensis* is rare in gardens near Cambridge, though "subwild" in Suffolk; it is always female.

Liparis Loeselii. The specimen mentioned in our former list as from Gamlingay has no doubt been accidentally transferred from one sheet of the Power Herbarium at Reigate to another: for a similar case has occurred with a specimen of *Malaxis paludosa* labelled Burwell Fen, where the same explanation may be given. In other words the specimens have probably been interchanged.

Orchis maculata var. *O'Kellyi*, Sawston Fen (Moss).

Habenaria (*Gymnadenia*) *Wahlenbergi*, Sawston Fen (Moss).

H. (*G.*) *densiflora*, Dernford Fen (Bab. Herbarium, *fide* Moss).

Iris fœtidissima. Several plants at Pampisford (Evans).

Juncus bufonius var. *fasciculatus* 1 (Upware, Adamson).

J. compressus. Again found in quantity at Fulbourn and Waterbeach (new stations) 1.

J. effusus 6.

J. subnodulosus 2.

Sparganium neglectum Beeby. 1 (Waterbeach, Moss).

Acorus Calamus 1. Add Upware (Moss).

Carex vulpina var. *nemorosa* 2.

Anthoxanthum Puellii 4 (Gamlingay, J. E. Little).

II. Gymnospermæ.

Juniperus communis. Add Roman Road, near Linton (Compton).

III. Bryophyta.

Musci.

The following list contains a few additions by Mr R. S. Adamson to the list of mosses of the county recorded by Rev. P. G. M. Rhodes in 1911.

The nomenclature followed is that of Dixon and Jamieson's *Student's Handbook of British Mosses*, Ed. 2, published in 1904. The divisions of the county are the geological ones used in our former list.

The following species are new to the county list with the exceptions of the species of *Sphagnum*, which, however, were only recorded on old authority. A species of *Sphagnum* also occurs on Chippenham Fen; but this Mr Adamson has not seen:

Sphagnum cymbifolium Ehrh. var. *squarrulosum* Nees and Hornsch. 4.

S. acutifolium Ehrh. var. *subnitens* Dixon 4.

Polytrichum gracile Dicks. 4.

Pottia intermedia Fürn. 3.

Tortula muralis Hedw. var. *aestiva* Brid. 6.

Barbula cylindrica Schp. 3.

Orthotrichum cupulatum Hoffm. 3.

Aulacomnium androgynum Schwaeg. 6.

Bryum inclinatum Bland. 4, 6.

B. intermedium Brid. 2.

B. erythrocarpum Schwaeg. 4.

Mnium affine Bland. 5, 6.

Brachythecium rivulare B. and S. 3, 4.

B. glareosum B. and S. 3.

Eurhynchium murale Milde 3.

Hypnum polygamum Schp. 1.

H. commutatum Hedw. 1.

In addition to the above, the following have been found in divisions of the county additional to those mentioned in the *Flora* or are confirmations of old records:

Catharinea undulata W. and M. 5.
Polytrichum juniperinum Willd. 6.
Dicranella heteromalla Schp. 4.
D. varia Schp. 4.
Pottia truncatula Linbd. 3.
P. minutula Lindb. 5.
Tortula Vahliana Wils. 6. On a bank near Kennett.
T. subulata Hedw. 4.
T. ruralis Ehrh. 1, 6.
Barbula fallax Mitt. 5.
B. unguiculata Hedw. 5.
Bryum cæspiticum L. 1. Recorded by Relhan.
B. capillare L. 5.
Mnium cuspidatum Hedw. 1, 6. Recorded by Relhan.
M. rostratum Schrad. 1. Recorded by Relhan.
Brachythecium albicans B. and S. 1. (Wisbech).
B. velutinum B. and S. 5.
B. plumosum B. and S. 2. Recorded by Relhan.
Eurhynchium swartzii Schp. 1.
E. striatum B. and S. 2. Recorded by Relhan.
Hypnum elodes Spruce 5.
Hypnum aduncum Hedw. var. *paternum* Sanio 5.
H. cupressiforme L. 1.
H. molluscum Hedw. 5.
H. schreberi Willd. 4. Previous record 5, Relhan.
Hylocomium splendens B. and S. 6. Previous record 5, Relhan.
H. squarrosum B. and S. 1.
H. triquetrum B. and S. 5.

Hepaticæ.

Owing to the combined influences of the low rainfall, the absence of hard rocks and the frequency of soils containing a high percentage of soluble salts, Liverworts are relatively rare in Cambridgeshire. The result of researches by Mr R. H. Compton of Gonville and Caius College is that no new species are added to the County List prepared by the Rev. P. G. M. Rhodes. Certain records, however, have been confirmed and new localities added, as the subjoined supplementary list will show.

Riccia fluitans L. 1. In ditches by the river and railway near Clayhithe.

Lunularia cruciata Dum. 1. At the water's edge, Byron's Pool, Trumpington.

Marchantia polymorpha L. 1. On walls and bridges, Backs of the Colleges, Cambridge.

Metzgeria furcata Lindb. 3. Wood on the Gog-Magog Hills, near Roman Road, Cambridge.

Pellia epiphylla Dum. 1. Hobson's Conduit, Cambridge; Bourn Brook, Grantchester. 4. Great Heath Wood, Gamlingay.

Lophozia turbinata Steph. 3. On bare chalk, Cherryhinton Chalk Pit.

Lophocolea bidentata Dum.
L. heterophylla Dum. } Widely distributed. 4. Great Heath Wood, Gamlingay. 5. Madingley Wood. 6. Kennett, banks and hedgerows.

Frullania dilatata Dum. 5. Gamlingay Wood.

A Note on the Food of Freshwater Fish. By J. T. SAUNDERS, B.A., Christ's College.

[*Read* 5 May 1913.]

VERY little is known about the food of freshwater fish, records in the past being very scanty and, owing to the difficulty of identifying remains in the stomach, sometimes inaccurate. Again, if an animal happens to be found two or three times in the stomach this was recorded as the chief food of the fish, without having any regard to the season of the year and the natural conditions under which the fish was living. This fault is very noticeable in semi-popular works like those of Tate Regan (*British Freshwater Fishes*, London, 1911) and Yarrell (*A History of the British Fishes*, London, 1836). Here we find it recorded that certain fish will eat such things as insect larvae, shrimps, small bivalves and the fry of other fishes. The natural conclusion that one draws from a statement such as this is that the fish swim about in search of food and snap up anything palatable that they meet. But this is far from being the case.

On opening up the alimentary canals of some sticklebacks (*Gasterosteus aculeatus*, L.) caught at Madingley, I found that their stomachs were full of Diatoms, but it was only one species of Diatom, viz. *Nitzschia sigmoidea*, Ehbr. It is true that there also occurred a few trichomes of *Oscillatoria nigra*, but they were so few that it is reasonable to suppose that they were ingested accidentally when the fish was seizing a Diatom. I have examined the stomachs of a great many sticklebacks at all times of the year caught in this pond and I found in them nothing but this Diatom, *Nitzschia sigmoidea*. All the specimens whose stomachs were full of Diatoms were caught with a worm and were large individuals, probably full grown. But finding it rather arduous to catch them singly with a worm I adopted the device of luring a number of sticklebacks over a net in the water by means of a worm. A great number of sticklebacks came to bite at the worm, and on quickly withdrawing the net fifteen to twenty could be caught at once. In this way fish of all sizes were taken.

An examination of the stomachs of fish that were not fully grown gave very different results from the examination of those that were full grown. I found at once that the younger fish do not exhibit such an exclusive preference for *N. sigmoidea* as do the older fish. *N. sigmoidea* is found only sometimes in their stomachs and it seems that the larger the fish the more restricted is its preference. The smaller ones are not so particular as the

larger and tend to be carnivorous; the smaller the fish the greater are its carnivorous propensities. Thus one finds that in a fish of about 4 cm. long the stomach and intestine will be full of insect larvae and small crustacea, while the rectum is distended with *Nitzschia sigmoidea* shells. In one of a smaller size the whole of the alimentary canal was full of insect larvae and small crustacea only, or we may find insect larvae and crustacea in the stomach and Peridinians in the intestine.

Two points are worth noticing here. The first is that the fish does not pick up anything digestible that comes handy but it confines its attention to one food only, or rather to one class of food. The stomach never contains a mixture, but is always full of one thing only. The second point is that it is capable of eating both animal and vegetable food.

As regards the animal food that is found in the stomach, it is a mixture of insect larvae and small crustacea of all kinds. These the fish evidently hunts for and catches by the use of its eyes only and not by the sense of smell. This may be amply proved in the following manner. If some sticklebacks be kept in an aquarium with some small waterbeetles, they will never attack the beetles, which are not an article of their diet. But if a number of Copepods are introduced into the aquarium the fish will at once attack them and eat them, at the same time they will often attack a waterbeetle by mistake, but they let the beetle go immediately, never swallowing it. It is only for a moment after the Copepods have been introduced that the beetles get attacked, the stickleback very soon learns to leave them alone. Sticklebacks will also snap at Copepods enclosed in a glass tube if this tube be placed in their aquarium.

But as regards the vegetable food I think that the stickleback must hunt for this by the sense of smell. It is difficult to see how the fish could get a meal that entirely consists of Peridinians or *N. sigmoidea* using only his eyes. And if he swam through the water with his mouth open he would surely engulf a mixture of organisms, and this is found to be not the case.

But so far I have been speaking of sticklebacks taken from one pond only, and from an examination of the contents of the stomachs of specimens caught in this pond we might assume that all young sticklebacks of about 4 cms. long are carnivorous, while the adults are vegetarian. This pond of which I have just been speaking is one of a series of ponds which have been formed in the excavations made by a former brick industry. They are all very close together, the banks between them being not more than a few yards wide. Yet in one pond it is found that all the adult sticklebacks feed only on a Diatom, *Nitzschia sigmoidea*, and in another all the adults are carnivorous and hunt for insect larvae and small

crustacea. Thus we see that the food of the stickleback may vary, but the variation is not a haphazard one for it effects equally all the individuals that live in one pond. It is something in the conditions under which a fish lives that determines its preference for a certain kind of food.

Under artificial conditions I found that the feeding habits of the fish completely changed. I could never get the large forms which fed on Diatoms to eat Diatoms, and I suspect this was because the Diatoms did not grow in their natural manner. These larger forms would however become carnivorous in aquaria and gave the impression that this was their natural method of feeding. Experiments in the laboratory are therefore useless to determine this point.

The fisherman, who catches fish with bread paste and gaudy flies, will probably ask how it is that such a thing as his bait which the fish cannot possibly have ever seen before proves so attractive. But the bait is not an article of diet. Every fisherman knows that there are times when the fish are feeding and times when they are not. Now my experiment of introducing Copepods into an aquarium and thus getting sticklebacks to snap at waterbeetles, shows that when fish are "on the feed" they will snap at anything that catches their eye, but unless they seize something palatable they will not swallow it. Therefore the fisherman angles when the fish are "on the feed," that is moving about in search of food, and he uses as a bait something that is likely to catch the fish's eye. The fish snaps at the bait and the fisherman strikes so as to drive the hook into its mouth. If the bait used were something that the fish ordinarily ate there would be no need for the exercise of any skill, for the fish would swallow it and inevitably get hooked. As it is the fisherman has to strike at the moment the fish snaps and before it has a chance to reject the bait. Herein lies the sport of fishing, for the more tentative the snap the greater will be the skill required to hook the fish ; the fish that snap so vigorously that they will hook themselves provide little sport for the angler.

We therefore come to two conclusions.

(i) Fish may feed on different things, but a meal always consists of one class of food only. This means that in order to ascertain the food of a fish we must open stomachs of fish taken at different seasons and under different conditions.

Additional evidence on this point is afforded by an observation, Dakin and Latarche (*Proc. of Royal Irish Ac.*, Vol. XXX. Sec. B, No. 3). They examined the contents of the stomachs of large numbers of Pollan (*Coregonus pollan*) and they remark: "The most extraordinary thing however was that on most occasions when Daphnids were found in the alimentary canals, practically

nothing else was present." I myself opened the stomachs of two pike (*Esox lucius*, L.) which were brought to me and found two roach in the one and one roach in the other. Other kinds of fish besides roach lived in the pond from which the pike were taken. This shows that both Pollan and pike as well as the stickleback like their meals to be composed of one thing only.

(ii) Preference for a certain kind of food varies according to locality.

Since all my observations have been made on sticklebacks taken from only three localities, I refrain from giving a list of the things they are capable of eating. It would be necessarily very incomplete.

Observations on Ticks: (a) Parthenogenesis, (b) Variation due to nutrition. By Professor NUTTALL.

[*Read* 5 May 1913.]

THE occurrence of parthenogenesis in ticks was recently observed by Aragão, in Brazil, in a new species of *Amblyomma* (*A. agamum*), the males of which have not as yet been discovered. Three complete generations of this tick have been raised experimentally and thousands of females were brought to maturity in the absence of males. This constitutes the first record of parthenogenesis in ticks.

The author described how he had succeeded in obtaining a parthenogenetic offspring from *Rhipicephalus bursa*, a species (prevalent on sheep in countries bordering the Mediterranean) in which both sexes occur in fairly equal numbers upon the host. Larval ticks issued in limited numbers from the eggs laid by unfertilized females.

Experiments were further recorded in which it was shown that the genus *Rhipicephalus* shows a considerable natural variation in size, and that imperfect feeding of the tick in its immature stages leads to the development of very small adults which, whilst fertile, are so different from the normal forms, that they could readily be taken for other species. It is only by determining the range of variability in a species under experimental conditions that the limits of a species in this respect can be determined and the making of bad species prevented.

The Division of Holosticha scutellum. By K. R. LEWIN, B.A. (Communicated by Professor NUTTALL.)

[*Read* 5 May 1913.]

THE account of the behaviour of the micronuclei at division, given by A. Gruber ("Weitere Beobachtungen an vielkernigen Infusorien," *Ber. Naturf. Ges. zu Freiburg I.B.* Bd. III. (1887), pp. 57—70) is not confirmed. In the period between divisions, *H. scutellum* possesses only a small number of micronuclei of about the size of the meganuclear segments, with which they have been confused. There is therefore no necessity to assume that numerous micronuclear divisions occur at the fission of the infusorian. Over the very limited series of preparations in which the micronuclei have been counted, a tendency is evident for the smaller numbers to occur at or near the stage of maximum concentration of the meganucleus. If this is significant, a reduction in the number of micronuclei must take place before division. There is, however, no evidence, nor any need to assume, that fusion of micronuclei is the way in which such a reduction would be accomplished.

Exhibition of living Termites. By Professor A. D. IMMS.

[*Read* 5 May 1913.]

THE author exhibited tubes containing living examples of the Termite *Archotermopsis wroughtoni* Desn. The Termites were obtained by him from the Kumam Himalayas, where they occur in dead trunks of the Chir pine (*Pinus longifolia*) at an altitude varying from about 4500 to 5800 ft. This species was described by Desneux in 1904 and was only previously known from Kashmir where it occurs in stumps of *Pinus excelsa.* The most interesting features are seen in the length of the cerci, which are composed of 7—8 joints in the sexual forms and of 6—7 joints in the soldiers and workers, in the possession of 5-jointed tarsi, and in the great development of the eyes. In the possession of these characters it is to be regarded as one of the most primitive of living Termites. Its nearest relationships are with two fossil species from the amber of Oeningen (Prussia) and the N. American genus *Termopsis,* Heer.

On the greatest value of a determinant whose constituents are limited. (Proof of Hadamard's theorem.) By Prof. A. C. DIXON, F.R.S.

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LET there be an array $\| a_{rs} \|$ of m rows and n columns ($n > m$) the constituent in the rth row and sth column being a_{rs}, a complex quantity, whose conjugate is b_{rs}.

Multiply the two arrays $\| a_{rs} \|$, $\| b_{rs} \|$ according to the ordinary rule, that is, form the determinant C, of order m, which has in its ith row and jth column the constituent

$$c_{ij} = \sum_{s=1}^{n} a_{is} b_{js} \quad (i, j = 1, 2 \ldots m).$$

C must be positive since it is the sum of products each formed with a determinant from $\| a_{rs} \|$ and the corresponding determinant from $\| b_{rs} \|$, that is, products of conjugate complex quantities.

The theorem to be proved is that C cannot exceed its leading term $\prod_{i=1}^{m} c_{ii}$, or say Π.

Assume this for $m-1$, and let C_{ij} be the coefficient of c_{ij} in C, that is, the product of the matrices

$$\| a_{rs} \| \text{ and } \| b_{rs} \|$$

with the ith and jth rows left out respectively, with the sign $(-1)^{i+j}$.

Thus the construction of C_{ij} is the same as that of c_{ij}, the places of a_{rs}, b_{rs} being taken by the first minors of the determinants of $\| a_{rs} \|$ and $\| b_{rs} \|$; the value of n is now different, being the number of determinants in a matrix of n columns and $m-1$ rows.

Since the theorem is true for $m-1$ we have

$$C_{ii} \leqslant \Pi \div c_{ii} \quad (i = 1, 2 \ldots m)$$

and also

$$\begin{vmatrix} C_{22} & C_{23} \ldots C_{2m} \\ C_{32} & C_{33} \ldots C_{3m} \\ \ldots\ldots & \ldots\ldots \\ C_{m2} \ldots\ldots & C_{mm} \end{vmatrix} \leqslant C_{22} \quad C_{33} \ldots C_{mm},$$

that is, $$c_{11}\, C^{m-2} \leqslant \Pi^{m-1} \div (c_{22}c_{33} \ldots c_{mm}),$$
whence $$C^{m-2} \leqslant \Pi^{m-2},$$
and if $m > 2$, $$C \leqslant \Pi,$$
for both aré positive.

Hence if true for $m-1$ the theorem holds for m when $m > 2$.
But when $m = 2$,

$$\begin{aligned} C &= c_{11}\, c_{22} - c_{12}\, c_{21} \\ &= c_{11}\, c_{22} - |c_{12}|^2 \\ &\leqslant c_{11}\, c_{22}. \end{aligned}$$

The theorem therefore holds when $m = 2, 3, 4 \ldots$ and universally.

When $n = m$, the result gives the theorem of Hadamard used by Fredholm in the theory of Integral Equations, that the absolute value of a determinant of order m cannot exceed the mth power of the absolute value of its greatest constituent multiplied by $m^{\frac{1}{2}m}$.

Expressions for the remainders when θ, θ^2, $\sin k\theta$, $\cos k\theta$ *are expanded in ascending powers of* $\sin\theta$. By Prof. A. C. DIXON, F.R.S.

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1. TAKE $\int_0^\theta \sin k(\theta - t)\sin^n t\,dt$, say u_n,

and integrate twice by parts. Thus

$$k^2 u_n = \Big[k\cos k(\theta-t)\sin^n t\Big]_0^\theta - kn\int_0^\theta \cos k(\theta-t)\sin^{n-1} t\cos t\,dt$$

$$= k\sin^n\theta + \Big[n\sin k(\theta-t)\sin^{n-1} t\cos t\Big]_0^\theta$$

$$- n\int_0^\theta \sin k(\theta-t)\{(n-1)\sin^{n-2} t\cos^2 t - \sin^n t\}\,dt$$

$$= k\sin^n\theta - n(n-1)u_{n-2} + n^2 u_n.$$

This holds when $n = 2, 3, 4 \ldots$ but when $n = 1, 0$ we have

$$k^2 u_1 = k\sin\theta - \sin k\theta + u_1,$$

$$k^2 u_0 = k(1 - \cos k\theta).$$

The general formula may be written

$$u_{n-2} = \frac{k}{n(n-1)}\sin^n\theta + \frac{n^2 - k^2}{n(n-1)}u_n.$$

Hence

$$\sin k\theta = k\sin\theta - (k^2-1)u_1$$

$$= k\sin\theta - \frac{k(k^2-1^2)}{3!}\sin^3\theta + \frac{(k^2-1^2)(k^2-3^2)}{3!}u_3$$

$$= \ldots\ldots$$

$$= k\sin\theta - \frac{k(k^2-1^2)}{3!}\sin^3\theta + \ldots + (-1)^n\frac{k(k^2-1^2)\ldots\{k^2-\overline{(2n-1)}^2\}}{(2n+1)!}$$

$$\times\left[\sin^{2n+1}\theta - \frac{k^2-(2n+1)^2}{k}u_{2n+1}\right] \ldots\ldots(1),$$

$$\cos k\theta = 1 - ku_0$$

$$= 1 - \frac{k^2}{2!}\sin^2\theta + \frac{k(k^2-2^2)}{2!}u_2$$

$$= \ldots\ldots$$

$$= 1 - \frac{k^2}{2!}\sin^2\theta + \frac{k^2(k^2-2^2)}{4!}\sin^4\theta \dots$$

$$+ (-1)^n \frac{k^2(k^2-2^2)\dots\{k^2-(2n-2)^2\}}{2n!}$$

$$\times \left[\sin^{2n}\theta - \frac{k^2-4n^2}{k}u_{2n}\right] \quad \dots\dots(2).$$

2. Similarly let

$$v_n = \int_0^\theta (\theta - t)\sin^n t\,dt,$$

and integrate by parts. Thus

$$nv_n = \int_0^\theta n(\theta-t)\sin^{n-1}t\,.\sin t\,dt$$

$$= -\Big[n(\theta-t)\sin^{n-1}t\cos t\Big]_0^\theta$$

$$+ n\int_0^\theta \{(n-1)(\theta-t)\sin^{n-2}t\cos^2 t - \sin^{n-1}t\cos t\}dt$$

$$= -\Big[n(\theta-t)\sin^{n-1}t\cos t + \sin^n t\Big]_0^\theta$$

$$+ n(n-1)\int_0^\theta (\theta-t)\sin^{n-2}t\cos^2 t\,dt$$

$$= -\sin^n\theta + n(n-1)(v_{n-2} - v_n),$$

and $v_{n-2} = \dfrac{1}{n(n-1)}\sin^n\theta + \dfrac{n}{n-1}v_n$.

This holds when $n = 2, 3, 4 \dots$, but when $n = 1, 0$ we have

$$v_1 = \theta - \sin\theta, \quad v_0 = \tfrac{1}{2}\theta^2.$$

Hence

$$\theta = \sin\theta + v_1$$

$$= \sin\theta + \frac{\sin^3\theta}{3!} + \frac{3}{2}v_3$$

$$= \dots\dots$$

$$= \sin\theta + \frac{1}{2}\frac{\sin^3\theta}{3} + \frac{1\,.\,3}{2\,.\,4}\frac{\sin^5\theta}{5} + \dots$$

$$+ \frac{1\,.\,3\dots(2n-1)}{2\,.\,4\dots 2n}\left[\frac{\sin^{2n+1}\theta}{2n+1} + (2n+1)v_{2n+1}\right] \quad \dots\dots(3),$$

and

$$\frac{1}{2}\theta^2 = \frac{1}{2}\sin^2\theta + 2v_2$$

$$= \frac{\sin^2\theta}{2} + \frac{2}{3}\frac{\sin^4\theta}{4} + \frac{2\,.\,4}{3}v_4$$

$$=\frac{\sin^2\theta}{2}+\frac{2}{3}\frac{\sin^4\theta}{4}+\frac{2.4}{3.5}\frac{\sin^6\theta}{6}+\dots$$
$$+\frac{2.4\dots(2n-2)}{3.5\dots(2n-1)}\left[\frac{\sin^{2n}\theta}{2n}+2n\,v_{2n}\right]\dots\dots(4).$$

3. In (1) (2) (3) (4) we have the expansions of $\sin k\theta$, $\cos k\theta$, θ and θ^2 in ascending powers of $\sin\theta$, with expressions for the remainders.

Now $$1.3<2^2,\qquad 2.4<3^2,$$
$$3.5<4^2,\qquad 4.6<5^2,$$
$$\dots\dots\qquad\dots\dots$$
$$(2n-1)(2n+1)<(2n)^2,\qquad 2n(2n-2)<(2n-1)^2.$$

Hence by multiplication the coefficient of v_{2n+1} in (3) is $<\sqrt{(2n+1)}$ and that of v_{2n} in (4) is $<2\sqrt{n}$.

Thus the remainders in (3) (4) tend to zero if $v_n\sqrt{n}$ does so, when $n\to\infty$.

Now if θ is between 0 and $\frac{\pi}{2}$ inclusive $\theta-t$ does not exceed $\frac{\pi}{2}-t$ which does not exceed $\cot t$.

Hence $$v_n<\int_0^\theta \sin^{n-1}t\cos t\,dt \text{ or } \sin^n\theta/n,$$
$$v_n\sqrt{n}<\sin^n\theta/\sqrt{n}$$
which tends to zero when n increases even when $\theta=\frac{1}{2}\pi$.

Thus the infinite series given by (3) (4) converge to θ and $\frac{1}{2}\theta^2$ respectively when θ is any real angle between $\pm\frac{1}{2}\pi$ inclusive.

4. In the series (1) (2) when k is real, if the factor
$$-m^2\quad(m=1,2,3\dots 2n+1)$$
is put in the place of k^2-m^2 in the coefficients of u_{2n+1}, ku_{2n} they reduce to those of v_{2n+1}, v_{2n} in (3) (4). Hence these coefficients in (1) (2) increase in a less ratio than those in (3) (4), at any rate when $n>\frac{1}{2}k$.

Also $$|\sin k(\theta-t)\div k|<(\theta-t),$$
so that $$u_n<kv_n.$$

Hence the validity of the infinite expansions (1) (2) follows when k is real and θ a real angle between $\pm\frac{1}{2}\pi$ inclusive.

5. The above is the elementary case. When k is not real, but θ is restricted as before, we can say that
$$|\sin k(\theta-t)|<\kappa(\theta-t),$$
where κ is a finite quantity, but generally is $>|k|$; then
$$|u_n|<\kappa v_n.$$

The coefficient of u_n or ku_n in (1) or (2) is that of v_n in (3) or (4) multiplied by the product of factors of the form $1-\dfrac{k^2}{n^2}$: these are known to form a convergent product. Hence the results are still true when k is imaginary.

6. When θ is imaginary, u_n and v_n may be treated by the following method, which would also apply if θ were real.

$$u_n = \int_0^\theta \sin k(\theta - t)\sec t \,.\, \sin^n t \cos t\, dt$$

$$= \Big[\sin k(\theta - t)\sec t \,.\, \sin^{n+1} t/(n+1)\Big]_0^\theta$$

$$- \frac{1}{n+1}\int_0^\theta \sin^{n+1} t\, \frac{d}{dt}\{\sin k(\theta-t)\sec t\}\, dt.$$

The terms at the limits vanish, and if g is the greatest value of $\dfrac{d}{dt}\{\sin k(\theta-t)\sec t\}$ on the path of integration, which is finite, we have

$$|u_n| < \frac{1}{n+1} g\, |\theta \sin^{n+1}\theta|$$

provided that $|\sin\theta|$ is the greatest value of $|\sin t|$ on the path of integration, which may be the straight line from 0 to θ.

Now if $\theta = \phi + \iota\psi$, $|\sin^2\theta| = \sin^2\phi + \sinh^2\psi$ which increases with the numerical value of ϕ or ψ, so long as ϕ is between $\pm\frac{1}{2}\pi$.

Thus the remainders in the series (1) (2) (3) (4) tend to zero when $n \to \infty$ provided that $|\sin\theta| \leqslant 1$ and the real part of θ is between $\pm\frac{1}{2}\pi$.

7. It is curious that the coefficient of $\sin^{2n+1}\theta$ in (1) tends to equality with $\frac{1}{2}n^{-\frac{3}{2}}\pi^{-\frac{1}{2}}k\cos\frac{1}{2}k\pi$ and that of $\sin^{2n}\theta$ in (2) to equality with $-\frac{1}{2}n^{-\frac{3}{2}}\pi^{-\frac{1}{2}}k\sin\frac{1}{2}k\pi$.

The expressions (1) (2) may be used to give a proof of the factorial expressions for the sine and cosine. In (1) (2) put π for θ. Thus

$$1 - \cos k\pi = \frac{k(2^2-k^2)(4^2-k^2)\dots(4n^2-k^2)}{2n\,!} u_{2n},$$

$$\sin k\pi = \frac{(1^2-k^2)(3^2-k^2)\dots\{(2n+1)^2-k^2\}}{(2n+1)!} u_{2n+1}.$$

Now when n tends to infinity the elements of u_n for which t is not nearly $\frac{1}{2}\pi$ may be neglected and thus u_n tends to equality with

$$\sin\tfrac{1}{2}k\pi \int_0^\pi \sin^n t\, dt.$$

This step is further examined below (§ 8). Thus

$$\sin \tfrac{1}{2}k\pi \ \text{ or } \ (1-\cos k\pi)/2 \sin \tfrac{1}{2}k\pi$$

is the limit when $n \to \infty$ of

$$\frac{k(2^2-k^2)(4^2-k^2)\dots(4n^2-k^2)}{2n!} \cdot \frac{1}{2}\int_0^\pi \sin^{2n} t\,dt$$

or of
$$\frac{k(2^2-k^2)\dots(4n^2-k^2)}{2n!}\,\frac{\pi}{2}\cdot\frac{1.3.5\dots(2n-1)}{2.4.6\dots 2n}$$

or of
$$\frac{1}{2}\pi k\left(1-\frac{k^2}{2^2}\right)\left(1-\frac{k^2}{4^2}\right)\dots\left(1-\frac{k^2}{4n^2}\right).$$

Also
$$\cos \tfrac{1}{2}k\pi \ \text{ or } \ \sin k\pi \div 2 \sin \tfrac{1}{2}k\pi$$

is the limit when $n \to \infty$ of

$$\frac{(1^2-k^2)(3^2-k^2)\dots\{(2n+1)^2-k^2\}}{(2n+1)!}\cdot\frac{2.4.6\dots 2n}{3.5.7\dots(2n+1)}$$

or of
$$\left(1-\frac{k^2}{1^2}\right)\left(1-\frac{k^2}{3^2}\right)\dots\left\{1-\frac{k^2}{(2n+1)^2}\right\}.$$

8. To justify the statement made as to u_n when $n \to \infty$, take the difference

$$u_n - \sin \tfrac{1}{2}k\pi \int_0^\pi \sin^n t\,dt$$

and write $\pi - t$ for t in u_n. The difference is then

$$\int_0^\pi \frac{\sin kt - \sin\frac{1}{2}k\pi}{\cos t}\sin^n t \cos t\,dt$$

$$= \left[\frac{\sin kt - \sin\frac{1}{2}k\pi}{\cos t}\cdot\frac{\sin^{n+1} t}{n+1}\right]_0^\pi$$

$$-\frac{1}{n+1}\int_0^\pi \sin^{n+1} t \,.\, \frac{d}{dt}\,\frac{\sin kt - \sin\frac{1}{2}k\pi}{\cos t}\,.\,dt.$$

The terms at the limits vanish, and thus the difference bears to $\int_0^\pi \sin^n t\,dt$ a ratio less than $\dfrac{h}{n+1}$ where h is the greatest absolute value of $\dfrac{d}{dt}\dfrac{\sin kt - \sin\frac{1}{2}k\pi}{\cos t}$ between the limits. Since h is finite, this proves the statement, unless k is an even integer, in which case the product expressions for $\sin \frac{1}{2}k\pi$ and $\cos \frac{1}{2}k\pi$ are evidently true.

A dust electrical machine. By W. A. Douglas Rudge, M.A., St John's College.

[*Read* 19 May 1913.]

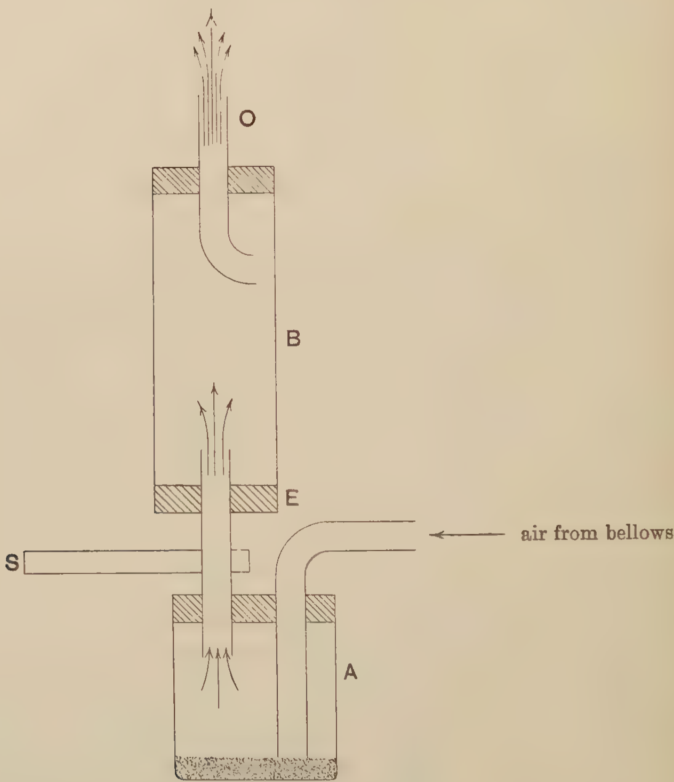
In the course of some work on atmospheric electricity carried on during the past three years, the author has shown that very considerable charges of electricity are given to the air during the raising of dust-clouds, whether by wind blowing over the surface of the earth, or by the motion of motor cars, etc. along dusty roads, or in general by raising a dust by any method. The kind of electrification varied with the nature of the dust itself, and the magnitude of the charge depended to some extent upon the fineness of the state of division. During dust-storms, hollow insulated vessels arranged to catch the dust may be charged to a potential of some thousands of volts. In a systematic study of the methods used for raising clouds, and determining the charge upon the dust particles and upon the air accompanying them, it was noticed that small sparks could occasionally be obtained from insulated vessels used to collect the dusts. Some simple pieces of apparatus have been constructed, by which it is possible to obtain considerable charges of electricity from the raising of a dust cloud. As shown in a previous paper* any kind of dust can be made to yield charges of electricity, and in the apparatus to be described, sand, road dust, flour, sulphur, iron filings, etc. may be employed as the working substance of the machine. The apparatus may be constructed very simply, but the form shown in the figure is the most satisfactory.

The machine consists of a chamber A in which the dust is raised by sending into it a current of air from the bellows. The dust is carried into the chamber B. This consists of a brass tube about 25×5 cms. insulated from the rest of the apparatus by an ebonite plug E. The dust escapes through O. In the dry African climate, the chamber B can be dispensed with, the tube leading from A serving as a collector and sparks will fly from it when a rapid current of dust is passing.

The effects produced by the machine are very remarkable with dry dust and a dry atmosphere, as the dryness allows of a very fine state of division of the dust. With an apparatus of the dimensions shown sparks up to 5 cms. in length have readily been obtained, and "brush" discharges will fly from the tube if the atmospherical conditions are suitable. The air escaping from

* *Phil. Mag.*, April, 1913.

O is very strongly charged, readily giving a potential gradient in a room of several hundred volts per metre. This charge upon the air is readily detected by aid of an insulated wire tipped with a radioactive substance, the wire rapidly taking the potential of the air in its neighbourhood, and if discharged will quickly become charged again. The air in a room may remain charged for more than half an hour after the dust has settled, so that it may be concluded that the air itself is actually charged.



The charge acquired by the apparatus has probably a two-fold origin (1) that due to the raising of the dust, and (2) that due to friction of the dust particles against the walls of the tube. At first it appeared as though only one charge was present upon the dust but recent experiments have shown that both kinds of electricity are present, one of them however predominating. Further experiments are now in progress.

On a mechanical vacuum tube regulator. By R. WHIDDINGTON, M.A., St John's College.

[*Read* 19 May 1913.]

IN several of the writer's previous papers mention has been made of a particular form of vacuum tube regulator. It is the object of this short paper to give a few experiments in connection with this device which not only serve to indicate the range of its usefulness but may perhaps help to throw a little light on its mode of action.

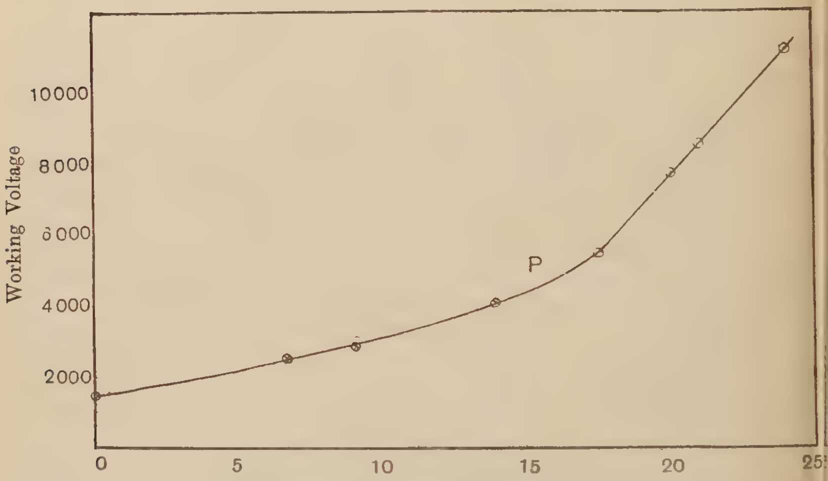
The usual method of altering the hardness of a vacuum tube is to vary the gas pressure within the tube. Under ordinary circumstances a diminution of pressure produces a hardening effect, or in other words the lower the pressure the faster the cathode rays shot off from the cathode—the more difficult it is to force current through the tube. This is only true however when the gas pressure is below a certain amount depending on the dimensions of the tube and its electrodes.

Winkelmann appears to have been one of the first to recognise the fact that the size, form, and position of the cathode in a discharge tube exerted a profound effect on the discharge. He found that it was possible by using tubes of small dimensions with the cathode in a confined space to produce quite fast cathode rays even when the pressure within the tube was as high as several mms. of mercury.

Of the many mechanical regulators based on this effect investigated by Winkelmann and others, perhaps the most useful is the one in which a sliding glass tube tightly fitting the cathode slides in and out, the position of this sheath determining largely the velocity of the rays. It is possible in this way to produce very considerable variations in the speed of cathode rays without in any way altering the gas pressure. This device was first described by Campbell Swinton and later by Wehnelt in 1903. There are many possible methods of varying the position of the sheath and one of the most convenient is to attach a small piece of iron to the sheath and influence it from without by a magnet. In this kind of way the sheath may be instantly adjusted to any desired point, the further the sheath is outdrawn the faster the cathode rays emitted from the cathode.

The graph gives some idea of the range of hardness to be expected from a tube fitted with such a regulator. The tube in this case was a litre bulb fitted with a cathode 1·5 cm. in diameter

mounted so as to project well within the bulb. The glass sheath was closely fitting and was controlled magnetically. It is important to select the glass sheath so that it is entirely free from air lines as otherwise the discharge passes along the lines and the tube runs unsteadily. The tube was evacuated to a convenient point and driven by a Mercedes influence machine which proved a fairly steady source of current, the voltage across the tube being measured by a Braun electrostatic voltmeter. In the graph the abscissae represent lengths of sheath outdrawn while the corresponding ordinates are the corresponding voltages across the tube. The pressure of gas within the tube was maintained constant throughout the experiment. Accurate readings could not



Length of Sheath (in mms.)
(Diameter of Cathode = 15 mms.)

be continued above 10,000 volts with the voltmeter but an equivalent spark gap indicated that the tube at its hardest was working at about 40,000 volts, beyond this point the tube became very unsteady in action.

It is noteworthy that beyond the point P on the graph the curve rises far more steeply. Observations carried out to above 10,000 volts with a spark gap indicate that the curve continues practically straight up to about 30,000 volts.

It seems very difficult, if indeed it is at present possible, to give an adequate explanation of the action of the sheath. This is a difficulty common to very many problems in connection with discharge tubes. What I propose to do then is to give a brief

account of a few experiments with the regulator which suggest one explanation and then to point out the complications which stand in the way of its complete acceptance.

The tube with which the following experiments were carried out was coil driven with a Lodge rectifier in series to prevent reversal of the current.

(1) When a vacuum tube is hardened by diminishing the gas pressure the hollow cylindrical cathode beam becomes more concentrated about its axis, which in the case of a circular plane disc cathode passes normally through the centre; at the same time the dark space lengthens out receding from the cathode. This further is exactly what happens when the sheath is drawn out over the cathode at a constant gas pressure, the cathode beam narrows down—a smaller part of the cathode functions—and the dark space lengthens out. If the pressure be not too low the sheath as it is caused to travel out may ultimately overtake the boundary of the dark space which is also travelling but at a slower pace. The interesting point to notice is that when once the sheath has overtaken the dark space boundary and is projecting into it no additional hardening effect can be produced by further drawing out the sheath.

(2) If the sheath is not too heavy and is running easily it will travel out of itself when the discharge is passing—in fact it may be made to rise against gravity if the tube be tilted so that the cathode points slightly upwards. If now the Lodge rectifier be cut out the coil may partly reverse and produce a spot of positive column on the cathode, if this happen the sheath will commence to slide back again to its original position the tube at the same time becoming softer. On switching in the rectifier again the sheath once more slides out and the tube hardens. It is sometimes necessary to tap the tube to get these mechanical effects but usually the vibration set up by the coil is sufficient to ensure easy movement.

(3) The effects described in both the above experiments can be obtained if the inside of the sheath is lightly silvered, not heavily enough to introduce the complications of an additional discharge but heavily enough to ensure that the sheath is at the same or nearly the same potential as the cathode itself.

(4) If the sheath be cut longitudinally into halves and only one half is used, the cathode beam is apparently repelled from the sheath, the rays pursuing a curved path when in the neighbourhood of the glass and proceeding in straight lines when beyond its influence.

These experiments taken alone without further evidence lead to the very natural conclusion that the sheath being negatively charged on the inside repels the cathode rays towards the centre

on being drawn out thus increasing the resistance of the tube. But even this apparently simple statement becomes complicated when we remember that the cathode beam is travelling in a strongly ionised gaseous layer so that the individual particles will be shielded from electrostatic action in some directions more than in others. It seems obvious in fact that any individual particle in the stream will only be repelled by the nearer portions of the charged sheath. No mere electrostatic explanation however can be sufficient for the system under consideration is not in electrical equilibrium.

Now the generally accepted theory of the discharge tube has it that the cathode and positive rays are inseparably connected. The cathode rays in their passage through the gas produce positive ions which move towards the cathode thus becoming the positive rays. When these positive rays strike the cathode they liberate electrons which shoot off under the electric force at the cathode and become cathode rays and the process is repeated so long as the potential difference between cathode and anode is maintained. The difficulty now arising is that if the sheath repels the negative rays towards the axis of the discharge it should also attract the positive rays outwards away from the axis. Unfortunately there is not sufficient data as to the velocity of these positive rays at different distances from the cathode for the question to be discussed quantitatively, but it is important to notice that the positive rays will always be under the action of the sheath for a very short distance for they acquire their velocity a short distance from the cathode and in any case a positive ray is not so long lived as a cathode ray owing to the ease with which it may become an uncharged atom by recombination. It would not be surprising therefore if the effect of the sheath on the cathode rays swamped the opposite effect on the positive rays. It is possible that this is the explanation of the sudden rise in the curve at P in the graph.

To summarize in conclusion the more obvious complicating effects which must be taken into account when attempting an explanation of this hardening action:

(1) The electrostatic screening action of the cathode beam.

(2) The similar screening action of the positive bundle which surrounds the cathode beam.

(3) The attraction which must exist between the positive and negative streams.

Some Plants new to the British Isles. By C. E. Moss, B.A., Emmanuel College.

[*Read* 28 April 1913.]

The following plants were briefly described and their geographical distribution indicated:

Alnus glutinosa.

(*a*) var. *macrocarpa.* Chippenham Fen, Cambridgeshire.

(*b*) var. *typica* (comb. nov.; ined.). The common form of the alder in southern England.

(*c*) var. *microcarpa.* The common form of the alder in northern England and in Scotland.

Ranunculus ficariæformis. Jersey, first found by Mr S. Guiton, and specimens sent to me by Mr E. W. Hunnybun.

Cheiranthus cheiri.

(*a*) var. *hortensis* (var. nov.; ined.). The cultivated form of the wallflower, and the one commonly occurring as a garden escape on old walls and sea-cliffs.

(*b*) var. *fruticulosus.* Perhaps the wild form of the wallflower. Rare, on old walls, e.g., Jersey.

(*c*) var. *angustior* (var. nov.; ined.). Closely allied to var. *fruticulosus.* Walls, Pevensey Castle, Sussex.

Arenaria trinervis.

(*a*) var. *typica* (var. nov.; ined.). The common form in Cambridgeshire.

(*b*) var. *pentandra.* Devonshire, Surrey.

Primula veris var. *suaveolens.* Suffolk and Cambridgeshire.

P. scotica var. *orkniensis* (var. nov.; ined.). First found by Mr Grant, of Orkney, and the characters of the fruit first elucidated by Mr E. W. Hunnybun.

Brunella laciniata × *vulgaris.* Hertfordshire, Cambridgeshire, Somerset.

Gymnadenia wahlenbergi. Cambridgeshire.

G. densiflora. In Herb. Babington, from Cambridgeshire.

On some new and rare Jurassic plants from Yorkshire:—Eretmophyllum, *a new type of Ginkgoalian leaf.* By H. Hamshaw Thomas, M.A., Downing College. Curator of the Botanical Museum, Cambridge.

[*Read* 28 April 1913.]

[Plates VI and VII.]

In the well-known Gristhorpe plant-bed, a number of isolated leaves occur, belonging to a type which has not been previously recognised. The plant-bed belongs to part of the Middle Estuarine Series of the Middle Jurassic and is exposed on the shore at the adjacent ends of Gristhorpe and Cayton Bays near Scarborough. I found most of my specimens in Cayton Bay, where they are locally plentiful. Two or three portions of leaves which appear to be closely allied to the Cayton forms have also been obtained from the Lower Estuarine beds at Whitby and though these exhibit some differences they may well be included in the same genus. All the leaves are beautifully preserved in shale, they were originally of firm texture and they can be readily detached from the shale, though the detached leaf is brittle and is easily broken into pieces. The examples shown in figs. 1 and 2 Pl. VI. were detached from the rock, gummed on to glass and photographed by transmitted light.

Description.

The leaves from the Gristhorpe bed, which I propose to call *Eretmophyllum* pubescens*, vary somewhat in shape but are usually oblanceolate sometimes becoming almost linear. They may be straight but are often slightly falcately curved or at least unsymmetrical with one margin straight and the other curved (cf. figs. 1 and 2). They are broadest towards the apex and taper gradually towards the base, the lamina passing into a broad petiole of varying length. The leaves differ somewhat in size; of the few complete examples the smaller are about 7 cms. long and 1 cm. broad, the larger 10 cms. long and about 2·5 cms. broad. Fragments of leaves more than 3 cms. broad have been found, which, when complete, may have exceeded a length of 10 cms. The average leaf is about 1·7 cms. broad and 9·5 cms. long,

* From ἐρετμόν = an oar or paddle and φύλλον = a leaf. I am indebted to Miss W. M. L. Hutchinson for suggesting this name.

including the petiole which measures from 1—3 cms. The margins of the lamina are entire and the apex either bluntly rounded or somewhat retuse (cf. fig. 2), in the latter case the apical notch is obliquely placed. Towards the base of the lamina the margins are frequently thickened on one face, the thickened portions passing into the petiole in the way usually seen in the leaves of the recent *Ginkgo biloba*. The petiole is somewhat expanded at its base.

The venation is usually very distinct. Two or three veins coming up from the petiole become distinguishable at the base of the lamina and soon dichotomise several times, the resulting divisions run parallel to each other through most of the length of the leaf (fig. 1), though in the more obovate forms they may again fork occasionally. Near the apex the veins converge somewhat. The veins are large and widely separated; they are from 1 mm. to 1·5 mm. apart. In a leaf of average size 13 veins occur in a width of 16 mm. Some of the leaves are now in a peculiar brown translucent condition and their veins can be often seen under the microscope to consist of a number of fine parallel dark strands, no doubt representing the original xylem or sclerenchyma strands.

Between the veins in some of the larger specimens a row of small spots or short lines may be observed which are visible to the naked eye and are 2—3 mm. apart. These are the secretory tracts which are seen in the corresponding position in recent Ginkgo leaves, which have been figured in the Jurassic *Ginkgo Obrutschewi* by Prof. Seward* and which I found to be present in some Ginkgo leaves from the Yorkshire Jurassic collected by Prof. Nathorst and now in the Stockholm Museum. When the leaves are macerated, these little secretory tracts yield small groups of cell-like structures which resist the action of the acid and might in some circumstances be taken for groups of spores.

The Cuticular Structure.

As is usually the case with stout leaves that separate readily from the matrix, the present specimens yield excellent cuticular preparations in which the outlines of the epidermal cells and the stomatal openings are clearly seen. In *E. pubescens* the upper and lower cuticles are very distinct. The upper epidermis was composed of more or less uniform cells and was devoid of stomata. The cells were irregularly angular or polygonal in shape, becoming somewhat elongated above the veins. Their walls, which are usually straight, sometimes show the slight undulations of the

* Seward (11), p. 46, Pl. IV. figs. 42, 43.

type seen in some recent Ginkgo leaves. In the centre of each cell is a short but conspicuous papilla (cf. fig. 4) and the presence of these structures gives to the cuticle a very characteristic appearance.

On the cuticle from the lower epidermis, the papillae are still more marked (fig. 3); the epidermal cells were again irregularly polygonal in shape. In these preparations we see the stomata very clearly, they are confined to the areas between the veins. The stomata are arranged in irregular rows and are easily distinguishable on account of the thickened subsidiary cells. The guard cells do not seem to have been thickened and were somewhat sunken, their outlines are now only indistinctly seen. Above and around them lie the subsidiary cells (fig. 5) four to seven in number, regularly arranged and with clearly defined walls; they are more or less uniformly thickened and sometimes possess small papillae, they show more clearly after staining with Fuchsine.

These stomatal structures show close resemblance to those of other Jurassic leaves allied to Ginkgo, and also to the stomata of the present-day leaves of this form*; the recent stomata, however, are less deeply sunken and not so much covered by the subsidiary cells.

The Whitby specimens.

In one of the plant beds on the Scar at Whitby I have found several portions of leaves which seem to belong to this genus. The largest piece is about 7 cms. long, 12 mm. broad and slightly falcate; though neither end is seen, it tapers slightly above and below. Another well-preserved fragment belonged to a somewhat broader leaf. The veins in these examples are about 1 mm. apart but do not clearly show any forking; a portion of one margin is slightly thickened as near the base of *E. pubescens*; secretory tracts have not yet been observed. The leaves were again stout and can be readily detached from the matrix. The epidermal structure of this form, which I propose to call *E. Whitbiense*, differs considerably from that of *E. pubescens*. When viewed under the binocular microscope, the upper surface is seen to be rough, the individual cells being more or less visible as convex projections. The areas above the veins are distinguishable by their more elongated cells but in addition to these, interstitial strands of elongated cells are seen between the veins, which sometimes appear to anastomose with each other, and may have corresponded with thickened hypoderm strands or small veins.

* Cf. Seward (11), Pl. v. figs. 59–62.

Similar interstitial strands have been recorded in Ginkgodium by Yokoyama* and in some Baieras†.

The cuticular preparations made from the leaves of this species are generally somewhat thicker than those from *E. pubescens.* The cells of the upper epidermis were strongly cuticularised above the veins and somewhat elongated (cf. Pl. VII. fig. 8), while the areas between the veins were composed of more rounded cells with a convex outer surface; in this region stomata are frequently seen (fig. 9, *st*), which are much smaller than those on the lower epidermis. The distinction between the cells above and between the veins is much greater than in the Gristhorpe specimens. The lower epidermis consists of fairly uniform squarish or hexagonal cells all considerably thickened (fig. 6). The stomata are seen in the areas between the veins but are smaller and fewer in number than in *E. pubescens*, they show five or six subsidiary cells of the same type as in that species (fig. 7), and these sometimes possess small papillate projections though they are not specially thickened. The guard cells were sunken, being just visible through the pore in the centre of the overarching subsidiary cells.

It will be seen from the foregoing description that the Whitby and Gristhorpe specimens exhibit some well-marked differences, chiefly in cuticular structure, but also in external appearance in well-preserved specimens. The resemblances and differences may be summarised as follows.

ERETMOPHYLLUM gen. nov.

Leaves oblanceolate to linear tapering into a distinct petiole. Apices rounded or retuse. Veins distant, dichotomising near base of lamina and more or less parallel above, slightly convergent near apex. Epidermal cells more or less rectangular or polygonal. Stomata with group of angular subsidiary cells regularly arranged round and above the guard cells.

ERETMOPHYLLUM PUBESCENS sp. nov.

Surface of leaves appearing more or less smooth. No interstitial veins or strands. Secretory tracts present between the veins. Epidermal cells of upper surface uniform, without stomata. Cells of upper and lower epidermis with short papillae. Stomata numerous on lower side of leaf.

ERETMOPHYLLUM WHITBIENSE sp. nov.

Upper surface of leaves rough. Interstitial veins or strands present between the veins. Cells of upper surface elongated

* Yokoyama (89), Pl. VIII. figs. 1, 1*a*, 14, 14*a*.

† Krasser (05), p. 24.

above the veins, stomata present in areas between the veins. Papillae absent. Lower epidermis smooth. Papillae only on subsidiary cells of stomata. Stomata small and not very numerous.

Relationship with other forms.

The genus *Eretmophyllum*, as has been already mentioned, possesses a number of features in common with the leaves of Ginkgo and must undoubtedly be classed among the Ginkgoales. The points of resemblance may be here summarised. Leaves broadest towards the apex, the lamina merging gradually into the petiole at the base, apical portion sometimes with a notch. Margins of the leaf thickened at the base. Veins distant, dichotomising. Secretory tracts between the veins. Epidermal cells rectangular or polygonal, their walls sometimes slightly sinuous. Stomata with sunken guard cells surrounded by five to seven somewhat thickened subsidiary cells arranged in a regular radial series.

The leaves differ from those of *Ginkgo* in being oblanceolate or linear, but approach those of *Ginkgodium* in outline. Many examples of the latter have been figured by Yokoyama* and I have also described specimens from South Russia† which may be referred to the same genus. *Ginkgodium*, however, is distinct in possessing shorter and comparatively broader leaves often deeply divided at the apex. A much more important distinction is seen in its nervation, the veins being fine, very numerous and parallel instead of spreading, they occasionally fork but do not converge at the base of the leaf, apparently springing from the thickened margin; interstitial nerves occur, but the venation is entirely different in character from that of the present genus.

In their form, their slightly falcate shape, and to some extent in their nervation, my leaves resemble those of *Feildenia* described by Heer‡ from the Miocene of Grinnell Land. The leaves of this genus, however, differ considerably from mine in their smaller size, closer venation and less distinct petiole. *Feildenia Mossiana* Hr. in its more ovate shape and coarser venation is the most comparable, but the curious convergence of the veins at the apex (if correctly figured) is very distinct. The veins in *Eretmophyllum* usually converge somewhat at the apex but never join in any way resembling Heer's figure§.

Fontaine|| has figured a leaf of somewhat similar character to

* Yokoyama (89), p. 56, Pl. II. fig. 4*c*, III. 7, VIII., IX. 1—10, XII. 14, 15.
† Thomas (11), Pl. IV. figs. 9—11.
‡ Heer (78), p. 21, Pl. I. figs. 3—11, VIII. 2*a*, 3*a*, 4, 5.
§ *Idem*, Pl. VIII. figs. 2*a*, 3*a*, 4.
|| Fontaine (89), Pl. LXXXV. figs. 5, 5*a*.

F. Mossiana from the Potomac beds of Virginia, with veins which fork in the lower part of the leaf, then run parallel and near the apex converge and join up on a central line. This type may be perhaps related to the Yorkshire form but the peculiar course of the veins at the apex, which according to Fontaine is clearly seen, is quite a distinguishing feature.

The only leaf bearing a close relationship to those before us, which I have found already figured, is the form recently described by Prof. Seward* from Afghanistan under the name of *Podozamites Saighanensis.* This leaf had a similar form and size to some of our examples, with a well-defined petiole and parallel veins 1 mm. apart, which branched near the petiole and converged slightly near the apex.

In referring this form to *Podozamites*, Prof. Seward pointed out that it could be compared with the leaves of Ginkgoales especially with the *Ginkgodium* type. It is extremely probable that this leaf should be placed in the genus *Eretmophyllum*, though until more material is forthcoming it is impossible to determine whether it is specifically as well as generically identical with either of the Yorkshire species.

It probably provides another example of a Jurassic genus with very discontinuous distribution.

Among the numerous examples of Podozamites which have been figured, a few show some slight resemblance to our present form, but none show the gradual narrowing of the lamina into a distinct petiole. Some of the examples of *P. lanceolatus* possess similar nervation and a small petiole†. The sporophylls recently described by Nathorst‡ as *Cycadocarpidium Swabii* possess distant veins which fork near the base of the lamina and thus allow of a somewhat limited comparison.

A comparison may also be made with the isolated leaves figured by Fontaine§ from Cape Lisburne, Alaska, under the name of *Nageiopsis longifolia* ? Font., but which according to Berry|| are in nowise related to *Nageiopsis*. Fontaine's figure 5 might possibly be an Eretmophyllum leaf figured upside down, and the shape of the other fragments, their distant veins almost parallel but sometimes forking, present some points of similarity.

It seems then that we are justified in regarding these Yorkshire leaves as types of a new genus of Ginkgoalian plants, which form a connecting link between *Ginkgo* or rather *Ginkgodium* and *Feildenia* or *Phoenicopsis* (if indeed these latter genera are members of the Ginkgoalian alliance).

* Seward, (12), p. 35, Pl. IV. fig. 53.
† Heer (77), Pl. XXVII. figs. 3, 4, 5*c*.
‡ Nathorst (11), p. 5, Pl. I. figs. 11—15.
§ Fontaine in Ward (05), Pl. XLV. fig. 1—5.
|| Berry, (10), p. 190.

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DESCRIPTION OF FIGURES.

PLATE VI. *E. pubescens.*

FIG. 1. Upper portion of a leaf of average size showing parallel venation, secretory tracts and shape of apex. Reduced to $\frac{5}{6}$ths.

FIG. 2. Almost complete small leaf showing unsymmetrical shape, retuse apex and forking venation. Like fig. 1 this photo was taken, by transmitted light, of the actual leaf which had been removed from the rock. $\times \frac{5}{6}$.

FIG. 3. Cuticle of lower epidermis showing stomata and papillate cells. $\times 40$.

FIG. 4. Cuticle of upper epidermis showing papillae and slightly sinuous walls of cells. $\times 150$.

FIG. 5. Cuticle of lower epidermis showing stomata with thickened subsidiary cells. $\times 150$.

PLATE VII. *E. Whitbiense.*

FIG. 6. Cuticle of lower epidermis. $\times 40$.

FIG. 7. Part of the same more highly magnified, showing stomata with surrounding subsidiary cells. $\times 150$.

FIG. 8. Cuticle of upper epidermis showing rows of elongated cells above veins or hypodermal strands. $\times 40$.

FIG. 9. Part of the same more highly magnified, showing thickened elongated cells above a vein and thinner portion with stomata (*st*). $\times 150$.

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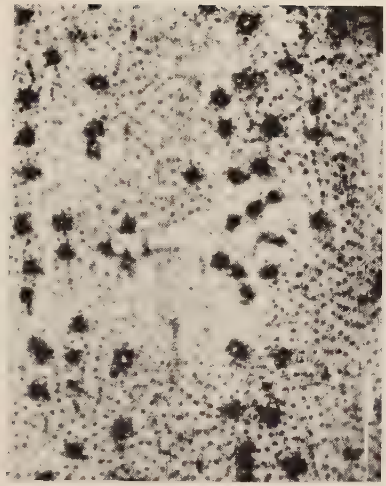

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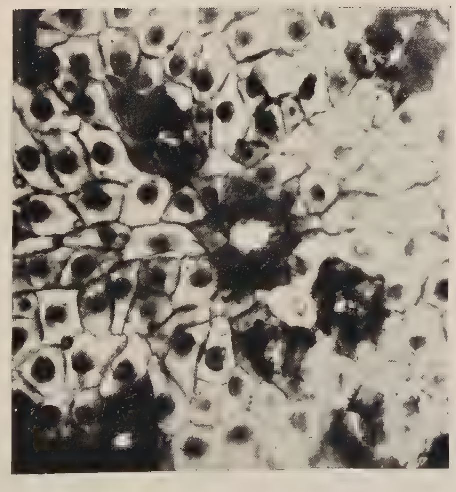

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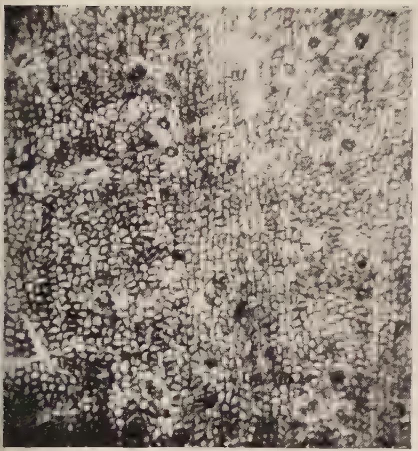
ERETMOPHYLLUM PUBESCENS.



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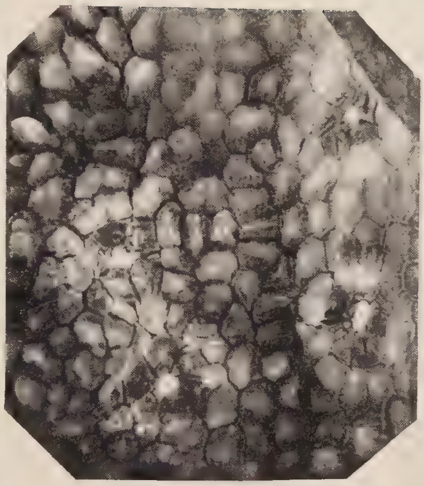

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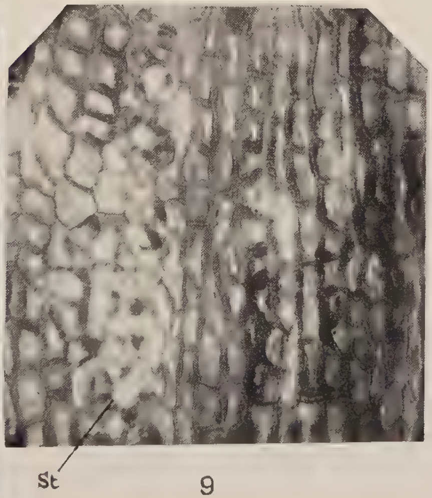

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ERETMOPHYLLUM WHITBIENSE.

The Unstable Nature of the Ion in a Gas. By R. D. KLEEMAN, D.Sc. (Adelaide), B.A., Emmanuel College.

[*Read* 19 May 1913.]

Theoretical considerations.

THE ions in a gas are usually assumed to be stable at constant temperature. The writer has pointed out*, however, that it follows from thermodynamical considerations that this can by no means be the case. It is shown that an ion cluster should be continually changing in complexity, the changes being governed by the same laws as chemical dissociation, namely by the laws of thermodynamics and the law of mass-action.

Some of the principal deductions made in the paper quoted will be mentioned here as an introduction to the experiments described in this paper. If v_0 denote the velocity of a free ion, v_1 that of a cluster formed by the elementary ion and a molecule, v_2 that of a cluster formed by the elementary ion and two molecules, and so on, and $c_0, c_1, c_2, \ldots$ denote respectively the concentrations of the ion clusters and the elementary ions, then the average velocity V, the quantity measured in practice, is given by

$$V = \left(v_0 + \frac{c_1}{c_0} v_1 + \frac{c_2}{c_0} v_2 + \ldots\right) \frac{1}{1 + \frac{c_1}{c_0} + \frac{c_2}{c_0} + \ldots}.$$

This equation may also be written

$$V = \left(v_0 + v_1 A_1 e^{-\int \frac{H_1}{T^2}.dT} + v_2 A_2 e^{-\int \frac{H_2}{T^2}.dT} + \ldots\right) \frac{1}{1 + A_1 e^{-\int \frac{H_1}{T^2}.dT} + \ldots},$$

where H_1 denotes the energy of formation of a cluster 1 from an elementary ion and a neutral molecule at the temperature T, H_2 the energy of formation of a cluster 2 from an elementary ion and two neutral molecules, and so on, and $A_1, A_2, \ldots$ are constants which depend only on the nature of the gas.

From considerations based upon the kinetic theory of gases it follows that the velocity of a stable cluster in a gas must depend on its mass, and decrease with an increase of its mass. The result obtained by Blanc and Wellisch† that the velocity of an ion in a gas does not depend much on the nature of the molecule from

* *Proc. Camb. Phil. Soc.* vol. XVI. pt. IV. p. 285.

† *C. R.* CXLVII. July 1908, pp. 39—42; *Proc. Roy. Soc.* Ser. A, LXXXII. July 31, 1909, pp. 500—517.

which it is derived may therefore be explained by the ion cluster continually changing in complexity, the effect of which might be to produce approximately the same average velocity in each case.

The ions made in a gas are initially elementary ions, and a certain time must therefore elapse after their production before all the possible kinds of clusters are formed and they are in equilibrium with one another. Therefore if the ionic velocity of freshly made ions is measured over a distance of such magnitude that all the possible clusters have not time to form, the velocity will be greater than that obtained over much larger distances. Moreover, the velocity will depend on the magnitude of the electric field applied. By means of this result the increase of the ionic velocity greater than inversely proportional to the pressure of the gas at low pressures was explained.

Experiments on the Ionisation by Collision with Positive Ions.

In the experiments on ionisation by collision the initial ionisation usually takes place in the powerful electric field in which new ions are produced by collision. The elementary ions are then seized upon by the electric field before they have time to attach themselves to neutral molecules, and given a velocity sufficiently large that ions by collision are produced, after which their chance of attaching themselves to neutral molecules is small. Therefore if the initial ions are made in a weak electric field so that they have a chance of forming clusters, and they are then drawn into the powerful field, we should expect that results would be obtained which differ in many respects from those obtained in the former case. The writer has carried out a set of experiments of this nature*. Fig. 1 indicates the experimental arrangement used. A is a plate connected to an electrometer, and B a wire gauze at a distance ·5 cm. from the plate. The chamber C, which was in electrical connection with the gauze, was connected with a battery of cells. Ions were made in the part a of the chamber and drawn through the gauze by the weak field existing between gauze and chamber due to some of the lines of force which end on the plate threading through the gauze on to the chamber wall. Negative or positive ions were drawn through the gauze accordingly as the chamber was raised to a negative or positive potential. It was found that the admixture of a gas to a gas of a different kind produced an effect which could only be explained by the nature of the positive ion depending on the nature of the atom or molecule from which it is produced. This result is of importance

* *Proc. Camb. Phil. Soc.* vol. XVI. pt. 7, p. 621.

Note. In the figures of this paper containing curves read 200 volts per division instead of 40, the small divisions not having come out in the photographs.

in connexion with experiments on the nature of the ions in a discharge tube. It is found that hydrogen atoms and molecules positively charged are always present in the tube, and their apparent number may even be increased by the addition of a gas whose molecules do not contain hydrogen. At first sight this would seem to indicate that the nature of the positive ion is independent of the nature of the atom from which it is produced. But this explanation is not conclusive, since the increase in the number of hydrogen atoms can also be explained thermodynamically. Hydrogen is always given off by the electrodes in the tube due to the discharge, and is therefore always initially present.

Evidence was also obtained that a molecule when ionised is sometimes split into its constituent atoms.

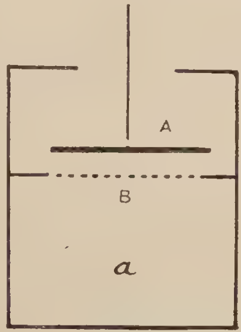


Fig. 1.

Experiments on the Ionisation by Collision with Negative Ions.

Further experiments were carried out along the same lines, especially with the negative ions, which will be described in this paper. For the experimental details (the same apparatus as before was used) the reader must consult the paper quoted.

Fig. 2 shows two curves obtained with negative ions when ether vapour was in the chamber. The curve A was obtained with the ions drawn through the gauze from the space adjacent to its lower side. The curve B was obtained after placing a quantity of radium near the chamber, in which case additional initial ions were made in the space between the gauze and plate by the γ rays of the radium. These ions are not given an opportunity of forming clusters. In the latter case the curve should be situated nearer to the zero than in the former, because in that case we are dealing with a larger amount of initial ionisation. That this must be so will at once be seen on doubling all the ordinates of the curve A, which amounts to doubling the number

of ions drawn through the gauze. It will be seen that the curve B is nearer to the zero than the curve A, as required. But the difference in position is much greater than warranted by the increase of the initial ionisation on bringing the radium near the chamber, which is very small since the curves almost coincide for small fields. It follows therefore that only a small proportion of the ions drawn through the gauze are in a state fit for the production of ions by collision, or the proportion of free ions to clusters is small in a number of ions in equilibrium in ether.

The following vapours were also examined in this way: ethyl propionate, methyl butyrate, acetylene, hydrogen, oxygen, nitrous oxide, carbon dioxide, air, carbon tetrachloride, ethyl chloride, chloroform, pentane, benzene, hexane, aldehyde, methyl formate,

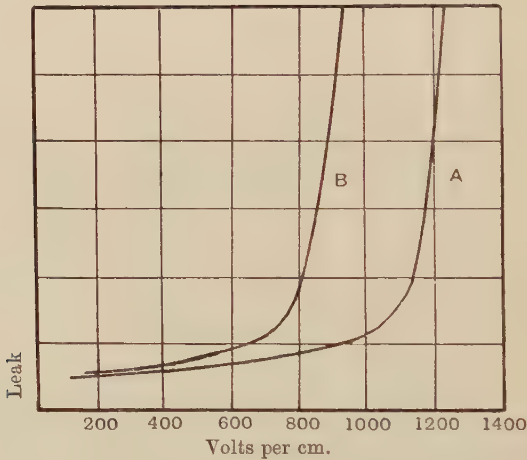


Fig. 2.

methyl bromide, methyl iodide, ethyl bromide, ethyl iodide, and carbon disulphide. The results obtained could to a certain extent be compared with one another, as will be explained. From the theory of clustering referred to at the beginning of the paper it follows that the number of free ions to clusters is independent of the pressure of the gas. The writer* has shown that the relative ionisation in gases whose molecules do not contain atoms of higher atomic weight than that of the chlorine atom is the same for γ and α rays, and in the case of the other gases the ionisation is greater in the latter case than in the former, but the ratio is always less than two. The initial numbers of free ions introduced by the γ and α rays respectively are therefore always proportional

* *Proc. Roy. Soc.* A, vol. LXXIX. p. 220, 1907.

to one another in the case of the former gases. It follows therefore that the ratio of the abscissae of two points one on each of the steep part of two curves of the type A and B corresponding to a common ordinate, should be approximately independent of the pressure of the gas. This was found to be the case. We may therefore as a first approximation take this ratio as a relative measure of the proportion of free ions to clusters in a gas. It was found that in the majority of the gases mentioned this ratio was approximately the same as that obtained with ether. The gases which showed a considerable deviation from this were carbon tetrachloride, carbon disulphide, benzene, air, oxygen, chloroform, and hydrogen, whose ratios were considerably greater than the ratio obtained with ether. Especially was this the case with carbon tetrachloride, carbon disulphide, and benzene; for these three gases the curves obtained almost coincided with one another. It follows therefore that the proportion of free ions to clusters in these particular gases is considerably greater than in the case of the other gases in which the relative ionisation for γ rays is the same as for α rays.

The separation of the curves obtained with the gases containing bromine and iodine atoms in their molecules was about the same as that obtained with ether. This separation is in part due to the greater relative ionisation of these gases with γ rays than with α rays. But the difference in ionisation accounted only for a small part of the separation. We may therefore conclude that these gases fall into line with the majority of the other gases.

It will be of interest here to consider theoretically the behaviour of ion clusters in gaseous mixtures. In one of the papers quoted it is shown that $c_n = t_n n_n$, where c_n denotes the concentration of the ion clusters of the type n, t_n the period of life of a cluster, n_n the number of clusters changing into other clusters per second. Now the quantities t_n and n_n depend directly on the nature of the collision between a cluster and the surrounding molecules. Thus when a cluster collides with a molecule the latter will have a dissolving or dissociating action upon the cluster due to electrostatic induction, chemical attraction, etc., apart from the kinematical effect of the collision. Therefore when two gases are mixed the ratio of free ions to clusters is not likely to be the mean of the ratios of the constituent gases, but may be much greater or much smaller, even if no new kinds of clusters are formed after mixture of the gases. Thus the behaviour of a mixture of gases can hardly be predicted from that of its constituents. If new kinds of clusters are formed after mixture of the gases, matters are still further complicated, and we may be quite sure then that the ratio is different from the mean of the ratios of the constituent gases.

Some experiments by Latty, Frank, and Townsend on the velocity and diffusion of ions have a bearing on the clustering effect under consideration. Thus Latty* found that the velocity of the negative ion in air *specially* dried is much greater than that usually observed. Moreover, the velocity did not vary inversely as the pressure of the air under these circumstances. For example, a velocity of 173 cm. per second was obtained for the negative ion in air at a pressure of 10 mm. of mercury on applying an electric force of ·5 volt per cm., which is about 100 times the velocity obtained under ordinary conditions. On increasing the electric field to ·9 volt per cm. the velocity rose to 1845 cm. per second. It appears therefore that a cluster in air composed of an elementary ion and water and possibly air molecules, is much more stable than when the cluster is composed of air molecules only.

Frank† found that the velocity of a negative ion in carefully purified argon is 206·3 cm. per second, but that it falls to 1·7 cm. per second on adding about one per cent. of oxygen. A cluster in argon composed of argon molecules only is thus very unstable and has a correspondingly short period of life, but its stability is greatly increased when oxygen molecules enter into its composition.

Townsend‡ found that when a gas has been subjected to a drying process for several days the rate of diffusion of the negative ions greatly increases, and does not obey the ordinary laws of diffusion. The effect seems to be greater with H_2 than with O_2, and much greater with these gases than with CO_2. A cluster in CO_2 composed of an elementary negative ion and CO_2 molecules thus seems to be comparatively stable.

When the ions are drawn through the gauze (see fig. 1) in experiments of the nature described in a previous part of the paper, a certain fraction (which may have any value lying between unity and $\frac{1}{\infty}$) should be, as we have seen, in the elementary state, and thus be in a state fit for the production of ionisation by collision if the electric field be sufficiently large. Further ions become elementary on their passage from the gauze to the plate, and the number of initial ions available for the production of ionisation by collision is thus increased. It will be of importance therefore to obtain a formula for the current observed under these conditions when the field is sufficiently large to produce fresh ions by collision of ions. The case when fresh ions are produced by collision of the negative ions only will be considered first.

Let K denote the total number of ions drawn through the

* *Proc. Roy. Soc.* A, vol. LXXXIV. 1910.
† *Verh. d. Deut. Phys. Gesell.* March, 1910, pp. 291—298.
‡ *Proc. Roy. Soc.* A, vol. LXXXV. pp. 25—29, 1911.

gauze of which k_1 are elementary negative ions and k_2 clusters of various complexities, and thus $K = k_1 + k_2$. The quantity K can be measured directly. The elementary ions give rise to a current $k_1 e^{l\alpha}$ by collision with neutral molecules*, where l denotes the distance between gauze and plate, and α the number of fresh ions produced by a negative ion per cm. of its path.

There is an additional current produced by some of the clusters becoming elementary ions during their passage from gauze to plate. Let N denote the number of clusters in a c.c. at a distance x from the plate. We then have $dc = -\frac{dN}{dt}\,.\,dx\,.\,e^{x\alpha}$, where dc denotes the current from a strip of gas of thickness dx and unit area in which $\frac{dN}{dt}\,.\,dx$ clusters become elementary ions per second. Now $\frac{dN}{dt} = -\eta N$, where η denotes the fraction of clusters becoming elementary ions per second. Integrating this equation we have $N = \frac{k_2}{V_1} e^{-t\eta}$, where t denotes the time the clusters take to pass over the distance $(l - x)$, and V_1 the average velocity of a cluster. The value of t is given by $t = \frac{l - x}{V_1}$. Therefore

$$-\frac{dN}{dt} = \frac{k_2}{V_1} \eta e^{-\frac{\eta(l-x)}{V_1}},$$

and thus

$$dc = \frac{k_2 \eta}{V_1}\,.\,dx\,.\,e^{\alpha x - \frac{\eta(l-x)}{V_1}}.$$

Integrating this equation between the limits $x = l$ and $x = 0$ we obtain

$$c = \frac{k_2 \eta}{V_1 \alpha + \eta}\{e^{l\alpha} - e^{-\eta \frac{l}{V_1}}\}.$$

The total current C is therefore given by the equation

$$C = \frac{k_2 \eta}{V_1 \alpha + \eta}\{e^{l\alpha} - e^{-\eta \frac{l}{V_1}}\} + k_1 e^{l\alpha} \quad \text{............(1)}.$$

If η is small in comparison with $V_1 \alpha$, $\eta \frac{l}{V_1}$ is small in comparison with $l\alpha$, and the equation becomes

$$C = \left\{\frac{k_2 \eta}{V_1 \alpha} + k_1\right\} e^{l\alpha} \quad \text{....................(2)}.$$

It will be seen that in the latter case the effect produced by the

* *The Theory of Ionisation by Collision*, p. 4. By Prof. J. S. Townsend.

dissociation of the clusters is equivalent to an additional number of free ions equal to $\frac{k_2\eta}{V_1\alpha}$ being drawn through the gauze.

In the paper on clustering of ions quoted it is shown that $tn = c$, where c denotes the concentration of the clusters per c.c., n the number changing into other clusters and elementary ions per second, and t the life of a cluster. If the clusters are considered in a body so that t denotes the average life of a cluster, $\eta c = n$, and therefore $t = \frac{1}{\eta}$.

It appears also that the dissociation of a cluster takes place after it has undergone a certain average number of collisions, and the time required for these collisions therefore varies inversely as the pressure of the gas, or η increases with increase of pressure of the gas. But the value of η probably also depends on the electric field applied to the gas when of great intensity. The field increases the violence of collision of the cluster with other molecules and consequently decreases its period of life. Since the kinetic energy given to a cluster between two consecutive collisions is proportional to the electric field and inversely proportional to the "mean free path" of the cluster, the effect of the field on the value of η may be expressed as a function of $\frac{X}{p}$. We may therefore write $\eta = p \cdot \phi_1\left(\frac{X}{p}\right)$, where the value of $\phi_1\left(\frac{X}{p}\right)$ increases with an increase of X, and consequently decreases with an increase of p.

If γ denote the fraction of elementary ions becoming clusters per second of a number of ions in equilibrium we have $\gamma k_1 = \eta k_2$, since the number of elementary ions becoming clusters per second must be equal to the number of clusters becoming elementary ions. The ratio of k_1 to k_2 is independent of the pressure of the gas when the ions are not subject to any external force such as an electric field. The effect of the electric field may as before be expressed in terms of $\frac{X}{p}$, or $\frac{k_1}{k_2} = \phi_2\left(\frac{X}{p}\right)$. It follows therefore that we may write $\gamma = p \cdot \phi_3\left(\frac{X}{p}\right)$.

The foregoing considerations will now be applied to some of the experimental results obtained. Let us first calculate the fraction of elementary ions drawn through the gauze that would have to be in the elementary state to account for the current due to ionisation by collision on the supposition that no clusters become elementary during their passage from gauze to plate. Thus in the case of CO_2 at a pressure of 9·5 mm. of mercury a current of 187 in arbitrary units was obtained corresponding to a field of

1360 volts per cm. From the curve for CO_2 giving* the relation between $\frac{X}{p}$ and $\frac{\alpha}{p}$ we obtain $\frac{\alpha}{p} = 1{\cdot}62$ corresponding to

$$\frac{X}{p} = \frac{1360}{9{\cdot}5} = 140,$$

and hence $\alpha = 15{\cdot}39$. Let n_0 denote the initial ionisation that would account for this current. Then we have $187 = n_0 e^{\cdot 5 \times 15 \cdot 39}$, and hence $n_0 = {\cdot}064$. The actual number of ions drawn through the gauze corresponding to a field of 1360 volts can most accurately be obtained by reversing the field and measuring the positive leak obtained. This leak was found to be equal to 25. Thus about ·2 % only of the ions drawn through the gauze need be in the elementary state to account for the current obtained. Actually a smaller fraction is in the free state since some of the clusters become elementary ions during their passage from gauze to plate, and augment the current due to collision of ions. The result obtained fits in with experiments described in a previous part of the paper, where it was shown that the position of the collision curve (fig. 2) with respect to the axes is greatly influenced by radium brought near the chamber, though the initial ionisation was not increased by an appreciable degree.

In the case of air (dried by bubbling it through strong H_2SO_4) at a pressure of 15 mm. of mercury a current of 172 was obtained corresponding to a field of 1440 volts per cm. Now $\frac{\alpha}{p} = {\cdot}85$ corresponding to $\frac{X}{p} = 96$, and hence $\alpha = 12{\cdot}75$. The value calculated for n_0 is thus ·37. The total current through the gauze corresponding to 1440 volts per cm. was equal to 20. Thus less than 2 % of the ions drawn through the gauze were in the free state. In a similar way it was shown that a small fraction only of the negative ions drawn through the gauze when H_2 was in the chamber are in the free state.

It might be suggested that the meshes of the gauze in causing the electric field to be non-uniform in its immediate neighbourhood give rise to the results obtained. Thus the effective distance between gauze and plate may be smaller or greater than that which actually exists. If however the distance between the plate and an imaginary plane corresponding to the collision current for a constant voltage per cm. is calculated, it is found to be only ·14 of the actual distance between gauze and plate. It is quite certain from this number that the effect is not due to the meshes of the gauze.

* *The Theory of Ionisation by Collision*, pp. 19—21. By Prof. J. S. Townsend.

The fraction of clusters drawn through the gauze that become elementary ions on their passage from gauze to plate must be small if the calculated fraction is small of the ions drawn through the gauze that must be in the elementary state to account for the total current observed due to ionisation by collision. If all the clusters were to become elementary ions before reaching the plate the current obtained would be very nearly equal to that obtained on the supposition that all the ions drawn through the gauze are in the elementary state. Since only a small fraction of the ions are in the elementary state it follows that the life of a cluster in the gases under consideration is greater than the time the cluster takes to pass from gauze to plate. If the velocity of a cluster is taken equal to the velocity of an ion measured in the usual way (strictly it must be less) the time of passage of a cluster in the experiments just mentioned is for CO_2 $\frac{\cdot 5 \times 9\cdot 5}{\cdot 85 \times 760} = \cdot 00735$ sec. and for air $\frac{\cdot 5 \times 15}{1\cdot 87 \times 760} = \cdot 00554$ sec. Thus the average periods of life of the clusters must be greater than these values, and the values of η consequently less than 66·45 and 90·2 respectively.

The values of $V_1\alpha$ in the cases under consideration are 1047 and 1150 respectively, and thus the values of η are small in comparison with the values of $V_1\alpha$. Equation (2) may therefore be used to calculate the values of η, if $\frac{\eta k_2}{\alpha V_1}$ is not small in comparison with k_1.

Thus for example in the case of air a current of 172 and 80 was obtained corresponding to a field of 1440 and 1320 volts respectively. The values of $\frac{\alpha}{p}$ corresponding to the values of $\frac{X}{p}$ are ·85 and ·715 respectively. The positive leaks corresponding to the above voltages were 20 and 18·5 respectively. If k_1 denote the number of free ions and k_2 the number of clusters corresponding to the leak 20, the number corresponding to the leak 18·5 are $k_1 \frac{18\cdot 5}{20}$ and $k_2 \frac{18\cdot 5}{20}$. The two simultaneous equations obtained from equation (2) and the equations $k_1 + k_2 = 20$ and $t_1\eta = 1$ then give $k_1 = \cdot 196$, $k_2 = 19\cdot 8$, $\eta = 1\cdot 134$, and $t_1 = \cdot 89$ sec. Thus about 1 % of the ions in equilibrium in air are in the elementary state; and the period of life of a cluster at a pressure of 15 mm. of mercury is of the order of one second. The value of γ is 115, and the corresponding period of life t_2 of an elementary ion therefore ·00872 sec. Since t_1 and t_2 are inversely proportional to the pressure, their values at atmospheric pressure are ·0176 and ·000172 second respectively.

It is of interest to obtain a value for the average number of collisions a negative cluster undergoes before becoming a simple ion. From the kinetic theory of gases we find that the number of collisions an air molecule undergoes with other molecules at a temperature of 30° is $1 \cdot 6 \times 10^{10}$ per second. A negative ion cluster in air at standard pressure therefore undergoes on the average a number of collisions of the order 10^8 per second before becoming a simple ion, and an elementary ion undergoes a number of collisions of the order 10^6 per second before it successfully forms a cluster.

The above calculations cannot be carried out with success in the case of CO_2; very likely $\eta \frac{k_2}{\alpha}$ is small in comparison with η. Townsend's experiments on diffusion suggest that η is probably smaller in CO_2 than in other gases. Better results might be obtained after making some improvements in the experimental arrangement. These would consist in having the distance between gauze and plate as large as is convenient, and using as large a potential difference as possible, the pressure of the gas being regulated to suit these conditions.

The numerical results obtained for air can only be approximately correct because the calculations are subject to considerable errors, as small errors in the data may appear as large errors in the ultimate results. Moreover, we have seen that the period of life of a cluster and elementary ion depends greatly on the dryness of the air. Latty's experiments suggest that the period of life of an elementary ion in air specially dried is much greater than that obtained by the writer. It appears also that under these circumstances the values of α would be very different from those obtained by Townsend. The dryness of the air in Townsend's and the author's experiments was however probably approximately the same, and as the velocity of an ion and other quantities are approximately independent of the dryness when it is not exceptionally great, the use of Townsend's values of α was not objectionable. Further the values of η and t_1 are probably also seriously affected by the electric field when it is so large that new ions are produced by collision. However, the results obtained give one some idea of the processes going on in an ionised gas, and the order of magnitude of the quantities involved.

The conclusions of a general nature that can be drawn from the author's experiments and those of other investigators are as follows. The period of life of an elementary negative ion or a cluster depends very much on the nature of the gas in which it is formed. It depends also very much on slight admixtures of a different gas. The period of life of a cluster in a gas at standard pressure may have a value lying between a few seconds and a

small fraction of a second. The order of magnitude of the period of life of an elementary negative ion is about $\frac{1}{100}$ that of a cluster.

The results obtained have an important bearing on the motion of a negative ion in a gas under the influence of an electric field. The average velocity V of the ion under unit electric field is obtained from an equation given at the beginning of the paper which may also be written

$$V = \frac{c_0 v_0 + c_a v_a}{c_0 + c_a},$$

where c_a denotes the concentration of the clusters (considered in a body) and v_a the average velocity of a cluster, and c_0 and v_0 these quantities corresponding to the elementary ions. The experiments of Latty on the velocity of the negative ion in air specially dried suggest that v_a is of the order $100V$. Since c_a, we have seen, is of the order $\frac{c_a}{100}$, the order of magnitude of $c_0 v_0$ is the same as that of $c_a v_a$, and the presence of free negative ions in a gas thus affects the average ionic velocity. Thus the various expressions for the velocity of an ion through a gas that have been obtained by different physicists on the supposition that the ion consists of a definite unchanging cluster of molecules cannot be regarded as representing what actually happens in the gas, but have value only as useful empirical formula. It is conceivable that the effect of the passage of the ion through cycles of clustering on its velocity might be the same as that of a cluster of average mass. It should also be noticed that these formulae usually rest on one or more additional assumptions, which are not likely to be realized in practice. One of them is that the motion of an ion due to the electric field is reduced to zero on each collision with a neutral molecule.

The results obtained have also an interesting bearing on the interpretation of the results obtained on ionisation by collision in the usual (Townsend's) experimental arrangement. We have seen that $\frac{k_1}{k_2} = \phi_2\left(\frac{X}{p}\right)$, where $\phi_2\left(\frac{X}{p}\right)$ increases with an increase of the field and consequently decreases with an increase of the pressure. But it probably varies only appreciably with the field when its strength is comparable with that necessary to produce ionisation by collision. But even when ionisation by collision occurs k_2 may not be zero, for sometimes the collision of an elementary ion with a molecule must be favourable for the formation of a cluster. But the life of such a cluster will of course be much smaller than when the field is weak. Thus the current when ionisation by collision takes place may be divided into two parts, a current

of elementary ions and a current of clusters. But the elementary ions only produce further ions by collision. Now the quantity α in the ordinary theory of ionisation by collision is calculated on the supposition that the whole current consists of elementary ions. The quantity α when it refers to the partial current of elementary ions is however the important one, it will in this case be denoted by α_1. The α's are connected by the equation $C_e \alpha_1 = \alpha (C_e + C_c)$, where C_e denotes the current of elementary ions and C_c that of the clusters. Since C_e increases while C_c decreases with increase of electric field, the value of α_1 differs most from that of α when ionisation by collision begins to come in, being always greater than α_1, while for much greater fields they may be practically equal to one another. The curves connecting $\frac{X}{p}$ and $\frac{\alpha_1}{p}$ would therefore be less concave towards the $\frac{X}{p}$ axis near the origin than the curves connecting $\frac{X}{p}$ and $\frac{\alpha}{p}$ obtained by Townsend. We cannot obtain any information about the relative values of C_e and C_c from measurements of the ionisation current, and it does not seem possible to devise a simple experiment by means of which they could be directly measured.

The velocity with which an electron is ejected from its parent atom has little influence on the ionisation by collision. It is practically only the first collision of the electron with a neutral molecule that would get the benefit of the initial velocity of the former, if the motion of the electron is in the same direction as that given to it by the electric field. On the average the electron undergoes hundreds of collisions before it reaches one of the electrodes, and the first collision is therefore comparatively of no importance.

But the ionisation by collision should be considerably affected by the direction of motion of the ejected electrons relative to that given to them by the electric field. Consider the two cases when the electric field acts in the same direction as the ejected electron is moving, and when it acts in the opposite direction. In the latter case the electron must be reduced to rest before a velocity can be given to it sufficiently large to produce ionisation by collision. The chance of a cluster being formed is therefore much greater in the latter case than in the former. On a cluster being formed it must run a distance x depending on its period of life and velocity before it becomes an elementary ion again, after which it can be used by the electric field for the production of new ions by collision. Thus in the latter case the collision current is smaller than in the former. The effect may roughly be said to

correspond to a distance A between the electrodes in the former case and a distance $A-x$ in the latter. Since the collision current increases rapidly with the distance of separation of the electrodes and the value of α, the asymmetry in the leaks obtained by reversing the field should also increase with these quantities. The writer* has carried out experiments on the ionisation by the α particle involving the foregoing principles, and found that the ejected electron has a component of motion in the same direction as the ionising α particle.

The case when the initial ionisation is produced on the surface of one of the electrodes in the field producing ionisation by collision requires consideration here. Since all the ions start out as simple ions, their condition near the surface is different from that some distance away, where there is equilibrium between the clusters formed and the simple ions. It is evident that when the breadth of the region in which equilibrium is produced between the clusters and simple ions is comparable with the distance between the electrodes, the value of the collision coefficient α should depend on the distance between the electrodes and the pressure of the gas. But α would obviously not vary abruptly with the distance of separation of the electrodes because the region in question has no well defined boundary.

Now Campbell† has carried out an investigation on the dependence of α on the distance of separation between the electrodes. The object of the investigation was to distinguish between two hypotheses as to the changes an ion undergoes when producing ionisation by collision, viz. (1) that the simple ion never forms a cluster during collision, (2) that sometimes a cluster is formed which is not broken up subsequently. If the latter hypothesis is true α should depend on the distance between the electrodes and the pressure of the gas. Campbell concludes from his experiments that no clusters are formed. Interpreting these experiments from the point of view that the ion cluster is unstable, it follows that either the region in which equilibrium between the simple ions and clusters is produced is small, or, that the period of life of a cluster is small in comparison with that of a simple ion in an electric field of intensity sufficiently large to produce ionisation by collision. It will be easily seen that in the latter case the effect of the region under discussion is negligible for all distances of separation of the electrodes. The latter explanation is probably the correct one.

It was found impossible to obtain any definite information about the period of life of an elementary positive ion or cluster.

* *Proc. Roy. Soc.* A, vol. LXXXIII. p. 195, 1909, and *Phil. Mag.* p. 198, July, 1912.

† *Phil. Mag.* p. 400, March, 1912.

This was principally due to the fact that the values of $\frac{\beta}{p}$ required, where β denotes the coefficient of collision for the positive ions, could not be obtained from Townsend's curves for small values of $\frac{X}{p}$. There is good reason to believe, however, from other experiments, that the period of life of a positive cluster is greater than that of a negative. The behaviour of the positive and negative rays in a discharge tube, which has been studied in great detail by Sir J. J. Thomson, Wien, and others, is evidence in support of this. Obviously we should obtain evidence of those clusters only whose period of life is greater than the time it takes them to travel the length of the tube under the electric field. The number of different clusters obtained would in that case depend on the pressure of the gas in the tube, its dimensions, etc., and this has been found in practice. Now as a rule the number of different negative clusters obtained is much smaller than the number of positive clusters, and may be explained by a difference in the period of the clusters. It will be of interest, however, to give the formula here by means of which the period of life of a positive cluster could be calculated. The production of dark and luminous spaces in a discharge tube under certain conditions must be in part regulated by the fact that the proportion of free ions to clusters increases as the ions pass into a stronger field. The principles involved will be brought out in deducing the formula in question.

Let K denote the number of positive ions drawn through the gauze when at a positive potential, of which k_1 are in the elementary state and k_2 clusters, so that $K = k_1 + k_2$. The ionisation current c_1 produced by the free ions k_1 is given by

$$c_1 = \frac{k_1 (\beta - \alpha) e^{(\beta - \alpha) l}}{\beta - \alpha e^{(\beta - \alpha) l}} \quad \text{......................(3)},$$

where β denotes the number of new ions made by a positive ion per cm. of its path. This formula is obtained by interchanging α and β in the formula used by Townsend in his investigations of the ionisation by collision of positive ions.

The ionisation produced indirectly by the clusters will be considered separately. Let S denote the number of clusters that cross per second a plane parallel to the plate at a distance x, and δ the fraction of the number of clusters becoming elementary ions per second. We then have $\frac{dS}{dt} = -S\delta$, which on integrating becomes $S = k_2 e^{-\delta t}$. Now $t = \frac{l - x}{V_2}$, where V_2 denotes the average

velocity of the positive clusters per second. Hence

$$S = k_2 e^{-\frac{\delta(l-x)}{V_2}} = k_2 A z^x,$$

where $A = e^{-\frac{l\delta}{V_2}}$ and $z = e^{\frac{\delta}{V_2}}$.

Let r' be the number of negative ions produced by collision in the layer of gas between the plate and the parallel plane at a distance x, and r the number of positive ions produced by collision between this plane and the gauze. Let c denote the current crossing the plane which gives rise to further ionisation by collision. Then we have

$$c = k_2(1 - Az^x) + r + r'.$$

The number of ions dr generated between the two planes at distances x and $x + dx$ is given by the equation

$$-dr = [\{k_2(1 - Az^x) + r\}\beta + r'\alpha]\,dx.$$

Substituting for r' from the foregoing equation we obtain

$$\frac{dr}{dx} + (\beta - \alpha)\,r = (\beta - \alpha)\,k_2(Az^x - 1) - c\alpha.$$

Multiplying by $e^{(\beta-\alpha)x}$ and integrating we obtain

$$re^{(\beta-\alpha)x} = -k_2 e^{(\beta-\alpha)x} + \frac{k_2(\beta-\alpha)\,A e^{\left(\frac{\delta}{V_2}+\beta-\alpha\right)x}}{\frac{\delta}{V_2}+\beta-\alpha} - \frac{c\alpha e^{(\beta-\alpha)x}}{\beta-\alpha} + C \ldots (4),$$

where C is an arbitrary constant. The value of the constant is obtained from the condition that $r = 0$ when $x = l$, which gives

$$C = \left(\frac{c\alpha + k_2(\beta-\alpha)}{\beta-\alpha}\right) e^{(\beta-\alpha)l} - \frac{k_2(\beta-\alpha)\,A e^{\left(\frac{\delta}{V_2}+\beta-\alpha\right)l}}{\frac{\delta}{V_2}+\beta-\alpha}.$$

From one of the foregoing equations we have $c = k_2(1 - A) + r$, when $x = 0$, where r is given by equation (4), and hence on substituting for r we obtain

$$c\left(\frac{\beta - \alpha e^{(\beta-\alpha)l}}{\beta-\alpha}\right) = \frac{k_2\delta_1}{\delta_1 + \beta V_2 - \alpha V_2}\left(e^{(\beta-\alpha)l} - e^{-\frac{l\delta_1}{V_2}}\right) \ldots\ldots (5).$$

The total current c_t is given by

$$c_t = c + c_1 \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (6),$$

where c_1 and c are given by equations (3) and (5) respectively. If δ is small in comparison with $(\beta-\alpha)V_2$, $\frac{l\delta}{V_2}$ is small in comparison with $(\beta-\alpha)l$, and in that case

$$C_t = \left(\frac{\delta k_2}{V(\beta-\alpha)} + k_1\right)\frac{(\beta-\alpha)e^{(\beta-\alpha)l}}{\beta-\alpha e^{(\beta-\alpha)l}}.$$

The effect of the unstability of the positive ion is then as if an additional number of free ions equal to $\frac{\delta k_2}{V_2(\beta-\alpha)}$ were drawn through the gauze.

In conclusion I wish to express my thanks to Prof. Sir J. J. Thomson for his kind interest in these experiments. It should also be mentioned that the expense of the principal apparatus was defrayed out of a grant of the Royal Society.

Note on the Absorption of Cathode Rays by Metallic Sheets. By R. Whiddington, M.A., St John's College.

[*Received* 17 June 1913.]

Last year I published an account of some experiments on the transmission of cathode rays through matter which showed that the law determining the diminution of velocity of cathode rays through matter was expressed by the relation

$$v_0^4 - v_x^4 = a \cdot x,$$

where

v_0 = the velocity of a beam of rays incident on a sheet of material of thickness x,

v_x = the greatest velocity in the emergent stream,

and a = a constant depending on the material of the sheet.

We are thus led to the conception of range of cathode rays, for if in the above expression we put $v_x = 0$ the value of v_0^4/a is the thickness of the material through which cathode rays of velocity v_0 can just penetrate.

In fact by using this expression we can easily determine the constant a by measuring the velocity v_0, below which no appreciable effect is produced on the emergent side of a sheet of thickness d; we can then substitute these values in $d = v_0^4/a$.

I have recently been making experiments with cathode rays of definite speeds in order to determine the *number* of the rays absorbed during their passage through metals.

Lenard, Seitz and others many years ago showed that using rays of a more or less definite speed, the law of absorption was exponential, being expressed by the relation

$$I = I_0 e^{-\lambda x},$$

where

I_0 = the cathode ray current incident on a metal sheet of thickness x,

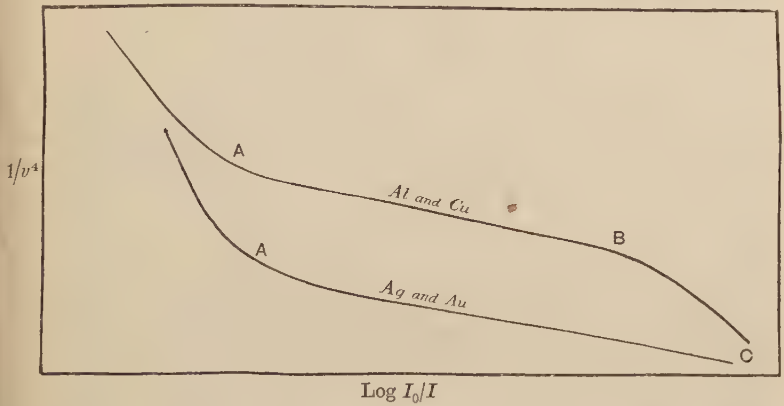
I = the corresponding emergent cathode ray current,

and λ = the absorption coefficient, a constant for any definite speed but varying inversely as the fourth power of the speed of the incident cathode rays.

I have found that in all cases I have examined this law is by no means universally applicable. In fact the experiments were originally commenced with the idea of discovering whether λ suffered an abrupt change when the material of the absorbing screen commenced to give out its fluorescent Röntgen radiation.

So far, experiments have been carried out using screens of Au, Ag, Cu and Al.

The curves given below are typical of two classes of absorption which have been obtained. In these curves $1/v^4$ is plotted along the y-axis while the corresponding values of $\log I_0/I$ are plotted



along the x-axis. The simpler type is the lower one obtained with Ag and Au sheets. It is clear that $\lambda \propto 1/v^4$ for velocities greater than that corresponding to the point A. The value of v for the point A is given quite as accurately as could be expected by the expression quoted above—$d = v^4/a$, and thus depends on the thickness of the sheet.

The other type has been obtained with Cu and Al sheets and shows in addition to the bend at A already explained another bend at B. The velocity corresponding to this point is in the case of Cu 6.2×10^9 cm./sec., which is the velocity above which the fluorescent X-radiation of Cu is excited. This is the effect which was being looked for, but it is not possible just at present to ascribe it definitely to the excitation of the fluorescent radiation since Al also shows a similar effect though not at the same velocity.

The experiments are being continued.

The Influence of Molecular Constitution and Temperature on Magnetic Susceptibility. Preliminary Note. By A. E. Oxley, B.A., Coutts Trotter Student, Trinity College.

[*Received* 14 July 1913.]

In a former paper* the author has shown how the observed departure from the Curie-Langevin laws for paramagnetism and diamagnetism may be explained in terms of the variation of the nature of the molecular complexes with temperature. Later, the influence of such aggregations has been considered by Holm†, Weiss and Piccard‡, and Piccard§; the last physicist showing that the variation of the diamagnetic susceptibility of water with the temperature can be interpreted as due to the presence of two types of complexes. The idea of the coexistence of different types of complexes has been satisfactorily applied to the case of aqueous solutions of salts of the ferromagnetic elements‖.

The present note is intended to contain a preliminary account of further experiments which have been made to test the effect of the presence of molecular complexes on the susceptibility.

We shall denote the specific diamagnetic susceptibility by χ.

About thirty organic and several inorganic substances have been investigated and the variation of χ with temperature has been examined over a range of 250° C. It is found in general¶ that there is a decrease in the value of χ during the passage from the liquid to the crystalline state. This decrease amounts to 5°/$_\circ$ (approx.) of the value of χ. If the substance supercools or passes into a gel as the temperature is lowered there is no discontinuity, but when crystallisation does take place the value of χ is found to decrease. If the substance is now heated χ remains constant until the normal melting point is reached but increases during fusion. A hysteresis loop due to temperature is thus obtained, similar to those which Hopkinson** discovered for the passage from the paramagnetic (non-crystalline) to the ferromagnetic (crystalline) state in the case of nickel-steels, the critical temperature in the latter case corresponding to the temperature of fusion of the diamagnetic crystals in the former case.

With regard to the continuity found when the substance passes

* *Proc. Camb. Phil. Soc.*, Vol. xvi. p. 486, March 1912.

† *Ark. för Mat.*, Stockholm, 8, 16, 1912.

‡ *Comptes Rendus*, t. 155, p. 1234.

§ *Comptes Rendus*, t. 155, p. 1497.

‖ *Proc. Camb. Phil. Soc.*, Vol. xvii. p. 65, Dec. 1912.

¶ There are a few exceptional cases which are not discussed here.

** *Proc. Roy. Soc.*, Vol. xlviii. p. 1, 1890; or Ewing's *Magnetic Induction in Iron and other metals* (third edition), p. 184 et seq.

from a liquid into a gel, it is interesting to note that Chaudier* has found that the specific magnetic rotation of such substances is continuous, a discontinuity appearing only when crystallisation sets in.

On the theory of diamagnetism developed by Langevin†, the above experiments indicate that in the quasi-chemical compounds formed during aggregation or crystallisation the internal structure of the molecule or atom is modified. This result was assumed in the investigation cited at the beginning of this note.

It will be observed that an exact interpretation of the experimental facts must necessarily be highly speculative in view of the present state of the electron theory of valency and further discussion of the quantitative results is reserved until an early date.

* *Comptes Rendus*, t. 156, p. 1529, May 1913.

† A complementary theory to that of Langevin has recently appeared dealing with the diamagnetism of conducting substances and the part played by free electrons (Schrödinger, *Sitz. d. k. Akad. Wien* cxxi. p. 1305, 1912). In dealing with organic liquids the diamagnetic effect due to this source is negligibly small.

Note. In the paper "The Variation of Magnetic Susceptibility with Temperature. Part II. On Aqueous Solutions" (*Proc. Camb. Phil. Soc.*, Vol. XVII. p. 65) the temperature coefficient given at the foot of p. 75 is due to du Bois. The recent work of Weiss and Piccard, referred to above, shows that this value is incorrect, but the change does not affect the numbers given in the tables on pp. 76—77.

It should be mentioned that the susceptibilities given by Jaeger and Meyer are referred to unit volume of the solutions. As, however, the density of a solution is very approximately a linear function of the absolute temperature, over the interval considered, the *specific* susceptibility is correctly expressed by the formula

$$\chi = \frac{A}{\vartheta} + B + C\vartheta,$$

where A, B and C are independent of the absolute temperature (ϑ). The conclusions reached concerning the presence of molecular complexes are of course unmodified.

A Chinese Flea-trap. By EDWARD HINDLE, B.A., Ph.D., Magdalene College, Cambridge, Assistant to the Quick Professor of Biology. (*Communicated by Professor Nuttall.*)

[*Read* 5 May 1913.]

THROUGH the kindness of Mr Stanley A. Stericker, we have recently been able to obtain from Cheng-tu, the capital of Sze-Chwan, an example of a flea-trap much used by the natives in that part of China.

The apparatus consists of two pieces of bamboo one inside the other. The outer bamboo is about one foot in length and $2\frac{1}{2}$ inches in diameter and is fenestrated in the manner shown in the accompanying photograph. The inner bamboo is of equal length but only about one inch in diameter, and is kept in position within the former by means of a short wooden plug.

The manner in which the apparatus is employed is as follows:

The two pieces of bamboo are first separated by removing the wooden plug. The inner bamboo is then coated with bird-lime, or some similar sticky substance, and put back in position within the fenestrated bamboo. The function of the latter is protective and prevents the sticky surface from coming in contact with any large objects. The whole trap can now be placed under bed-clothes, or amongst rugs, etc., and any fleas that get on to the surface of the inner bamboo at once stick to the bird-lime and are thus caught.

The apparatus is said to be a very efficient flea-trap, and considering its simplicity it might be used with advantage during plague epidemics, in order to catch any fleas, rat or human, within houses. Considering the importance of the rat-flea in the transmission of plague, the employment of a simple and effective flea-trap, such as the one described above, would probably have a decided effect on the spread of the disease.

Some methods of measuring the surface tension of soap films. By G. F. C. Searle, Sc.D., F.R.S., University Lecturer in Experimental Physics, Fellow of Peterhouse.

[*Read* 19 May 1913.]

§ 1. *Introduction.* The following paper gives an account of some methods employed in my practical class at the Cavendish Laboratory for the measurement of the surface tension of films of soap solution. The first two methods have been in use for some years, but I have included them in the hope of making the paper more useful to teachers of practical physics. The apparatus may, without loss of efficiency, be constructed in quite "home-made" style or may be built up of elements at hand in most laboratories.

§ 2. *Torsion balance method.* Let $ABCD$ (Fig. 1) be a rectangular frame of thin wire, the plane of the rectangle being

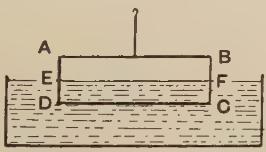


Fig. 1.

vertical. If the frame be dipped into a soap solution and then be partially withdrawn so that the horizontal surface of the solution cuts the frame at E and F, a film will be formed, which will fill the area $ABFE$. This film will pull the frame downwards. If the surface tension of the solution be T dynes *per centimetre**, and if the distance EF be l cm., the downward pull on the frame will be $2Tl$ dynes, *each* surface of the film contributing Tl dynes. If the downward pull be measured, the surface tension can be calculated.

The pull of the soap film is easily measured by aid of the simple torsion balance shown in Fig. 2. This was designed, in conjunction with Mr W. G. Pye, as a more convenient form of the apparatus originally constructed at the Cavendish Laboratory.

The base of the balance is a tripod stand furnished with a levelling screw. From this stand rises an adjustable vertical rod carrying a stiff metal frame, across which is stretched a torsion wire; the tension of the wire can be adjusted by a screw. A double-ended beam is attached to the wire. The long arm of the

* So many elastic constants are expressed in dynes *per square centimetre* that students easily fall into the error of stating surface tensions in dynes per square cm.

beam is pointed to serve as an index, and this index point moves near a vertical scale divided to millimetres; the short arm carries an adjustable counterpoise for bringing the beam to a horizontal position. To secure a good connexion of the beam to the wire, the beam is clamped to a short metal tube of small bore, through which the wire passes and to which the wire is soldered. Near the pointed end of the beam is cut a small notch which serves to define the position of a hook supporting a small scale pan. Below the scale pan hangs the wire frame on which the film is formed. This frame is about 8 cm. in length and 3 cm. in height.

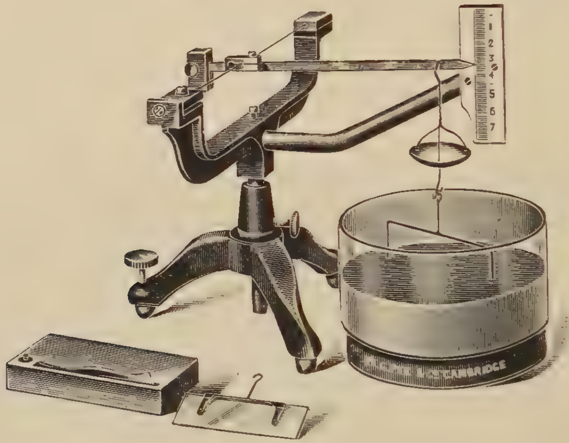


Fig. 2.

The sensitivity of the torsion balance depends upon the thickness of the torsion wire; by using wires of different thicknesses a wide range of sensitivity can be covered. For the present purpose, it is convenient to use a torsion wire such that one deci-gramme in the scale pan gives a deflexion of about 2·5 mm.

A beaker containing soap solution is placed so that the frame dips into it, and the height of the torsion balance is adjusted so that when the frame is drawn down by the film, the film is one or two centimetres in height. There must be enough solution in the beaker to allow the frame to be completely immersed when necessary.

The measurements are taken as follows:—The end of the balance arm is depressed so as to completely immerse the frame. The arm is then allowed to rise, when a film will be formed between the emergent part of the frame and the horizontal surface of the solution. The solution will then begin to drain off from the wire and from the film itself, and the scale reading

of the index will change slightly, but will reach a steady value after a short time. This steady reading is recorded. The reading must be taken to $\frac{1}{10}$ mm.; a short focus lens mounted in a clip may be used to give the necessary optical assistance.

The film is now broken. The arm then rises but is brought back to the position it had when the film was intact by placing a mass m grms. in the pan. If necessary, the exact mass required is obtained by interpolation from two scale readings, the mass being a little too large for one reading and a little too small for the other. A "rider" may also be used to obtain the fine adjustment. If its mass be r grms., its effect is equivalent to that of a mass rx/d grms. placed in the pan, where d and x are the distances from the axis of the torsion wire to the notch and to the point of suspension of the rider. Since the volume of the wire below the surface of the solution is the same as when the film was intact, the upward thrust of the solution is the same in each case.

Hence the weight, mg dynes, of the mass in the pan is equal to the force which was exerted by the film. Thus

$$mg = 2Tl$$

or

$$T = \frac{mg}{2l} \text{ dynes per cm.} \quad \text{.....................(1)},$$

where l is the distance EF (Fig. 1) between the points where the frame cuts the solution. This is the shortest distance between the two circles in which the plane of the surface intersects the two wires.

After each pair of readings, the height of the torsion balance may be slightly changed or a small mass may be placed in the pan. In this way a number of independent readings may be obtained.

§ 3. *Practical example.* The following results were obtained in an experiment:

Width of frame $= l = 8{\cdot}00$ cm. Temperature about 19° C.

Reading with film unbroken	Readings with film broken		
	0·40 grm. in pan	0·45 grm. in pan	m
3·83 cm.	3·79 cm.	3·85 cm.	0·433 grm.
3·87	3·82	3·88	0·442
3·90	3·85	3·90	0·450
4·00	3·94	4·00	0·450

Mean value of $m = 0{\cdot}444$ grm.

Hence $$T = \frac{mg}{2l} = \frac{0{\cdot}444 \times 981}{2 \times 8{\cdot}00} = 27{\cdot}22 \text{ dynes per cm.}$$

§ 4. *Measurement of surface tension of water.* The balance may also be used to determine the surface tension of water or any other transparent liquid which will not form persistent films. A thin rectangular glass plate is held in a clip (Fig. 2) by which it is suspended below the scale pan instead of the wire frame. The glass slips sold for microscope slides are convenient. The plate is adjusted in the clip so that when it is suspended its lower edge is horizontal. The plate is then allowed to dip into the liquid in the beaker and the balance is raised until the lower edge of the plate is exactly in the plane of the undisturbed part of the surface of the liquid; the fine adjustment is conveniently made by the levelling screw in the base of the instrument.

The scale reading of the index is now taken and then the beaker of liquid is removed and the plate is dried by filter paper. A mass m grms. is then placed in the pan to bring the index to the first reading. If the length of the plate be l cm. and its thickness be a cm., the surface tension is given by

$$T = \frac{mg}{2(l+a)} \text{ dynes per cm.}$$

If water be used, the surface should be free from grease. The beaker should be cleaned with potash and should be filled with water freshly drawn from the tap. The glass plate should also be cleaned with potash.

§ 5. *Thread method.* The surface tension of a soap solution can be found by this method with very simple apparatus.

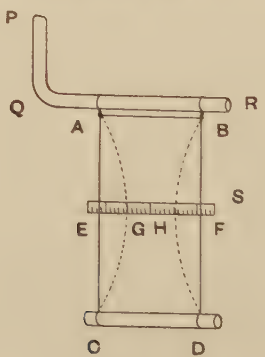


Fig. 3.

On the horizontal arm QR (Fig. 3) of a bent glass rod PQR slide two rings A, B. Through eyes on these rings passes a thread whose ends are attached to a glass rod CD. By adjusting the rings and the thread the distances AB and CD can be made equal, and the rod can be made to hang with its axis horizontal.

The points C, D should be at equal distances from the corresponding ends of the rod. The distance AC should be three or four times the distance AB.

If the whole system be dipped into soap solution and be then withdrawn, a film will be formed in the area bounded by the thread and the lower rod CD. The threads now take the form of curves AGC, BHD, which we shall show are arcs of circles. The vertical part PQ of the bent rod is fixed in a clamp and a horizontal scale S is placed close to the film, and the distance GH between the two points on the threads where the tangents are vertical is determined from the scale readings of the threads. The film is then broken and the scale readings of the threads are again taken. Before the first pair of readings is taken as much as possible of the solution adhering to the lower rod is removed by filter paper so that the supported mass may be as nearly as possible the rod alone.

Let the distance EF (Fig. 3) between the threads when they are vertical be a cm. and let the distance between the points GH when the threads are curved be b cm. Let the mass of the rod CD be m grms.; the mass of the threads and of the film may be neglected. Let the tension of the threads at G and H be N dynes.

The weight of the part of the system below a horizontal plane through G, H is mg dynes, and this is supported by the stresses which act across the plane. The force due to the film is $2Tb$ dynes, since there are *two* faces to the film, and the force due to the threads is $2N$ dynes. Hence the equation of equilibrium is

$$2Tb + 2N = mg \quad \text{.......................(2).}$$

Since the weight of the threads is negligible and since the force exerted by the film on any element of either thread is at right angles to the element, it follows that the tension of each thread is constant and equal to N dynes.

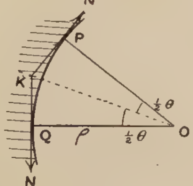


Fig. 4.

Let P, Q (Fig. 4) be two neighbouring points on the thread, and let the radius of curvature of the arc PQ be ρ cm. Let PQ

subtend an angle θ radians at O, the centre of curvature. Then $PQ=\rho\theta$.

The force which the other parts of the thread exert upon PQ is $2N\sin\frac{1}{2}\theta$ in the direction KO, where K is the point of intersection of the tangents at P and Q. The force which the film exerts on PQ is in the direction OK and lies between $2T.PQ$ and $2T.PQ\cos\frac{1}{2}\theta$ dynes or between $2T\rho\theta$ and $2T\rho\theta\cos\frac{1}{2}\theta$ dynes. Since the force due to the thread balances the force due to the film, we see that N lies between

$$2T\rho\frac{\frac{1}{2}\theta}{\sin\frac{1}{2}\theta} \text{ and } 2T\rho\frac{\frac{1}{2}\theta}{\sin\frac{1}{2}\theta}\cos\frac{1}{2}\theta.$$

When θ approaches zero, the limit of $\frac{1}{2}\theta/\sin\frac{1}{2}\theta$ is unity and the limit of $\cos\frac{1}{2}\theta$ is also unity. Hence

$$N=2T\rho \text{ dynes} \quad\text{.......................(3).}$$

Since N and T are constant, ρ also is constant and hence the threads form arcs of circles.

The equilibrium equation now becomes

$$T(2b+4\rho)=mg,$$

or

$$T=\frac{mg}{2b+4\rho} \quad\text{.........................(4).}$$

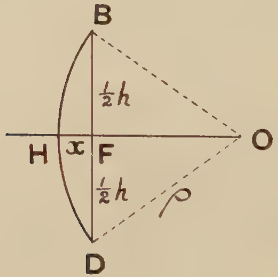


Fig. 5.

The radius of curvature, ρ, of the threads must now be found. Let $x=\frac{1}{2}(a-b)$ so that (Figs. 4, 5) $x=HF=EG$. Then, from Fig. 5, if $BD=h$ cm.

$$x(2\rho-x)=\tfrac{1}{4}h^2,$$

or

$$\rho=\frac{h^2}{8x}+\frac{x}{2} \quad\text{.........................(5).}$$

The vertical distance $h=BD$ is measured *while the film is unbroken.*

If the length of each thread, i.e. the length of the arc BHD, be l cm., the radius can be found from an *approximate* formula (6). Thus, if the angle BHO be ϕ, we have $\phi = l/2\rho$. But

$$\frac{x}{\rho} = 1 - \cos\phi = 1 - \left(1 - \frac{\phi^2}{1.2} + \frac{\phi^4}{1.2.3.4} - \ldots\right) = \frac{\phi^2}{2}\left(1 - \frac{\phi^2}{12} + \ldots\right)$$

$$= \frac{l^2}{8\rho^2}\left(1 - \frac{l^2}{48\rho^2} + \ldots\right).$$

Hence $$\rho = \frac{l^2}{8x}\left(1 - \frac{l^2}{48\rho^2} + \ldots\right).$$

The first approximation is $\rho = l^2/8x$. Using this in the term $l^2/48\rho^2$, we have as a second approximation

$$\rho = \frac{l^2}{8x}\left(1 - \frac{4}{3}\frac{x^2}{l^2} + \ldots\right) = \frac{l^2}{8x} - \frac{x}{6} + \ldots \quad \ldots\ldots\ldots(6).$$

§ 6. *Practical example.* The following is a record of an experiment by Mr C. E. Simmons.

Distance EF between threads when vertical $= a = 1{\cdot}68$ cm.
Minimum distance GH between threads when curved $= b = 1{\cdot}15$ cm.
Vertical distance BD when film is unbroken $= h = 7{\cdot}40$ cm.
Mass of glass rod $CD = m = 2{\cdot}94$ grms.

Hence $$x = \tfrac{1}{2}(a - b) = 0{\cdot}265 \text{ cm.},$$

and $$\rho = \frac{h^2}{8x} + \frac{x}{2} = \frac{7{\cdot}40^2}{8 \times 0{\cdot}265} + \frac{0{\cdot}265}{2} = 25{\cdot}83 + 0{\cdot}13 = 25{\cdot}96 \text{ cm.}$$

Hence, by (4),

$$T = \frac{mg}{2b + 4\rho} = \frac{2{\cdot}94 \times 981}{2{\cdot}30 + 103{\cdot}84} = 27{\cdot}17 \text{ dynes per cm.}$$

§ 7. *Viscosity potentiometer method.* In this method the pressure excess due to a spherical soap film is measured by aid of what may be called a "viscosity potentiometer." Air from a gasometer G (Fig. 6) flows through two tubes AB, CD, which are connected in series by the joint BC. The pressure at A

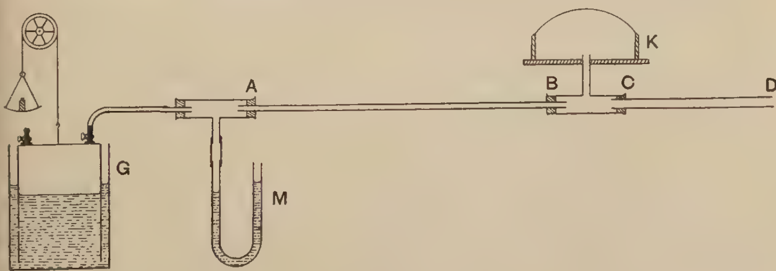


Fig. 6.

is measured by the water manometer* M; the end D is open to the atmosphere. From the junction BC a side-tube leads to a cup K with a horizontal circular rim on which a soap film is placed. On account of the viscosity of the air there is a fall of pressure along each tube, and for a given flow of air, the fall of pressure in either tube is proportional to the length of the tube and inversely proportional to the fourth power of its internal radius, provided the flow is so slow that stream-line motion exists. The excess of the pressure in the cup K above that of the atmosphere causes the film to rise above the rim and to take the form of part of a spherical surface. From the distance, h cm., of the highest point of the film above the plane of the rim and from the radius, c cm., of the rim, the radius, r cm., of the spherical surface is deduced. Thus

$$h(2r-h)=c^2,$$

or

$$r=\frac{c^2+h^2}{2h} \qquad (7).$$

If the length of the tube DC be l_1 cm. and its internal radius be a_1 cm. and if l_2, a_2 be corresponding quantities for the tube BA, and if the pressure excess at B be p and that at A be P, then

$$\frac{P-p}{p}=\frac{a_1^4}{l_1}\cdot\frac{l_2}{a_2^4} \qquad (8).$$

It is sometimes convenient to form the portion AB of two or more tubes arranged in series. If the lengths and internal radii of these be l_2, l_3, ... cm. and a_2, a_3, ... cm., then the equation becomes

$$\frac{P-p}{p}=\frac{a_1^4}{l_1}\left(\frac{l_2}{a_2^4}+\frac{l_3}{a_3^4}+\dots\right) \qquad (9).$$

From (8) or (9) the value of p can be found in terms of P, the pressure excess observed on the gauge M. It is convenient to arrange the tubes so that P is about 100 times p. The internal radii of the tubes may be found by means of mercury. For accurate work they should be calibrated†.

The gasometer G is formed of a cylindrical can 16 cm. in diameter and 24 cm. in height. Part of its weight is supported by a string which passes over a ball-bearing pulley and carries a pan and weights. By varying these weights the pressure excess in the gasometer can be adjusted. The can is furnished with two gas-fitter's taps, as shown in Fig. 6. The lower rim of the can is

* A petroleum manometer would be better, but the density of the liquid must be found.

† For details of this process see G. F. C. Searle, "A simple method of determining the viscosity of air." *Proc. Camb. Phil. Soc.* Vol. XVII. p. 183.

loaded with lead so that the equilibrium of the can is stable when the can is floating with its axis vertical. The can is connected to the joint A by a piece of flexible rubber tube. Since the walls of the can are thin, the pressure excess diminishes only very slowly as the can sinks in the cistern.

The internal radii of the flow tubes must not be too small. With tubes of small radii, the flow of air is so small that a considerable time elapses before the film reaches its full height, and there is a danger that the film may break before the necessary measurements can be made.

§ 8. *The bubble holder.* The details of the arrangement on which the spherical film is formed and measured are shown in Fig. 7. A brass plate 8·5 cm. in diameter is carried by a tripod;

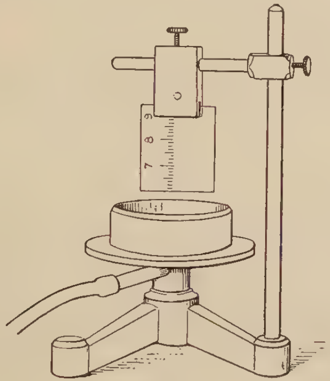


Fig. 7.

a central opening in the plate communicates with the tube by which a connexion is made with the joint BC (Fig. 6). A ring rests upon this plate. The upper end is bevelled so that the edge lies on the internal cylindrical surface of the ring; this secures a definite base for the spherical film. The joint between the ring and the plate is made air tight by a little of the soap solution. This arrangement allows a number of rings of different radii to be used and also allows the ring to be adjusted on the table. A rod rising from the tripod base is furnished with an adjustable horizontal arm which carries a clamp holding a glass scale divided to millimetres. It is convenient to adjust the scale and ring so that, when the spherical film is formed, the lower edge of the scale may be as near as possible to the highest point of the film. There is then little error due to parallax if the film and scale are observed through a telescope set at the same level as the top of the film, and the film and the scale will be both in focus at the same time. A flame placed behind the film at a sufficient distance to avoid the

effects of its heat or a mirror reflecting the sky light may be used to give the necessary illumination.

A film is formed on the ring by dipping a card into the solution and then drawing the edge of the card gently across the ring.

The position of the zero of the glass scale relative to the plane of the rim of the ring is found by placing a steel scale of known width on the rim with the plane of the scale vertical and then observing the reading of the upper edge of the steel scale on the glass scale.

An alternative method of determining the height of the vertex of the film is to use a horizontal microscope provided with a vertical motion and to "set" the microscope first on the vertex of the film and then on the rim of the ring. The difference of the readings gives the distance h.

§ 9. *Practical details.* A series of measurements may be taken by varying the counterpoise of the gasometer, and determining r, the radius of the spherical film, in each case. The value of Mr, where M cm. is the difference of level of the manometer columns, is found in each case, and the mean of these values is used in calculating the surface tension. Since the pressure in the gasometer slowly diminishes, the observations for r and M should be made as nearly as possible simultaneously.

§ 10. *Geometry of the curved film.* When a curved film is formed on a circular rim by a pressure excess p, it must be a surface of revolution about the axis of the rim, and this surface is easily shown to be part of a sphere. Let VMO (Fig. 8) be the axis

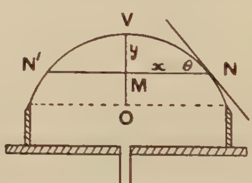


Fig. 8.

of the rim of the cup, and V the vertex of the film; the figure is a section of the system by a plane through VO. Let NMN' be the trace of a plane normal to VO and let $MN = x$ and $VM = y$. Let the tangent at N to the section of the film make the angle θ with MN. Since the force on the film NVN' due to the surface tension on *both* sides of the film acting along the circle NN', in which the plane NMN' cuts the film, is equal to the force due to the pressure excess p acting on the circular area of radius MN, the equation of equilibrium is

$$2T \sin \theta \,.\, 2\pi x = p \,.\, \pi x^2,$$

or
$$x=\frac{4T}{p}\sin\theta=r\sin\theta,$$

where $r=4T/p$. But $dy/dx=\tan\theta$ and hence

$$\frac{dy}{dx}=\frac{\sin\theta}{\sqrt{1-\sin^2\theta}}=\frac{x}{\sqrt{r^2-x^2}}.$$

Thus
$$y=-\sqrt{r^2-x^2}+K.$$

Choosing the constant K so that $y=0$ when $x=0$, we have

$$(y-r)^2+x^2=r^2,$$

which represents a circle of radius r whose centre lies on VO at a distance r below V. Hence the film is spherical and has the radius

$$r=4T/p.$$

The surface tension is therefore given by

$$T=\tfrac{1}{4}rp \quad\text{..............................}(10).$$

As the height, h, of the vertex of the film above the rim of the ring is gradually increased from zero to c, the radius of the rim, the radius of the bubble diminishes from infinity until it reaches c. If h be further increased, the radius of the bubble increases also. Hence the pressure excess must not be greater than $4T/c$, for if the height of the bubble be made to exceed c, the pressure which it can resist will become smaller as the radius of the bubble increases, with the result that, if the bubble be supplied with air, it will swell until it bursts.

§ 11. *Practical example.* The following results were obtained by G. F. C. Searle and A. J. Berry. Two tubes in series were used between the manometer and the bubble stand.

Length of tube (1) or CD (Fig. 6) $=l_1=28{\cdot}8$ cm.
Mass of mercury filling tube $=81{\cdot}8$ grms.
Density of mercury $=13{\cdot}55$ grms. per c.c.

Square of radius of tube $=a_1^2=\dfrac{81{\cdot}8}{13{\cdot}55\times\pi\times 28{\cdot}8}=0{\cdot}06672$ cm.^{2}

Hence $a_1=0{\cdot}2583$ cm. and $a_1^4=4{\cdot}452\times10^{-3}$ cm.^{4}

Thus
$$\frac{a_1^4}{l_1}=\frac{4{\cdot}452\times10^{-3}}{28{\cdot}8}=1{\cdot}546\times10^{-4}\text{ cm.}^3$$

By similar measurements on tubes (2) and (3)

$$l_2=120{\cdot}0\text{ cm.},\quad a_2=0{\cdot}1437\text{ cm.},\quad a_2^4=4{\cdot}259\times10^{-4}\text{ cm.}^4$$

$$l_3=158{\cdot}2\text{ cm.},\quad a_3=0{\cdot}1479\text{ cm.},\quad a_3^4=4{\cdot}784\times10^{-4}\text{ cm.}^4$$

Hence $\frac{l_2}{a_2^4}=2{\cdot}817\times10^5\,\text{cm.}^{-3}$ and $\frac{l_3}{a_3^4}=3{\cdot}307\times10^5\,\text{cm.}^{-3}$

Thus, by (9), § 7,

$$\frac{P-p}{p}=\frac{a_1^4}{l_1}\left(\frac{l_2}{a_2^4}+\frac{l_3}{a_3^4}\right)=1{\cdot}546\times10^{-4}\times6{\cdot}124\times10^5=94{\cdot}66.$$

Hence $$\frac{p}{P}=\frac{1}{95{\cdot}66}=1{\cdot}045\times10^{-2}.$$

Radius of rim of cup $=c=2{\cdot}90$ cm.

The table gives the results of a number of observations for various values of the difference of level, M cm., of the water in the limbs of the manometer.

Difference of water levels M	Height of vertex above rim h	Radius of bubble $r=\frac{c^2+h^2}{2h}$	Mr
2·53 cm.	1·28 cm.	3·92 cm.	9·93 cm.^{2}
2·64	1·42	3·67	9·69
2·93	1·73	3·30	9·66
2·99	1·68	3·34	9·99
3·11	1·90	3·16	9·84
3·29	2·38	2·96	9·73
3·36	2·31	2·97	10·00

Mean 9·83

Since $P=981\,.\,\rho M$, where ρ = density of water = 1 grm. cm.$^{-3}$, we have $Pr=981Mr$, and hence, by (10),

$$T=\tfrac{1}{4}rp=\tfrac{1}{4}\frac{p}{P}\cdot Pr=\tfrac{1}{4}\times1{\cdot}045\times10^{-2}\times9{\cdot}83\times981=25{\cdot}19 \text{ dynes per cm.}$$

It will be seen that there are some inconsistencies among the readings. They probably arise from small variations of pressure in the gasometer due to friction in the pulley or capillary action: these variations do not instantaneously cause changes of the radius of the bubble. Better results would probably be obtained with a larger gasometer. Greater steadiness would be obtained by inserting a piece of small bore tube between the gasometer and the manometer joint and then using a higher pressure in the gasometer. A petroleum gauge would be better than a water gauge as it is less affected by capillarity.

§ 12. *Buoyancy method.* This method was suggested by the plan adopted by Mr J. D. Fry in the calibration of his new Micro-manometer*; it depends upon the difference of density between cold and hot air at the same pressure. The method is instructive as it helps the student to realise the presence of the atmosphere. A metal tube $ABCD$ (Fig. 9) has the two portions AB, CD, each a

* J. D. Fry, *Philosophical Magazine*, April 1913, p. 494.

few centimetres long, at right angles to the main portion BC which is about one metre in length. This bent tube is surrounded by a second tube FG which is used as a steam jacket for heating the tube $ABCD$. The parts AB and CD are horizontal and the tube may, if desired, be rotated about CD as a horizontal axis. The inner tube passes out of the steam jacket through rubber bungs. The opening D is connected by a horizontal rubber tube DE to the cup K on which a bubble is to be formed; the same bubble holder is used as in § 8. The end A remains open.

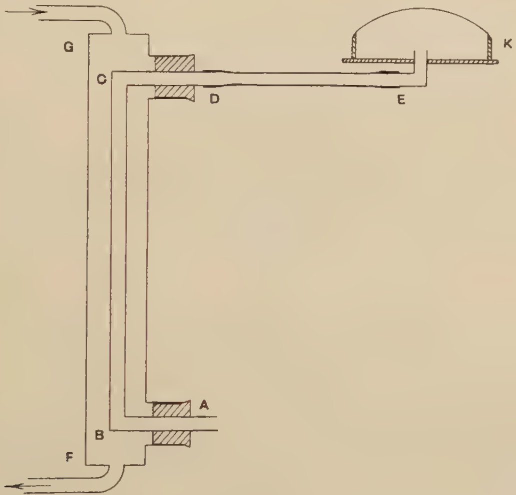


Fig. 9.

Let the temperature of the atmosphere in the neighbourhood of the apparatus be $t_1°$ K., i.e. $t_1°$ on the Kelvin or "absolute" scale, and let the temperature of the air in the tube BC be $t_2°$ K. Let P dynes cm.$^{-2}$ be the atmospheric pressure at the level of A. Let the density of air at the normal pressure p_0 and the normal temperature $t_0°$ K. be ρ_0 grm. cm.$^{-3}$, let the density of air in the atmosphere at the level of A be ρ_1 and that at B in the tube be ρ_2. Let the height of D above A be z cm. Then, if we neglect the very small changes of density due to differences of level, the pressure in the atmosphere at the level of CD is $P - g\rho_1 z$*, while the pressure in the tube at C is $P - g\rho_2 z$. If the horizontal tube DE is long enough to ensure that the bubble stand K is not heated by the steam jacket, the fall of pressure between E and the bubble is, to all the accuracy required, the same as that which occurs in

* The exact expression is $Pe^{-g\rho_1 z/P}$.

the atmosphere over the same difference of level*. Hence p, the pressure excess within the bubble, is given by

$$p = gz(\rho_1 - \rho_2) \text{ dyne cm.}^{-2} \quad \text{.................(11)},$$

where
$$\rho_1 = \frac{\rho_0 P t_0}{p_0 t_1}, \quad \rho_2 = \frac{\rho_0 P t_0}{p_0 t_2}.$$

When the radius, r cm., of the bubble is known, the surface tension is calculated by

$$T = \tfrac{1}{4} rp.$$

§ 13. *Practical details.* The bends at B and C are formed as in Fig. 10. The end of the long tube is soldered into a block of

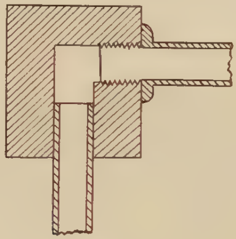


Fig. 10.

brass. The short side tube screws into the block, the joint being made tight by a flange and a leather washer. The short tube must be stout to withstand the strain involved in making the joint tight. The side tubes are screwed into the blocks after the long tube has been placed within the steam jacket. Steam is supplied by a small boiler; the waste steam from the jacket should be led away clear of the apparatus. The inner tube is about 0·8 cm. in diameter and the tube forming the steam jacket about 2·5 cm. in diameter.

The bubble is formed on the ring and the observations for its radius are made just as in §§ 7, 8. The temperature of the surrounding air is observed by a thermometer.

If, as suggested by Mr J. D. Fry, the tube $ABCD$ be turned about CD as a horizontal axis, the difference of level, z, between D and A can be changed and with it the pressure excess in the bubble. The difference of level can be calculated in terms of the distance AD if the tube be mounted so that the inclination of the plane ACD to the vertical can be measured.

* It will be easily seen that no care need be taken to ensure that those parts of the tube DE *which are at atmospheric temperature* shall be in the same horizontal plane as CD.

To ensure that the interior of the tube $ABCD$ is dry, air may be blown through it while it is heated.

§ 14. *Practical example.* The following results were obtained by G. F. C. Searle and A. J. Berry.

Barometric height = 76·53 cm.
Temperature of atmosphere $= t_1 = 273 + 19{\cdot}5 = 292{\cdot}5°$ K.
Temperature of steam $= t_2 = 273 + 100{\cdot}2 = 373{\cdot}2°$ K.
Height of heated column $= z = 95{\cdot}8$ cm.
Density of air at normal pressure and temperature $= \rho_0 = 1{\cdot}293 \times 10^{-3}$ grm. cm.$^{-3}$

Hence

$$\rho_1 = 1{\cdot}293 \times 10^{-3} \times \frac{76{\cdot}53}{76{\cdot}00} \times \frac{273}{292{\cdot}5} = 1{\cdot}2152 \times 10^{-3} \text{ grm. cm.}^{-3},$$

$$\rho_2 = 1{\cdot}293 \times 10^{-3} \times \frac{76{\cdot}53}{76{\cdot}00} \times \frac{273}{373{\cdot}2} = 0{\cdot}9524 \times 10^{-3} \text{ grm. cm.}^{-3}$$

The pressure excess in the bubble is thus

$$p = gz(\rho_1 - \rho_2) = 981 \times 95{\cdot}8 \times 0{\cdot}2628 \times 10^{-3} = 24{\cdot}70 \text{ dyne cm.}^{-2}$$

The following values of the distance, h, of the vertex of the bubble from the plane of the rim were found with five different films:

1·09, 1·10, 1·09, 1·10, 1·10 cm.

Mean value of $h = 1{\cdot}096$ cm.
Radius of rim $= c = 2{\cdot}90$ cm.
Mean radius of bubble $= r = (c^2 + h^2)/2h = 4{\cdot}385$ cm.
Hence we find for the surface tension

$$T = \tfrac{1}{4} rp = \tfrac{1}{4} \times 4{\cdot}385 \times 24{\cdot}70 = 27{\cdot}07 \text{ dynes per cm.}$$

§ 15. *Comparison of results.* The results given by the various methods are as follows:

Method	Surface tension
Torsion balance	27·22 dyne cm.$^{-1}$
Thread	27·17
Viscosity potentiometer	25·19
Buoyancy	27·07

PROCEEDINGS AT THE MEETINGS HELD DURING THE SESSION 1912—1913.

ANNUAL GENERAL MEETING.

October 28th, 1912.

In the Comparative Anatomy Lecture Room.

DR DUCKWORTH, IN THE CHAIR.

The following were elected Officers for the ensuing year:

President:

Dr Shipley, Master of Christ's College.

Vice-Presidents:

Prof. Hopkinson.
Prof. Wood.
Prof. Pope.

Treasurer:

Prof. Hobson.

Secretaries:

Mr A. Wood.
Mr F. A. Potts.
Mr G. H. Hardy.

Other Members of the Council:

Mr R. H. Rastall.
Dr Lucas.
Dr Newell Arber.
Prof. Sir J. J. Thomson.
Mr J. E. Purvis.
Mr R. P. Gregory.
Dr Cobbett.
Mr J. Mercer.
Dr Marshall.
Mr G. R. Mines.
Dr Barnes.
Mr F. J. M. Stratton.

The following were elected Fellows of the Society:

J. W. Lesley, B.A., Emmanuel College.
J. T. Saunders, B.A., Christ's College.
P. G. M. Dunlop, M.A., Gonville and Caius College.

The following were elected Associates:

J. K. Robertson, Emmanuel College.
S. N. Maitra, Emmanuel College.

The following Communications were made:

1. Anthropometric data collected by Professor Stanley Gardiner in the Maldive Islands. By Dr DUCKWORTH.

2. Preliminary Note on the Inheritance of Self-Sterility in *Reseda odorata*. By R. H. COMPTON, M.A., Gonville and Caius College.

3. The effects of hypertonic solutions upon the eggs of *Echinus*. By J. GRAY, B.A., King's College. (Communicated by Mr F. A. Potts.)

4. Pulsus alternans. By G. R. MINES, M.A., Sidney Sussex College.

November 11*th*, 1912.

In the Cavendish Laboratory.

PROFESSOR SIR J. J. THOMSON, IN THE CHAIR.

The following were elected Associates:

G. B. Hony, Christ's College.
H. Smith, Emmanuel College.

The following Communications were made:

1. On the theory of the motion of charged ions through gases. By Professor Sir J. J. THOMSON.

2. On a simple method of determining the viscosity of air. By Dr G. F. C. SEARLE, Peterhouse.

3. Note on the Röntgen Radiation from Cathode particles traversing a gas. By R. WHIDDINGTON, M.A., St John's College.

4. The Diffraction of Short Electromagnetic Waves by a Crystal. By W. L. BRAGG, B.A., Trinity College. (Communicated by Professor Sir J. J. Thomson.)

5. Experiments on the Electrical Discharge in Helium and Neon. By H. E. WATSON. (Communicated by Professor Sir J. J. Thomson.)

6. Some Diophantine Impossibilities. By H. C. Pocklington, M.A., St John's College.

7. A class of Integral Functions defined by Taylor's Series. By G. N. Watson, M.A., Trinity College.

8. Notes on the Volatilization of certain Binary Alloys in high Vacua. By A. J. Berry, B.A., Downing College.

November 25th, 1912.

In the Sedgwick Museum.

Dr Shipley, President, in the Chair.

The following was elected a Fellow of the Society:

H. Hartridge, M.A., King's College.

The following were elected Associates:

R. Rossi, Trinity College.
S. P. Heath, Fitzwilliam Hall.

The following Communications were made:

1. The Gravels of East Anglia. By Professor Hughes.

2. The Meres of Breckland. By Dr Marr.

3. On the earlier Mesozoic Floras of New Zealand. By Dr Newell Arber.

4. The mineral composition of some Cambridgeshire sands and gravels. By R. H. Rastall, M.A., Christ's College.

5. A remarkable instance of complete rock-disintegration by weathering. By F. H. Hatch.

January 27th, 1913.

In the Cavendish Laboratory.

Mr J. E. Purvis, in the Chair.

The following were elected Fellows of the Society:

J. Gray, B.A., King's College.
T. C. Nicholas, B.A., Trinity College.

The following was elected an Associate:

S. Manlik, Non-Coll.

The following Communications were made:

1. Further applications of positive rays to the study of chemical problems. By Professor Sir J. J. THOMSON.

2. The Atomic Constants and the Property of Substances. By R. D. KLEEMAN, B.A., Emmanuel College.

3. Some Diophantine Impossibilities. By H. C. POCKLINGTON, M.A., St John's College.

4. Magnetic Susceptibility with Temperature. Part II, On aqueous Solutions. By A. E. OXLEY, B.A., Trinity College.

5. The Properties of Liquids connected with the surface Tension. By R. D. KLEEMAN, B.A., Emmanuel College.

February 10*th*, 1913.

In the Comparative Anatomy Lecture Room.

DR SHIPLEY, PRESIDENT, IN THE CHAIR.

The following were elected Fellows of the Society:

W. H. Mills, M.A., Jesus College.
R. Thirkill, M.A., Clare College.

The following was elected an Associate:

F. Balfour Browne, M.A. (Oxon.).

The following Communications were made:

1. Note on the respiratory movements of *Torpedo ocellata.* By G. R. MINES, M.A., Sidney Sussex College.

2. The swarming of *Odontosyllis.* By F. A. POTTS, M.A., Trinity Hall.

3. Observations on *Polyporus squamosus.* By S. R. PRICE, M.A., Clare College.

4. Note on the Composition of some Pleistocene Sands near Newmarket. By R. H. RASTALL, M.A., Christ's College.

February 24*th*, 1913.

In the University Chemical Laboratory.

PROFESSOR POPE, VICE-PRESIDENT, IN THE CHAIR.

The following were elected Fellows of the Society:

John Christie, B.A., Trinity College.
G. J. Hill, M.A., Peterhouse.

The following Communications were made:

1. The ten Stereoisomeric Tetrahydroquinaldinomethylenecamphors. By Professor POPE and Mr J. READ.

2. The chemical and bacterial condition of the Cam above and below the sewage effluent outfall. By J. E. PURVIS, M.A., St John's College and A. E. RAYNER, M.A., Gonville and Caius College.

3. Some experiments on the slow combustion of Coal Dust. By F. E. E. LAMPLOUGH, M.A., Trinity College and Miss A. M. HILL.

4. The Oxidation of Ferrous Salts. By F. R. ENNOS, B.A., St John's College. (Communicated by Mr C. T. Heycock.)

5. On the optically active semicarbazone and benzoylphenylhydrazone of *cyclo*-hexanone-4-carboxylic acid. By W. H. MILLS, M.A., Jesus College and Miss A. M. BAIN.

6. Experiments illustrating "Flare Spots" in Photography. By Dr G. F. C. SEARLE, Peterhouse.

7. Note on the effect of heating Paraformaldehyde with a trace of sulphuric acid. By P. G. M. DUNLOP, M.A., Gonville and Caius College.

April 28*th*, 1913.

In the Botanical Laboratory.

DR SHIPLEY, PRESIDENT, IN THE CHAIR.

The following were elected Fellows of the Society:

R. H. Peters, B.A., Gonville and Caius College.
C. M. Sleeman, M.A., Queens' College.

The following were elected Associates:

C. G. L. Wolffe, M.D. (McGill University).
J. Burtt-Davy, Non-Coll.

The following Communications were made:

1. Notes on additions to the Flora of Cambridgeshire. By A. H. EVANS, M.A., Clare College.

2. On some new and rare Jurassic plants from Yorkshire. By H. HAMSHAW THOMAS, M.A., Downing College.

3. Some Varieties of Plants new to the British Isles. By C. E. MOSS, B.A., Emmanuel College.

4. Observations on *Hirneola auricula-judae* Berk (Jew's ear). By the Rev. M. J. Le Goc, B.A., Fitzwilliam Hall. (Communicated by Mr F. T. Brooks.)

5. (1) On the greatest value of a determinant whose constituents are limited.
   (2) Expressions for the remainders when θ, θ', $\sin k\theta$, $\cos k\theta$ are expanded in ascending powers of θ.

By Professor A. C. Dixon.

May 5th, 1913.

In the New Medical Schools.

Professor Nuttall, in the Chair.

The following was elected a Fellow of the Society:

N. P. McCleland, M.A., Pembroke College.

The following Communications were made:

1. Observations on Ticks:
   (*a*) Parthenogenesis,
   (*b*) Variation due to nutrition.

By Professor Nuttall.

2. Exhibition of a Chinese flea-trap. By E. Hindle, B.A., Magdalene College. (Communicated by Professor Nuttall.)

3. Exhibition of living Termites. By Professor A. D. Imms.

4. The Division of *Holosticha scutellum*. By K. R. Lewin, B.A., Trinity College. (Communicated by Professor Nuttall.)

5. Sarcosporidia in an African Mouse Bird. By H. B. Fantham, B.A., Christ's College.

6. Note on the food of freshwater Fish. By J. T. Saunders, B.A., Christ's College.

May 19*th*, 1913.

In the Cavendish Laboratory.

DR SHIPLEY, PRESIDENT, IN THE CHAIR.

The following were elected Fellows of the Society:

P. G. Bailey, M.A., Clare College.
Franklin Kidd, B.A., St John's College.
Rev. M. J. Le Goc, B.A., Fitzwilliam Hall.
G. Udny Yule, M.A., St John's College.

The following were elected Associates:

F. E. Baxandall.
C. P. Butler.
W. Moss.
W. E. Rolston.

The following Communications were made:

1. (1) Some methods of measuring the surface tension of soap films.

(2) A simple method of testing lens systems for aberration. By Dr G. F. C. SEARLE, Peterhouse.

2. On the Unstable Nature of the Ion in a Gas. By R. D. KLEEMAN, B.A., Emmanuel College.

3. A Dust Electrical Machine. By W. A. DOUGLAS RUDGE, M.A., St John's College.

4. A mechanical vacuum tube regulator. By R. WHIDDINGTON, M.A., St John's College.

5. The hydrodynamical theory of lubrication with special reference to air as a lubricant. By W. J. HARRISON, M.A., Clare College.

ERRATA

Sarcocystis colii, n. sp.

Page 224, line 14, *for* "lack of," *read* "apparent lack of."

Explanation of Plate V. *After* "Dorsal aspect...*Sarcocystis colii*," *insert* *a.* = trophozoites. (Miescher's tubes.) Approximately natural size.

PROCEEDINGS

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A possible connexion between abnormal sex-limited transmission and sterility. By L. DONCASTER, Sc.D., King's College.

[*Read* 17 November 1913.]

In a recent paper* I suggested that the rare tortoiseshell male cat might be produced by the failure of the normal sex-limited transmission of the yellow factor from the male parent. When a yellow (orange) male cat is mated with a black female, the normal result is that all the female offspring are tortoiseshells, all the males black, showing that the yellow factor is sex-limited in its transmission by the male, and goes only into gametes which will give rise to females. Some cases, however, are recorded of tortoiseshell males being produced from yellow sires, and I suggested that these arise by the occasional failure of the sex-limited transmission, with the result that the yellow factor is transmitted to a male. Such a male, receiving yellow from the male parent and black from the female, would be a tortoiseshell. If the black factor is also sex-limited in the male, as suggested by Little†, then a tortoiseshell male could also arise, by failure of the sex-limited transmission, from a black male by yellow or tortoiseshell female, some cases of which have been recorded. It therefore seems probable that the rare tortoiseshell male is produced only by the abnormal transmission from the sire to a male child of a character which normally goes into female producing gametes, and that the tortoiseshell male contains two positive factors (those for both yellow and black), instead of either one or the other as in normal male cats.

* "On Sex-limited Inheritance in Cats." *Journ. of Genetics*, III, 1913, p. 11.
† C. C. Little, *Science*, May 17, 1912.

In the paper referred to, I mentioned that I had in my possession a tortoiseshell tom, which I was mating with black females in the hope of testing this hypothesis. The experiment has now been in progress for about nine months; the tom has mated, apparently successfully, with each of four females several times, but none of them have become pregnant. (The females have had respectively two, three, five and five periods of 'heat' during the time, and at each period there have been one or more apparently normal copulations.)

It seems fairly clear, therefore, since three of the four females have previously borne kittens to other sires, that the tom-cat is sterile. This is confirmed by facts pointed out to me by Dr F. H. A. Marshall. On examining one of the females about four weeks after the last pairing, he found that the mammae were swollen and on pressure exuded a small amount of milk. This continued for about two weeks, after which the exudation became less and finally disappeared. Now it is known that there may be correlation between the development of corpora lutea and mammary hypertrophy, and Marshall and Hammond have found that in rabbits from which the uterus has been excised so that pregnancy is impossible, slight lactation may supervene after ovulation, owing, apparently, to the presence of corpora lutea in the ovary, and that a period of 'heat' does not recur until this lactation has ceased. Further, Longley* has shown that in the cat ovulation only occurs after copulation. If, then, the slight lactation observed was caused, as in the rabbit, by the presence of corpora lutea consequent on ovulation, the copulation must have been sufficiently normal to cause ovulation, which does not take place in its absence. That ovulation occurred is also suggested by the rather widely separated periods of heat. Cats in the breeding season usually come on heat every three weeks or oftener; my females, after pairing with the tortoiseshell tom, did not come on again as a rule for at least five weeks, and usually not for six weeks or even more. When all these facts are taken together, there seems to be little doubt that the tom is sterile, although he is normally formed and has the sexual instincts strongly developed.

If this were an isolated case, it might be of no importance, but there are hardly any records of offspring of tortoiseshell males, and the few that exist are perhaps not wholly above suspicion. I believe that two such cats at least are well known never to have become parents, and I know of another case in addition in which a tortoiseshell tom was paired but no pregnancy followed. It seems, therefore, that the abnormal constitution of a tortoiseshell tom with regard to its inherited characters may be connected with

* W. H. Longley, "Maturation of the Egg and Ovulation in the Domestic Cat." *Amer. Journ. Anat.* XII. 1911, p. 139.

sterility. If the sterility is due, directly or indirectly, to the failure of sex-limited transmission on the part of one of the parents, we should expect other cases in which there is an exception to the normal sex-limited transmission to be sterile also, and this summer I have bred two specimens of this kind in the moth *Abraxas grossulariata*. Usually the *grossulariata* female transmits the *gross.* character only to her sons, so that when mated with a *lacticolor* male (from which the *gross.* character is absent), the male offspring are *gross.*, the females *lact.* I have had very rare exceptions to this rule in preceding years, but none in which I could entirely exclude the possibility that the *gross.* females were really wild females of which the larvae had accidentally been introduced with the food. This year, however, I have in one family from the mating *gross.* ♀ × *lact.* ♂ in addition to 21 *gross.* males and 14 *lact.* females, two *gross.* females to which it is impossible to ascribe such an accidental origin, for they have peculiar features which I have only seen in the strain to which the family belongs. I intend to discuss elsewhere the importance of these exceptions to the normal sex-limited transmission; in the present connexion, the interesting fact about them is that both were quite sterile. Both paired normally; one laid no eggs at all, and the other laid only 22 eggs, none of which developed. Sterility in the strain to which they belong is not very rare; this summer, in 56 matings with females more or less related to the moths in question, 10 were nearly or quite sterile. On the other hand, I paired four normal sisters of the exceptional females, and all were fully fertile. It is thus not very probable that both the exceptional females should have been sterile, if the sterility had no connexion with their abnormal hereditary constitution. The facts, therefore, seem to indicate that when, by failure of sex-limited transmission, an individual arises which receives from one parent a character which it normally receives only from the other, that individual tends to be sterile.

The Flight of the House-Fly. By EDWARD HINDLE, B.A., Ph.D., F.L.S., Assistant to the Quick Professor of Biology; Magdalene College, Cambridge. (Communicated by Mr C. WARBURTON.)

[*Read* 17 November 1913.]

During the months of July, August, and September, 1912, the author, in conjunction with Mr Gordon Merriman, conducted an extensive series of experiments on the range of flight of *Musca domestica* Linn. in the town of Cambridge. In the course of these experiments upwards of 25,000 flies have been liberated under very variable meteorological conditions, and about 50 observation stations were employed for their recovery.

In all cases the flies used in these experiments were either caught in balloon traps, or directly netted. The method of obtaining flies by breeding was abandoned, as it was almost impossible to obtain them without numerous other species of insects, and also on account of the possible objections to such artificially bred flies.

Prior to being liberated the flies were kept for about 24 hours in cages made of mosquito netting and were fed on brown sugar, the moisture being supplied by a layer of damp sand. By this method it was assured that they had emerged sufficiently long to allow the full development of their chitinous exoskeleton, necessary in order to obtain the full power of flight.

Preparatory to colouration the insects were transferred from the mosquito cages into wire balloon traps. This transference was effected as follows:—the loose sides of the mosquito netting cage were tied round the bottom of the balloon trap. The latter was then held towards the light and the whole of the cage surrounded with a black cloth. Owing to the strong attraction of the light, the flies all made their way towards the brightly illuminated balloon trap and in passing through the small hole in the bottom of the latter, it was possible to make accurate counts of them, as not more than two or three were able to pass through at the same time. When about 1500 flies had entered the balloon trap it was closed, then removed and another trap fixed in its place.

The most satisfactory mode of marking the flies was found to be that devised by Nuttall (*vide* Jepson, 1909), and this was employed in all these experiments. The balloon trap containing the flies was placed in a large brown paper bag containing a handful of powdered blackboard chalk, coloured either red, orange,

or yellow. The mouth of the bag was then closed and the whole gently shaken for one or two minutes so that the flies were thoroughly dusted with the chalk. The balloon was then removed and, after being taken to the point selected for the liberation, the trap was opened and the flies allowed to escape in any direction they chose. The flies were recovered either by means of fly-papers or balloon traps, several of which were exposed at the various observation stations. The traps and papers were examined for several successive days after the liberation of a number of coloured flies and as the observation stations extended as far as 900 yards from the point of liberation, comprising both thick and sparsely populated localities, an accurate idea of the distribution of the insects was thus obtained. Full meteorological data were kindly supplied by Messrs W. E. Pain, Chemists, Sidney St., Cambridge. Their observations were made in the centre of the town and in consequence indicate the exact conditions under which the flies travelled.

Discussion of results.

Unfortunately, nearly all our experiments in Cambridge were seriously handicapped by the great difficulty of obtaining flies in sufficient numbers and also by the adverse meteorological conditions. Throughout August the weather was so bad that from the nineteenth to the thirty-first not a single fly could be liberated. During the early part of September nearly all the flies became infected with *Empusa muscae* and this, in conjunction with the cold weather, brought the investigation to a sudden end. In the earlier experiments we should have preferred to have liberated at least double the number of flies, but owing to the difficulty of procuring them this was impossible. Our results, therefore, are not as complete as we could have wished.

Nevertheless, owing chiefly to the large number of stations employed for the recovery of the flies and their being situated in every direction, we have been able to obtain certain definite results.

The most striking feature is the marked effect of the direction of the wind on the courses taken by the flies. After a careful examination of all our results, we can state definitely that flies tend to travel either directly *against* or *across* the wind. The only exceptions to this rule were those recovered within a radius of about 150 yards from the point of liberation and probably these flies were individuals that had merely selected the first shelter they could find. These results differ considerably from those of Copeman, Howlett and Merriman (1911) who found that the flies tended to go *with* the wind, but it should be remembered

that not only were these investigators working in open country, but also their traps were set solely at stations to the west of the point of liberation and consequently none of the flies that flew in other directions would be recovered.

Owing to lack of opportunities we have been unable to decide why, in our experiments, the flies tended to travel either against or across the wind. Two explanations seem possible.

(1) The flies may direct their flight against any current of air to which they are subjected. This property is known as positive anemotropism and is possessed by some other insects and birds. In view, however, of the contrary results obtained by Copeman, Howlett and Merriman (1911) we cannot come to definite conclusions on this point and further experiments are required to determine if other factors than wind-direction may influence the direction of flight.

(2) The flies may travel against the wind, being attracted by any odours it may convey from a source of food. A point in favour of this supposition is the nature of the stations at which flies were recovered after they had travelled any distance. These comprised a butcher's shop, public-houses and a restaurant, all of which places gave off odours that are notoriously attractive to flies.

The maximum distance travelled by any of the flies we liberated in Cambridge was 770 yards, which is considerably less than that covered by those liberated in the open country at Postwick, in one case as much as 1700 yards. This difference may be attributed to the absence of shelter in the case of the Postwick flies, whereas in Cambridge food and shelter were always plentiful. On the whole, we do not think it likely that as a rule flies travel more than a quarter of a mile in thickly housed areas. Throughout our experiments the only individual that exceeded this distance had travelled 770 yards, of which a large part was across open fen land.

The chief factors influencing the dispersal of the flies are temperature, weather, and the time of day when the insects are liberated. The effect of temperature is very marked, as when it is low the flies become torpid and seek the first available shelter. Fine weather is also a necessary condition for long flights, as rain at once drives the flies under cover. The ideal conditions for an experiment are two or three days of fine warm weather, during which the flies can make their flight, succeeded by a wet or showery day, when they are driven indoors and thus can be recorded at the various stations.

With regard to the altitude of the point of liberation, flies set free from the roof* tended to disperse slightly better than those

* A height of about 45 feet.

liberated from the ground, but the differences were not very considerable.

When the insects are liberated late in the afternoon, they do not disperse in as great numbers as those liberated during the morning, but the distances travelled are not inferior.

With regard to the vertical flight of the house-fly, although we have found no means of estimating the maximum, nevertheless, when liberating them from the ground, we have frequently observed the flies at once mount almost vertically upwards to a known height of 45 feet.

Summary.

(1) House-flies tend to travel either *against* or *across* the wind. This direction may be directly determined by the action of the wind, or indirectly, owing to the flies being attracted by any odours it may convey from a source of food.

(2) The chief conditions favouring the dispersal of flies are fine weather and a warm temperature; the nature of the locality is another considerable factor, as in towns flies do not travel as far as in open country, this being probably due to the food and shelter afforded by the houses.

(3) Under experimental conditions, the height at which the flies are liberated and also the time of day influence the dispersal of the insects. When set free in the afternoon they do not scatter so well as when liberated in the morning.

(4) From our experiments the usual maximum flight in thickly housed localities seems to be about a quarter of a mile, but in one case a single fly was recovered at a distance of 770 yards. It should be noted, however, that part of this distance was across open fen land.

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On the Dependence of the Relative Ionisation in various Gases by β Rays on their Velocity, and its bearing on the Ionisation produced by γ Rays. By R. D. KLEEMAN, B.A. (Emmanuel College), D.Sc. (Adelaide).

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Experiments which give some information whether the relative ionisation by β rays in various gases depends on the velocity of the rays have already been carried out by the writer*, who measured the relative ionisation per c.c. in various gases by the β rays of actinium and uranium. The experiments shewed, as far as they went, that the relative ionisation is independent of the velocity of the β ray. Further experiments have been carried out on the subject and will be described in this paper.

A beam of heterogeneous β rays was obtained by placing a quantity of radium in a thin glass tube at one of the entrances of a bore-hole ·6 cm. in diameter passing through a lead block about 7 cm. square. The beam was allowed to pass into an ionisation chamber through an aluminium window ·16 mm. thick. By means of a magnetic field whose lines of force were at right angles to the bore-hole the beam of rays could be "hardened" to any desired extent by bending some of the slower moving rays aside. It was found that the gradual hardening of the beam produced no change in the relative ionisation of the gases in the chamber. The ionisation of methyl iodide and hydrogen in terms of that of air was measured, since these gases differ from one another considerably in the nature of the atoms they contain. As an illustration of the experiments it may be quoted that in one case the ionisation of hydrogen in terms of that of air was ·156 without the magnetic field, and ·151 with the magnetic field. The magnetic field in this particular case reduced the ionisation in air and hydrogen to about one-half of their previous values. In the experiments the ionisation of air and hydrogen were first measured without a magnetic field on, putting air into the chamber first. Then leaving the hydrogen in the chamber and putting the magnetic field on, the ionisation of the hydrogen was measured under the new conditions. Lastly, putting air into the chamber its ionisation was measured with the same magnetic field on. This process was repeated several times and the mean of the results taken. When methyl iodide mixed with a certain proportion of air was used instead of hydrogen in the experiment

* *Proc. Roy. Society*, A, vol. 83, p. 530 (1910).

mentioned, the relative ionisations with and without a magnetic field were 1·97 and 1·82 respectively. The relation between velocity and number of β rays from radium has been investigated by Pashen. It appears from this that in my experimental arrangement the velocities of the rays producing the larger part of the ionisation probably lay between 1×10^{10} and $2{\cdot}9\times10^{10}$ cm. per second. It would of course have been preferable to obtain a spectrum of the rays by means of a magnetic field and investigate the ionisation produced at different parts. If any indications had been obtained that the relative ionisation depended on the velocity of the β ray, an elaborate experiment of this nature would have been carried out. It would, however, be desirable to carry out experiments on relative ionisation, using the cathode rays obtained in a discharge tube, or those ejected from metals by X rays, since their velocity is considerably less than the majority of the β rays from radium. An experiment of this nature is about to be carried out.

The results obtained have an important bearing on the nature of the process of the ionisation in a chamber through which γ rays are allowed to pass. The ionisation in question may be divided into three parts, viz. (*a*) the ionisation produced by the β rays given off the walls of the vessel, (*b*) that by the secondary γ rays given off the walls, (*c*) and that due to the absorption of the primary γ rays by the gas in the vessel. The ionisation under (*c*) consists of the ions produced by the β rays ejected from the molecules of the gas under the action of the primary γ rays, and possibly of direct ionisation by the primary γ rays. According to some experiments of C. T. R. Wilson*, however, the latter part is comparatively small or non-existent. We shall return to this point later. It can be easily shewn that the ionisation under (*c*) is small in comparison with that under (*a*). For the total absorption of the primary γ rays by the gas is very much smaller than that by the walls of the vessel. And since the mass of matter per cm.² of the walls of the vessel through which a β ray is able to penetrate corresponds to probably hundreds of times that of the mass per cm.² of the gas in the vessel, a much larger number of β rays will cross the chamber which originate in its walls than which originate in the gas. The ionisation under (*b*) has usually not been deemed worthy of much or any consideration by physicists who have worked with γ rays. It is by no means of negligible magnitude. This will appear from a study of the ionisation by γ rays of different hardness. The writer† has shewn, using secondary γ rays, that the ionisation in methyl iodide relative to that in air greatly increases with the

* *Proc. Roy. Society*, A, vol. 87, p. 277 (1912).
† *Ibid.*, vol. 82, p. 358 (1909).

softness of the rays. This fact has recently been used by Rutherford to isolate the different groups of the γ rays of radium. Now according to the results obtained in this paper this could not be due to differences in the nature of the β radiation given off the walls of the vessel. It could not have been due to some of the soft β radiation from the walls being totally absorbed by the gas, since the total ionisation depends little on the nature of the gas*. Therefore since the ionisation under (c) is small in comparison with that under (a), this increase must be due to an increase of the ionisation under (b).

It appears from the experiments by C. T. R. Wilson mentioned that β rays produce secondary β rays of small velocity only, i.e. δ rays. Bragg† and the writer‡ have shewn that the velocity of the secondary β rays produced by γ rays decreases with the absorbability of the latter. It follows from this that if the β and γ rays of radium are allowed to fall upon a plate of material, and soft γ radiation is produced in it, soft β radiation should emerge from the surface of the plate due to this soft γ radiation. The writer§ has shewn the existence of this soft β radiation, some of which is totally absorbed in about 2 cm. of air at atmospheric pressure. The energy of the soft γ radiation producing the ionisation under (b) is probably much less than that of the primary γ rays. But since it is much more absorbable the ionisation it produces could easily be greater. It is to be expected then that if a narrow beam of γ rays is sent through an ionisation chamber and the β rays curled up by means of a magnetic field, an experiment the writer‖ has carried out, there should still be considerable ionisation left in the chamber. Bragg has objected to this experiment, because it is difficult to curl up all the β rays to the same extent owing to their cutting the lines of magnetic force at different angles. Undoubtedly this objection might be raised, but still I think that the diminution with increase of magnetic field should have been greater under the particular circumstances than that obtained. Bragg and Madsen¶ have carried out some experiments which they thought incidentally shewed that the ionisation produced by secondary γ rays in the vicinity of material on which primary γ rays are allowed to fall is small in comparison with that produced by the secondary β rays. They placed plates of material of different thicknesses in increasing order of magnitude across a beam of γ rays and measured the ionisation produced on the side of the plate where the γ rays

* *Proc. Roy. Society*, A, vol. 84, p. 17 (1910).
† *Phil. Mag.*, Dec. 1908.
‡ *Proc. Roy. Society*, A, vol. 82, p. 128 (1909).
§ *loco. cit.*
‖ *Proc. Camb. Phil. Society*, vol. xv. Pt II. p. 169 (1909).
¶ *Phil. Mag.*, Dec. 1908, p. 932.

emerged. Since they got no further increase in ionisation after a thickness of material giving the maximum β radiation had been reached, they concluded that the secondary γ rays produce no appreciable ionisation. But it must be taken into account that since the energy of the secondary γ radiation is probably smaller than that of the β radiation, the absorbability of the secondary γ rays must be of the same order of magnitude as that of the β rays to produce an appreciable ionising effect. This secondary γ radiation would thus reach a maximum for a thickness of material certainly not much greater than that giving the maximum β radiation. An increase in the ionisation with the thickness of material cannot therefore be expected, and Bragg and Madsen's conclusions do not therefore definitely follow from these experimental results. It should be pointed out that the soft secondary γ rays, whose coefficient of absorption is the same as that of the β rays, are eliminated in measurements of the coefficient of absorption of secondary γ rays. For the thickness of the wall of the ionisation chamber is usually such as to absorb all the β rays falling upon it and consequently all the secondary γ rays of the same absorbability. Measurements by various observers have shewn that the secondary γ rays as a whole are much more absorbable than the primary γ rays. Thus if carbon is taken as the radiating and lead as the absorbing material the writer* has shewn that the coefficient of absorption for the secondary γ rays is about 10 times the magnitude of that of the primary rays. Much softer rays are thus likely to exist which are eliminated in the way explained. However, taking all the evidence into account, there is no doubt that the ionisation produced by the secondary γ rays from the walls of the ionisation chamber is less than that produced by the β rays when a gas containing atoms of low atomic weight such as air is in the chamber. Probably in that case it is of the order of 20 per cent. of the total ionisation. The percentage probably increases with the softness of the primary γ rays. When, however, a heavy gas like methyl iodide is in the chamber it may be very much greater, since the ionisation of heavy gases relative to air increases greatly with the softness of the rays; thus the ionisation of methyl iodide relative to that of air is about 6 for the primary γ rays of radium calculated from absorption data, while for X rays it is about 80. Bragg† has given a formula which expresses the ionisation in a chamber through which γ rays are allowed to pass in terms of other quantities. This formula does not take into account the production of secondary γ rays in the walls of the vessel. It is necessary to add two terms to the formula, one term expressing

* *Phil. Mag.*, May 1908, p. 652.
† *Ibid.*, Sept. 1910, p. 402.

the ionisation due to the absorption of the secondary γ rays by the gas in the chamber, and the other the ionisation produced by the β rays produced by the secondary γ rays in the walls of the chamber.

When the softness of the primary γ radiation is increased, that of the secondary radiation from the walls of the chamber must be increased also. Thus the behaviour of the relative ionisations of the gases in the chamber with a variation of the softness of the primary γ rays, is an index of a general nature of the variation of the chance of an atom becoming the source of a β ray under the influence of γ pulses with the softness of the latter. Some experiments* that I have carried out previously have a bearing on this point. It was found that the ionisation of a gas relative to that of air increased with the softness of the γ rays when the molecules contained atoms heavier than those in air. But it decreased with the softness of the rays in the case of hydrogen. This fits in with the results obtained by Crowther† with X rays of different hardness. The results are thus evidence of the identical nature of γ and X rays.

It will be of interest here to consider more closely the nature of the process of the ionisation by γ or X rays. According to the experiments of C. T. R. Wilson mentioned the ionisation is produced by the β rays ejected by the rays. In these experiments photographs of the ions were obtained by making them serve as nuclei to a fog in a cloud-producing apparatus of appropriate construction. Previously, however, Crowther‡ had shewn that the ionisation by X rays is largely direct ionisation. The method he used involved the principle that the ionisation produced, varies directly as the pressure, while that produced by the secondary β rays varies as the square of the pressure. Beatty§ later found that the amount of direct ionisation greatly depended on the nature of the gas and hardness of the rays. These experiments do not fall into line with those of C. T. R. Wilson. But the disagreement is, I think, more apparent than real. An inspection of the photographs obtained by C. T. R. Wilson will shew that a blotch occurs at the beginning of each streak which shews the ions produced along the course of a β or cathode ray. It may be mentioned that a blotch also occurs at the end of each streak, being due to the zig-zag path and intense ionisation by the cathode particle before it ceases to ionise. In the former case it seems to be most likely due to the cathode particle ionising one or more atoms of its parent

* *loco. cit.*

† *Proc. Roy. Society*, A, vol. 82, p. 115 (1909).

‡ *Ibid.*, p. 103 (1909).

§ *Ibid.*, vol. 85, p. 230 (1911).

molecule. Besides, the recoil atom may also help to ionise the atoms of the molecule and to break it up, and also to ionise other molecules like an α particle. Now all the ions that are made from the molecules which give rise to the cathode rays appear as direct ionisation since it varies proportionally to the pressure. So does all the ionisation produced by the recoil atom, since its path would be extremely short in comparison with the diameter of chamber even at very low pressures. Thus the ionisation that varies directly as the pressure may not be of negligible magnitude in comparison with the whole ionisation, especially at low pressures. Bragg* has used a method to investigate the point under discussion which is entirely different from that used by Crowther and Beatty. He concludes that all the ionisation is produced by the secondary cathode rays ejected by the X rays. His method would not shew as direct ionisation that produced from the parent molecules of the cathode particles. Thus a discrepancy between his and Crowther's and Beatty's experiments may be expected.

Some experimenters have investigated the variation of the ionisation by γ rays of a gas with its pressure over a great range of pressures. But this can furnish little if any reliable information about the exact nature of the process of ionisation. For the ionisation produced by the secondary β rays from the walls of the vessel, that of the molecules producing the secondary β rays, and the ionisation produced by the recoil atom, if any, varies directly as the pressure. The experiment does not isolate these different ionisations. The remainder of the ionisation varies as the square of the pressure.

But this subdivision of the ionisation holds only if all the β rays end their life in the walls of the vessel. Strictly the ionisation produced by the secondary β rays from the gas which do not cross the chamber or are reflected by the walls of the vessel and end their life in the gas, varies approximately proportionally to the pressure over small ranges of pressure. It increases strictly somewhat more rapidly than proportionally to the pressure. The ionisation by the β rays from the walls of the vessel which end their life in the gas of the vessel is approximately constant for small variations of the pressure, but strictly increases with the pressure.

Thus one part of the ionisation produced by the soft γ rays from the walls of the vessel varies as the square of the pressure of the gas, and the other part approximately directly as the pressure. The latter part would increase with increase of the softness of the rays and the pressure of the gas. That part of the ionisation

* *Phil. Mag.*, Sept. 1910, p. 406.

varies as the square of the pressure does not violate the conclusions drawn by the writer from the experiments on soft secondary γ rays. It was found that the ionisation was proportional to the pressure over the range of pressures used. Any deviation from this at other pressures could not seriously affect the results, since the ionisation in a gas was compared with approximately the same mass of air and then reduced to standard pressure. Thus the scattering and absorption of the β rays by the gas was approximately the same in the two cases. The principal reason for doing this at the time was that it rendered the ionisations in most cases nearly equal to one another.

I have carried out some experiments with the object of detecting the ionisation that is probably produced by the recoil atom or molecule when a β ray is ejected. They depended on the principle that the ions produced by the recoil atom would probably shew initial recombination like those made by the α particle. The difficulty is to get rid of the ionisation by β rays from extraneous sources. The experiments were not a success, probably owing to this difficulty not having been overcome to a sufficient extent. A shallow cylindrical ionisation chamber was used whose walls consisted of tightly stretched tissue-paper over a skeleton chamber of thin aluminium wire. A mixed beam of β and γ rays from a quantity of radium was passed through the chamber. The current was measured for fields of different strengths when the beam was largely freed from β rays by means of a strong magnetic field, and when it was not so modified. The current saturation curves obtained were the same in the two cases. It could be shewn from energy considerations that when the beam was free from β rays the ionisation due to the recoil atom must have been less than 10 °/。 of the whole ionisation. From the outset, therefore, the experiment had not much chance of success. Another method is about to be tried.

Note on a Dynamical system illustrating Fluorescence. By N. P. McCLELAND, M.A., Pembroke College.

[*Read* 27 October 1913.]

The characteristic feature of the phenomenon of fluorescence is that the period of the exciting force differs from that of the induced vibration.

No simple dynamical system appears to have been brought forward hitherto in which this condition is fulfilled, consequently it appears of interest to suggest the following, which is founded on a well-known model of the atom.

Suppose a particle (of unit mass) revolves in a circular orbit about a fixed point, there being no resistance to the motion, but that forces exist which damp vibrations along the radius vector.

The law of attraction is here taken to be that of the inverse square but a similar result will be obtained under any law which permits stable motion. Let this system be acted on by a periodic force in its own plane, the disturbance being small.

Let this force be $A\sin(pt+\epsilon)$, acting parallel to $\theta=0$.

Let $r=r_0+\rho$, $\theta=\omega t+\psi$ be the coordinates at time t. A, ρ and ψ are supposed always small.

We have for the disturbed motion

$$\ddot{r}+k\dot{r}-r\dot{\theta}^2=-\frac{r_0{}^3\omega^2}{r^2}+A\sin(pt+\epsilon)\cos(\omega t+\psi)$$

$$+\frac{1}{r}\frac{d}{dt}(r^2\dot{\theta})=-A\sin(pt+\epsilon)\sin(\omega t+\psi).$$

Substituting $r_0+\rho$ for r, $\omega t+\psi$ for θ and keeping in only terms of the 1st order we obtain

$$\ddot{\rho}+k\dot{\rho}-\rho\omega^2-2r_0\omega\dot{\psi}=2\rho\omega^2+A\sin(pt+\epsilon)\cos\omega t \quad \text{...(i)},$$

and $$r_0\ddot{\psi}+2\omega\dot{\rho}=-A\sin(pt+\epsilon)\sin\omega t \quad \text{............(ii)}.$$

The last becomes on integration

$$r_0\dot{\psi}+2\omega\rho=\frac{A}{2}\left(\frac{\sin\{(p-\omega)t+\epsilon\}}{p-\omega}-\frac{\sin\{(p+\omega)t+\epsilon\}}{p+\omega}\right)$$
$$+\text{const.......(iii)}.$$

Substituting for $\dot{\psi}$ in (i) we obtain

$$\ddot{\rho}+k\dot{\rho}-3\rho\omega^2-2\omega\left(\frac{A}{2}\left[\frac{\sin\{(p-\omega)t+\epsilon\}}{p-\omega}-\frac{\sin\{(p+\omega)t+\epsilon\}}{p+\omega}\right]\right.$$

$$\left.-2\omega\rho+\text{const.}\right)=A\sin(pt+\epsilon)\cos\omega t\;\ldots\text{(iv)}$$

which easily reduces to

$$\ddot{\rho}+k\dot{\rho}+\omega^2\rho=\frac{A}{2}\left[\frac{p+\omega}{p-\omega}\sin\{(p-\omega)t+\epsilon\}\right.$$

$$\left.+\frac{p-\omega}{p+\omega}\sin\{(p+\omega)t+\epsilon\}\right]+\text{const.}\;\ldots\text{(v)}.$$

The disturbance therefore consists of two trains of waves of different period, and it can be seen that these do not combine to give a single train of the same period as the initial disturbance.

On the Presence of certain Lines of Magnesium in Stellar Spectra. By F. E. Baxandall, A.R.C.Sc. (Communicated by Professor Newall.)

[*Read* 24 November 1913]

In a recent paper* by Professor Fowler on "New Series of Lines in the Spark Spectrum of Magnesium," four magnesium spark lines are recorded which do not belong to the series. The same four lines had previously been given by Fowler and Payn†, but in the later paper more accurate wave-lengths are given. The series lines, with the exception of λ 4481·3, are outside the region usually obtained in photographs of stellar spectra, but the four lines mentioned occur in a part of the spectrum available for investigation, their wave-lengths being 4384·86, 4390·80, 4428·20 and 4434·20. In a "Catalogue of 470 of the Brighter Stars," published in 1902 by the Solar Physics Committee, a record is given of the wave-lengths of lines in the spectra of various stellar types. Reference to this shows that weak lines are recorded at or near the positions of Fowler's magnesium lines in the spectra of α Cygni and α Canis Majoris, as follows:

Mg. Spark Lines (Fowler)	*α Cygni*	*α Canis Majoris*	*Remarks*
4384·86	—	4384·7 (1)	
4390·80	4391·0 (2·3)	4391·0 (2)	Stellar line probably chiefly due to proto-titanium 4391·19.
4428·20	4428·7 (1)	4428·7 (1)	
4434·20	4434·4 (1)	4434·4 (<1)	

Although the magnesium line 4390·80 may enter into the composition of the stellar line 4391·0, the latter is probably due chiefly to proto-titanium line 4391·19, as all the other prominent enhanced lines of titanium are represented in these stellar spectra. The remaining lines have not hitherto been assigned to a terrestrial source, and it seems highly probable that they are the stellar analogues of the magnesium lines. The largest divergence in wave-length is 0·5 tenth-metre (line 4428·20), but there is a possibility of such an error occurring in the measurement of weak stellar lines which, under the magnification used in the measuring instrument, are very difficult to see.

Since the publication of the stellar records referred to, better photographs of the spectra have been obtained at South Kensington, and new measures of the wave-lengths of the lines have been made. The fiducial lines used in the determination of the positions are Fe 4383·72, pFe 4417·00 and pTi 4444·0, all of which are

* *Proc. Roy. Soc.* vol. LXXXIX. p. 133, 1913. † *ib.* vol. LXXII. pp. 253—257.

sharply defined lines in the spectrum of α Canis Majoris. From measures on these and the stellar lines under discussion and subsequent use of Hartmann's interpolation formula the resulting wave-lengths for the three unknown lines measured are

4384·7, 4428·4, 4434·3.

Fowler's wave-lengths for the three magnesium lines are

4384·86, 4428·20, 4434·20.

The differences here are no greater than one would expect, allowing for the weakness of the stellar lines. As a check on the accuracy of the resulting wave-lengths, another line was measured which was known to be identical with the laboratory line Fe 4404·88. The deduced wave-length for this stellar line was 4404·9. It is fairly certain, then, that the wave-lengths estimated for the weaker unknown stellar lines are correct to within $\pm$ 0·2 tenth-metres.

Fowler's line 4384·86 falls in position between two well-authenticated lines in the α Canis Majoris spectrum. The first is 4383·72, a very strong arc and spark line of iron, the other 4385·55, an enhanced iron line. In the stellar spectrum these two lines are nearly equal in intensity, and the line 4384·7 between them makes a close and almost equally spaced triplet with them, and there is no doubt about the separation of these lines in the best spectra of Sirius photographed at South Kensington with two Henry prisms of refracting angle 45° and aperture 6 inches. Reference to the best spectra of this star in the Cambridge series of photographs used for radial-velocity work abundantly verifies the existence of the extra line.

The line at wave-length 4384·7 was not recorded in the α Cygni spectrum in the publication previously referred to, but an examination of the most recent Kensington photographs and the Cambridge radial-velocity plates leaves no doubt as to its occurrence in that spectrum. Taking the three lines (4383·72, the extra line, and 4385·55) as they occur in stellar spectra, the interval on the red side is a little smaller than the other and the wave-length of the middle line is 4384·7. There can be no doubt about the wave-lengths of the two outside lines, which are identical with the two solar lines 4383·72, 4385·55. If the wave-length recorded by Fowler for the magnesium line is correct, and if this line really does occur in the stellar spectrum, the resulting stellar triplet formed by this line and the iron and proto-iron lines should be spaced in the proportion of 11·4 to 6·9, the larger space being on the more refrangible side. As previously stated, the space on the violet side is greater than the other in the stellar spectrum, but certainly not in the required proportion, assuming that Fowler's wave-length is correct.

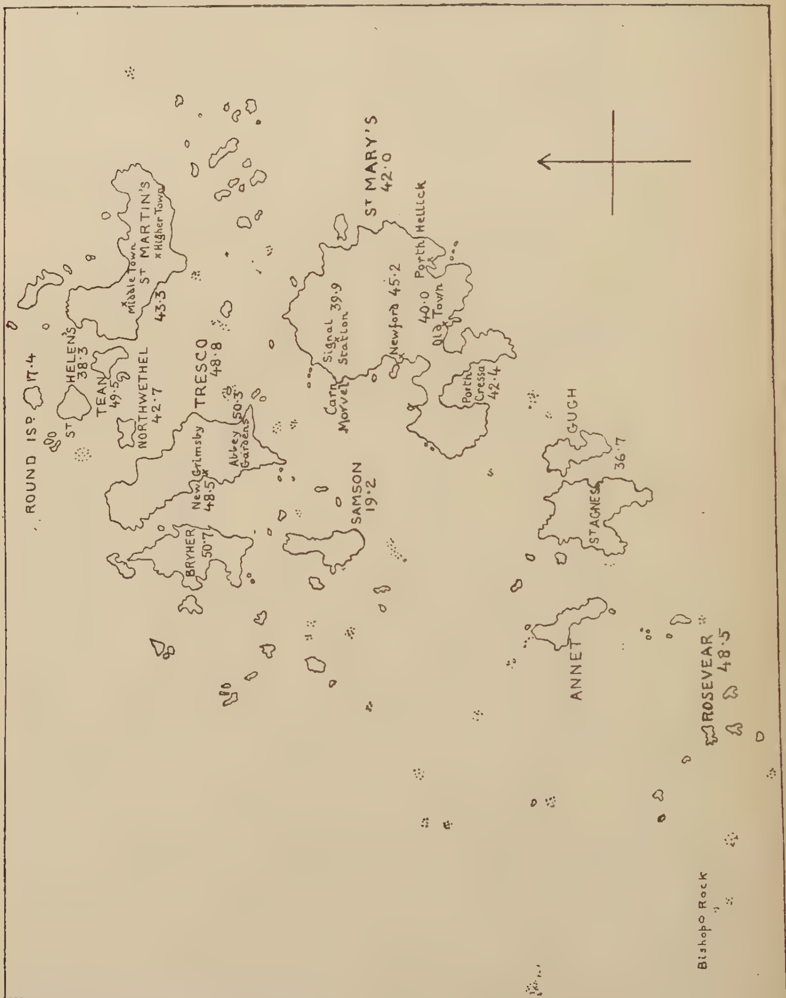
It would appear then that either (1) the laboratory line and the stellar line are not identical, or (2) the wave-length of the laboratory line as recorded by Fowler is from 0·1 to 0·2 tenth-metre too high. In stating these alternatives, it is assumed that the two outside stellar lines are identical with the solar lines mentioned, but there is practically no doubt on this point.

The 4481 line of magnesium, which is a very strong line in Fowler's spectrum giving the magnesium lines under discussion, is, in stellar spectra, about at its maximum in such stars as α Cygni and α Canis Majoris, in which these other magnesium lines appear to exist.

The proportions of the sexes of Forficula auricularia *in the Scilly Islands.* By H. H. BRINDLEY, M.A., St John's College.

[*Read* 17 November 1913]

IN the *Proceedings*, vol. XVI. pt. 8, 1912, p. 674, I summarised the results of enumerating the sexes of adult individuals of *Forficula auricularia*, the Common Earwig, in collections obtained from



The figures indicate the percentage of male *Forficula auricularia*.

29 localities in the British Isles. It was explained that this study of the proportions of the sexes arose incidentally to an enquiry,

still in progress, on the dimorphism of the forceps in the adult male. In the table showing the proportions of the sexes in collections made up to 1911 the extreme instances are both from the Scilly Islands; Round Island, the granite islet at the north of the group, having 16·1 per cent. of males; while the comparatively large and cultivated island of Tresco has the highest male percentage so far observed, viz. 59·7. Both collections were made in August and September 1911. Mr E. J. Burgess Sopp, F.E.S., had kindly sent me the sex proportions of several collections made by himself in four English counties, and in his list also Tresco has the highest male percentage, 55·5 (collection made in 1903). In view of these results I went to the islands in the second half of August in 1912, in company with Mr F. A. Potts of Trinity Hall and Mr J. T. Saunders of Christ's College, to whom I am indebted for great assistance in the task of collecting as many earwigs as possible. Collections were made in all the inhabited and in five of the uninhabited islands. We have to thank Mr T. Algernon Dorrien-Smith, the Lord Proprietor of the islands, for his kind hospitality and for local information which much facilitated the earwig collecting and the other zoological work which partly occupied the time of Messrs Potts and Saunders.

The figures in the sketch map show the percentages of males in the collections made in August 1912, and also in one islet, Rosevear, which we did not visit, and in one locality, Porth Cressa, the S.W. inlet of St Mary's. These were searched for me by our boatman and his family in September 1913. The following table includes the collections shown in the sketch map and also all others from the Scilly Islands which are in my hands.

From certain of the islands the total number of adult specimens is too small to accept the proportions of the sexes calculated therefrom without reserve. It has been a frequent experience that the proportions vary a good deal when the sexes are taken haphazard from a collecting bottle in which they have been mixed together, at least until a total of 300 has been exceeded. But on the islands which yielded a low total it was obvious that prolonged search would be necessary to capture say a thousand adults. A little experience keeps the searchers to the right spots and we left the smaller islands feeling that the hunt had been thorough: our boatman and his son were provided with a killing bottle on each occasion and they added considerably to our collections.

If we set aside the islands in which the total number of adults collected is less than 300, there is still much evidence that the proportions of the sexes vary widely, and in the group as a whole the range is as considerable as in collections made in Great Britain in localities between Edinburgh in the north and Poole Harbour and West Cornwall in the south.

Locality	Year	Total ♂ and ♀ adults	Percentage of males
Round Island	1911	3655	16·1
" "	1912	2016	17·4
Samson	1912	172	19·2
St Agnes and Gugh	1912	466	37·8
St Helen's	1912	120	38·3
St Mary's; between Carn Morvel and Signal Station	1912	587	39·9
" between Porth Hellick and Old Town	1912	368	40·0
" total collections	1912	1610	42·0
" Porth Cressa	1913	1330	42·4
Northwethel	1912	206	42·7
St Martin's	1912	823	43·3
St Mary's; Newford Strand and Island ...	1912	655	45·2
Rosevear	1913	2153	48·5
Tresco; New Grimsby	1912	1650	48·5
" total collections	1912	2020	48·8
Tean	1912	196	49·5
Tresco; Abbey Gardens	1912	370	50·3
" " "	1913	356	50·6
Bryher	1912	1052	50·7
Tresco; Abbey Gardens	1911	330	59·7

It was pointed out in my previous paper* that the percentage of males from the same locality is found to vary for different years in collections made in the same months and large enough for reliance to be placed on the percentages. In some cases the variation is greater than might be expected, thus in collections kindly made for me by Professor R. C. Punnett, at Bobbing, Kent, in 1903, 1904 and 1906 there was a range of 9·2. In Tresco Abbey Gardens, Scilly Isles, the male percentages for 1903, 1911, 1912 and 1913 were 55·5†, 59·7, 50·3 and 50·6. This range is considerable, but in all cases the male percentage is above the average, and forms a striking contrast with 16·1 and 17·4 in the large collections made for me at Round Island by the light-keepers in two successive years. The collection from Porth Cressa in St Mary's in 1913, 42·4, agrees closely with the average 42·0 of collections from three other localities in the same island in 1912.

* *Loc. cit.* p. 677.

† Figure kindly furnished to me by Mr E. J. Burgess Sopp from a small collection (146 adults) made in August, 1903.

The variation between different years is not wide enough to discredit the conclusion that in some of the Scilly Islands the percentage of males is habitually low, while in others it is above the average.

The proportions of the sexes observed in Scilly have very little relation to the mutual positions of the islands. Comment on these points may be deferred till after a brief mention of the situations in which earwigs were found most abundantly. The Scilly Islands are granite with much blown sand, which is especially abundant on Tresco. There is an outcrop of altered Killas on White Island, N.W. of St Martin's, and on the latter a patch of gravel which is possibly of Eocene date. Glacial deposits occur on the larger islands, especially in the north of the group. There is a little alluvium on St Mary's*.

The whole group of islands is included in a parallelogram of 6×8 miles, and St Mary's, the largest island, does not exceed two miles in its longest diameter.

Taking the islands in an order which is roughly N.E. to S.W., the most northerly is

Round Island.—This dome-like mass of granite is 135 ft. high and inaccessible save for the Trinity House steps cut on the S. side. Its only inhabitants are the three light-keepers. The commonest plants are *Armeria maritima*, *Cochlearia officinalis* and *Mesembryanthemum edule.* There is no turf. The light-keepers throw their potato peelings and other kitchen refuse down the N.E. slope, and this midden swarms with earwigs under detached stones and in old meat tins. They are not numerous on the rest of the islet and there is no doubt that their food is mainly the kitchen refuse. They are mostly large-bodied and the high male is exceedingly common. The presence of man has apparently favoured their increase in this spot. (In the above and other references to the average size of body in adults, the statement covers both sexes)

St Helen's.—This island is also dome-like, but is about twice the size of Round Island and has much turf with scattered stones, while the rocky shore of its S. side has a fringe of bracken fern which grows high and thickly. On this fringe is the ruined "Pest House," an old quarantine hospital. There are now no inhabitants. Prolonged search all round the island discovered earwigs only in this neighbourhood. They were not numerous and nearly all were found under fallen slates and masonry in and about the Pest House. These earwigs were mostly fairly large and high males were common.

Tean.—A low irregular island uninhabited save by grazing cattle. It is nearly all turf-covered and has a fringe of blown

* G. Barrow, *Geological Survey Memoir; Isles of Scilly*, 1906.

sand as well as a large beach on its N.E. side. The comparatively few earwigs were chiefly under stones on the sand near the margin of the turf. High males were not infrequent, but there were more small-bodied individuals than on the two above islands.

St Martin's.—This inhabited island is well cultivated on its S. side, where the slopes are cut up into small root fields and pastures by numerous stone walls. There are *Escallonia* shelter hedges at the S.W. end. The higher land is largely pastures. The second highest point in the islands is on St Martin's, a spot 160 ft. high at the E. end, on which the Day Mark is erected. Earwigs were numerous in the stone walls. They were on the average much smaller than those of the islands already mentioned, and "low" males were common. The sand beaches on the N. side were not searched.

Northwethel.—This islet resembles Tean in its general features, but has less turf and more blown sand. Earwigs were not very numerous and all were found under stones lying on the sand. In size they resembled the specimens from Tean.

Tresco.—This large and inhabited island is chiefly pasture with many root fields, and it possesses a special feature in the sub-tropical gardens of The Abbey at the S. end. Collections were made in two localities. (i) *New Grimsby*, the western village. Earwigs were fairly abundant in the stone walls above the houses and very numerous in the scattered pieces of rotting wreck wood lying on the turf near the beach. Breaking open the wood with a chisel turned out swarms of earwigs. The majority had large bodies and "high" males were common: "low" males were much more frequent than on Round Island. (ii) *The Abbey Gardens.* Here earwigs have been caught for me for three successive years by Mr James Jenkin, the head gardener. They do not appear to be specially numerous, and, compared with those from New Grimsby, half a mile distant, they are small-bodied, with few if any "high" males.

Bryher.—This island is inhabited and is chiefly pasture with scattered stones and a certain number of root fields. It has one turfy hill. Earwigs were caught in numbers under stones in and near the cultivated fields and less abundantly under stones on the turf. They were of medium size, and the "high" male was not conspicuous.

Samson.—An island composed of two turfy hills with scattered stones. Each hill is about 100 ft. high. They are divided from each other by an extensive beach of blown sand with numerous stones which runs across from shore to shore. There is a wide belt of very thick and high bracken on the E. side, in and near which are the ruins of four or five houses, one of which was inhabited till 1855. *Salsola kali* is abundant on Samson. The island is now

used only for grazing. The earwigs were found under stones on the turf on the S.W. part of the island and on the blown sand beach in the middle. They were much the same size as those on Bryher.

St Mary's.—In this, the largest island, much of the land is under cultivation and almost all the remainder is pasture. There is a good deal of bracken on the N.E. side. Almost all the likely spots were searched for earwigs, and they appeared to be most numerous in the following districts:—

(i) Between *Carn Morvel* and the *Coast Guard Signal Station*, which is placed on the highest point, 165 ft., in the Scilly Islands. The walls of the pastures and turnip fields on the slopes up from Carn Morvel to the Signal Station were full of earwigs.

(ii) *Newford Strand* and *Island.* The Strand is a beach of blown sand followed by flat turf with a few houses. Earwigs were fairly plentiful under stones where sand and turf meet, as on Tean. Newford Island is connected with the Strand at low water. It is a small pasture walled round, and a certain number of earwigs were captured in the wall and under stones on the turf.

(iii) Between *Porth Hellick* and *Old Town.* A few earwigs were found under stones on the sandy beach of Porth Hellick, the majority were captured in the walls of pastures by the road side.

(iv) *Porth Cressa.* This bay is surrounded by gardens and pastures. The earwigs from all localities on St Mary's included many "high" males, and the average size of body probably exceeds that of individuals from St Martin's and Tresco Abbey.

St Agnes and *Gugh.*—The latter is a turfy, uninhabited islet connected with St Agnes by a sandy beach save near high water, thus it may be regarded as part of St Agnes. Earwigs were found under stones on the sand and turf near their meeting point. St Agnes has many houses and is like St Mary's in its cultivation. The walls of the pastures and root fields were fairly populated with earwigs, which were on the whole of about the same size as those from St Martin's and therefore smaller than the St Mary's individuals.

Annet.—This island of 90 acres has no high land. It is uninhabited. It is chiefly turf with much *Armeria maritima.* The north end is rocky and there are outcrops and stones lying about in many places. The whole of the soil is undermined by puffin burrows, for it is the regular breeding place of this bird in the Scilly Islands. Here no earwigs at all were found after prolonged searching by five persons. If there are any on Annet they are certainly in very small numbers.

Rosevear.—This rocky islet, with the smaller Rosevean and Gorregan, forms an isolated group about two miles E. of the Bishop Rock. Mr C. J. King, of St Mary's, informs me that when

he visited Rosevear in October 1912 the giant mallow (*Lavatera arborea*) was so high that a short man could walk hidden among it. *Armeria maritima* is one of the commonest plants. Rosevear bears the ruined huts of the builders of the present Bishop Lighthouse, who inhabited it from 1850 to 1858. In 1912 and when he made a collection for me in September 1913 my boatman, Mr S. Jenkin, reported the islet swarming with earwigs. The specimens are nearly all large and the males conspicuously "high." Among them is the longest pair of forceps which I have obtained or heard of, viz. 12·25 mm. The Round Island collection of 1911 contained one male with forceps 11·0 mm. In Mr W. Bateson's collection from the Farn Islands in 1892, by which he showed that the male earwig is dimorphic in respect of its forceps, the highest males have the value 9·0 mm.*

The dimorphism of the male does not fall within the scope of the present paper; all that need be said is that in the Scilly Islands both "low" and "high" males occur, and that in some of the islands the "high" individual is strikingly in excess. The necessary measurements are as yet too incomplete for saying anything about the extent to which the two kinds of males are present in the Scilly Islands, and it should be borne in mind that the above notes on both the dimorphism of the forceps and the general size of body are from a comparatively superficial examination of the collections.

It is obvious that the Common Earwig is very abundant in the islands and that it exhibits well-marked local differences in respect of (*a*) numerical proportions of the sexes, (*b*) body size of adult, (*c*) percentage of "high" males. (*b*) and (*c*) naturally to some extent vary together: as pointed out by Bateson, the "high" male has a large body and the "low" male a small one, but the body dimensions give a monomorphic curve; the male earwig appears to be dimorphic only as regards its forceps. No explanation is at present forthcoming of the local variations mentioned above and we remain in the dark with regard to their causes as to those of local races in general. As remarked on p. 329, the percentage of males bears very slight or no relation to the mutual positions of the islands. Bryher and Tresco yielded collections large enough to indicate the sex proportions with probable accuracy, the islands are contiguous and in both the male percentage is high. On the other hand Samson, almost as near and separated from Bryher by very shoal water, has very few males. The estimate is based on a small total collection, but I am inclined to believe that the male percentage is really particularly low on this island, for we made it the object of prolonged search

* Bateson and Brindley, *Proc. Zool. Soc. Lond.*, Nov. 15, 1892, p. 585.

and came away with the conviction that its earwig population is sparse. The sketch map renders it unnecessary to particularise other instances of the same kind. The thought arises as to how long the Scilly Islands have had their earwig population. The mainland is 25 miles away at its nearest, and though the Common Earwig hardly ever uses its wings it is conceivable that individuals have been blown across from Cornwall now and then. But an earwig with furled wings, though it readily drops in order to seek shelter, is not easily blown from a spot it intends to hold on to. Floating vegetable matter and soil on the feet of birds may have helped to introduce earwigs into the islands along with other non-flying invertebrates, such as woodlice and earthworms. Probably, however, man has been an important factor in carrying earwigs to the Scilly Isles: their habit of concealing themselves in folded clothes and in crevices of all kinds greatly assists their passive transport. We do not know how long ago man settled on the larger islands, one can say no more than that his arrival was pre-historic. With regard to Round Island and Rosevear the suggestion may be hazarded that the extraordinary abundance of earwigs they possess may be partly the result of recent human settlement. On Round Island there is little doubt that they find abundant nourishment in the light-keepers' rubbish heap, but on Rosevear there has been no such food supply for more than sixty years. I am unable to say anything as to what the Rosevear earwigs feed on at the present time. It is striking that these two islets should stand out as inhabited by earwigs with large bodies and great frequency of "high" males, while the numerical proportions of the sexes differ so much. It does not appear likely that food more varied than the wild vegetation is a cause of the greater size, for in the Farn Islands in 1907 Mr Potts and myself found that earwigs from the two uninhabited Widerpens were larger than those caught on the Inner Farn, which at that time had a poultry farm as well as the light-keepers' houses. Again, on Tresco, the specimens from New Grimsby were large while those from the Abbey Gardens comparatively small, both were living near human habitations but in different conditions as regards vegetation and soil. No more can be said with approach to certainty than that the specimens obtained from cultivated ground were generally smaller than those from rocky and wild localities. It may well be that local races are in process of evolution. The absence of earwigs on Annet is peculiar. Our search was sufficiently thorough to convince us that either earwigs have not obtained a footing there or that they exist in quite small numbers in isolated spots. Its nearness to a large island well stocked with earwigs and its vegetation gave the expectation that plenty would be found. This island furnishes

an argument against wind-carriage being an important factor in distributing them. Though uninhabited it is frequently visited by man, so that from time to time earwigs must be imported accidentally. It may be that this insect is a parallel case with that of cockroaches, which are apt to occur in vast numbers in one spot and yet spread slowly from village to village and even from house to house. The expectation is, however, that earwigs as indigenous European insects living normally in the open would spread more easily than cockroaches, which for the most part seem to have followed man from warmer regions. Annet is the island specially chosen by puffins as their breeding place: it is just possible that in some unknown way the presence of this bird is inimical to earwigs.

We had no opportunity of searching the Eastern Isles lying between St Martin's and St Mary's. All are small, but many of them have turf and plenty of other vegetation.

The present study of the earwigs of the Scilly Isles as a whole does no more than bring to light the facts recited, but they suggest that the group is a favourable and easily accessible locality for a full investigation as to sex-inheritance, influence of parasites and of environmental conditions.

About eight earwigs were taken, most of them on St Martin's, which were infested by a large Nematode or Gordiid worm, at present unidentified. This worm had its two ends hidden in the abdomen and its coiled body projecting from between the terga, which were much forced apart. The hosts seemed fairly active and well nourished.

In the latter part of August the earwigs of the Scilly Islands are, as on the mainland, nearly all adult. Nymphs were collected by us, but not with so much care as the adults; their smaller size renders them more difficult to secure.

Notes on the Breeding of Forficula auricularia. By H. H. Brindley, M.A., St John's College.

[*Read* 17 November 1913]

In a previous paper* I summarised what is known with regard to the oviposition, hatching and the duration of the immature life of the Common Earwig. It was mentioned that no record could be found of this species being raised to maturity from the egg in captivity, and that Mr Potts and myself had failed to do so with eggs laid by earwigs brought to Cambridge from the Farn Islands in 1907 and 1908.

In October 1912 I received a large number of living adults from Round Island, Scilly Isles. They were collected in September, and probably most of the females had paired, for while searching Gunwalloe Cove on The Lizard, in the same month of 1912, it was very common to find a male and a female together under stones.

About 110 females were isolated in plaster of Paris cells averaging $2\frac{1}{2}$ in. wide by $1\frac{1}{2}$ in. deep, and covered by watch glasses. 20 more were isolated in flower pots. In an endeavour to diminish the risk of septic infection and attack by fungi, coconut fibre was the only substance used for lining the cells and flower pots. The coconut fibre was kept fairly damp and as far as possible uniformly so. A small piece of washed potato without any skin adhering was the only food given, and this was renewed twice weekly. This diet was suggested by the quantity of potato peelings in the light-keepers' rubbish heap from which the earwigs were captured.

All the cells and flower pots were kept in a room in the Zoological Laboratory, Cambridge, as far as possible from the hot air supply, though the temperature was on the average considerably above the winter temperature in the open on the Scilly Islands. The isolated earwigs were somewhat sluggish and did not eat the potato slips much. Some hid under the latter, some buried themselves first below the surface of the coconut fibre, while a good many remained on its surface.

It was not possible to examine all the cells daily, but an endeavour to look at all at least twice a week was made. So the days which follow should no doubt in many cases be slightly antedated. Counting the number of eggs in a clutch was rather neglected, in the fear that much disturbance of the heap in which they were laid might diminish the chances of hatching.

* *Proc. Camb. Phil. Soc.* vol. xvi. part 8, 1912, p. 674.

In the following table, showing the results obtained, 0 in the second column indicates that the eggs disappeared without hatching (some were attacked by mould, but the disappearance of the rest could not be explained). The figures in brackets after dates indicate the approximate number of eggs or individuals.

The young when first observed were very small, having a body length of about 5 mm. Thus they were probably all in either the first or second instar, as newly hatched earwigs are 4 mm. long as a rule.

	Eggs found	Eggs seen hatching	Young found	Young last seen alive	Adult stage attained
1	—	—	Jan. 18 (23)	Feb. 7 (1)	—
2	—	—	Jan. 18 (8)	Apr. 10 (2)	—
3	—	—	Jan. 19 (8)	Feb. 17 (1)	—
4	—	—	Jan. 20	Mar. 8 (1)	—
5	—	—	Jan. 23	Feb. 3 (several)	—
6	—	—	Jan. 23	May 9 (1)	—
7	—	—	Jan. 24	—	June 17 (1 ♂, 1 ♀); June 23 (3 more ♀s); July 4 (1 more ♀)
8	—	—	Jan. 27	Mar. 22 (1)	—
9	—	—	Jan. 28 (12)	—	July 17 (1 ♀)
10	—	—	Feb. 3 (*c.* 10)	—	June 25 (1 ♀)
11	—	—	Feb. 7 (1)	Apr. 29 (1)	—
12	—	Jan. 21	Jan. 21	Feb. 7 (several)	—
13	—	Jan. 31	Jan. 31	—	June 23 (1 ♀)
14	Jan. 19 (37)	—	—	Jan. 31 (1)	—
15	Jan. 19 (19)	—	Feb. 13 (1); Feb. 18 (5)	Mar. 15 (several)	—
16	Jan. 19	—	Jan. 31 (7)	May 9	—
17	Jan. 19 (12)	0	0	—	—
18	Jan. 24	0	0	—	—
19	Jan. 24	0	0	—	—
20	Jan. 25	0	0	—	—
21	Jan. 30	0	0	—	—

Thus greater success was obtained than in the previous attempts, for in four families all stages from oviposition to maturity occurred in the laboratory.

I am indebted to Mr C. B. Williams, of Clare College, for details of a case in which he was successful in obtaining mature individuals from the egg, hatching and subsequent stages being passed through in the laboratory of the John Innes Horticultural Institute, Merton. The year was the same, 1913, as my own observations. The dates were:

March 8: female and 20 to 30 eggs found in a pine stump.

March 14—16: hatching took place, and the young were fed on potato and flower petals.

August 20: 4 became adult.

(In 1912, April 28, Mr Williams found two very small and apparently newly hatched young, and by August 28 both were adult. They lived till about March 17, 1913)

The table of results obtained in the Cambridge Laboratory shows that out of about 130 Round Island females 21 certainly laid eggs (probably more did so and the fact escaped observation, but the examination of the cells was sufficiently frequent to render it unlikely that more than a very few clutches were missed). Four of the broods observed produced adult earwigs. The adults were nine in number, one male and eight females, which is curiously near the proportions given by Round Island earwigs collected in large quantities. The usual number of eggs laid by one female may be calculated very roughly by taking the average of 37, 19, and 12, the instances in which the eggs were counted with approximate accuracy; this average is 23 and is about the number usually found. Supposing that all the 130 females which were put into cells had laid 23 eggs each, there would have been 2990. Actually, 21 females laid eggs, and if 23 was the average number, potentially 483 adults were produced. But only nine were actually found, which is 1·86 per cent. producing adult individuals of all the eggs laid. Supposing that all the 130 females which were isolated in cells had each laid 23 eggs, that all had hatched and all the young had reached maturity, 2990 adults would have been produced: nine in 2990 is 0·3 per cent. So great a failure to lay eggs at all and so great a mortality in infancy as appear in this case might be attributed to the artificial conditions under which the females were placed in October and continued in for five months or more before they began to lay: the conditions of moisture, ventilation, diet and the substances they lived among were all more or less abnormal. On the other hand, is it likely that the mortality among the immature is to any high degree less among earwigs in the wild state? I am inclined to doubt it. Again, in the artificial conditions of a laboratory the eggs and insects were out of the reach of various factors, both physical and organised, which affect adversely their development under natural conditions. On the other hand it seems probable that the artificial conditions had something to do with the great failure to lay eggs, only 21 doing so out of 130 females presumably fertilised seems a very low proportion. Still, a certain number died in the first two or three weeks of their life in the cells, some may never have been fertilised and the new conditions may have inhibited the power of laying. As regards the failure of several of the clutches to hatch, the premature oviposition probably brought about by the

high temperature and other special features of life in confinement may have rendered the eggs incapable of development. We do not know the usual month for oviposition or how long is the period of immaturity in the Scilly Islands, apparently these facts are not established on extensive evidence for any country. A brief review of the published statements in this connection was attempted in my previous paper. In the neighbourhood of Cambridge it is likely that oviposition takes place in March or April, that the eggs hatch in May, and that the offspring become adult in July or August. There is little doubt that the appearance of eggs as early as January is premature for even so temperate a climate as that of the Scilly Islands, especially when it is borne in mind that a fair number of nymphs are found there in the latter half of August. That laboratory conditions encourage premature oviposition is well known for more than one order of insects. Many of the earwigs brought to Cambridge from the Farn Islands began to lay about the middle of October. The same acceleration is well shown by the further history of case 7 in the above table. The male and the five females, which became adult between June 17 and July 4, were placed together in a large dish on coconut fibre and supplied with green food as well as potatoes. On November 4, 13 young were found, apparently in the first instar. The eggs were not observed, so it is uncertain if the offspring of one or of more of the females was found, for all the latter were living and apparently healthy on this day. The young were isolated and four at least were alive and active on December 21—there is no doubt that some had died. Thus these grand-children of a female brought from Round Island in September 1912 were hatched probably seven or eight months earlier than the third generation would be in a wild state. A batch of living adults was procured from New Grimsby, Tresco, early in October 1913, and about 44 females were isolated on coconut fibre in flower pots. On October 28 a batch of eggs was found. So far these have not hatched.

Survival of males through the winter.

In my previous paper (p. 678) the doubt as to what extent this occurs under normal circumstances was alluded to. In the subsequent experiments the point has not been examined in detail, partly because a large number of the males have been killed and preserved for measurements of the forceps, and partly because the artificial conditions of a laboratory militate against a satisfactory conclusion.

Maternal care of the eggs.

The eggs of earwigs laid in the wild state are usually found in a little pit excavated and covered in about an inch below the surface, or else in convenient crevices in vegetation. In the cells and flower pots in which the Round Island females were kept most of the clutches were laid on the surface of the coconut fibre, but in many cases the mother protected them by a thin covering of fibre so that they lay in a small pit immediately under the surface. Whether she made the excavation before or after oviposition was not ascertained, the act of laying itself was not observed in any case. There is no doubt that the mother watches over the eggs, as has been stated by various authors, but the assertion, made from time to time, that she guards the young seems to have no foundation. The newly hatched are active and begin feeding in a few hours, possibly less, after becoming free from the egg membrane, while the mother displays no interest in them. Before hatching her behaviour is very different. The female either covers the little pile of eggs with her body or else keeps her head towards them with the antennae playing over them. Possibly the second attitude is the result of her being disturbed rather than the natural one. If driven away from the pile of eggs, for they are usually laid in a heap resembling a pile of round-shot, in a few minutes she has returned and is seen diligently bringing the eggs together with the first pair of legs. This accords with Camerano's observation* quoted in my previous paper.

Hatching.

This was observed in the case of two clutches of eggs. Shortly before rupture of the egg membrane the position of the head is seen easily by the black eyes, the only pigmented part of the young earwig, showing through the membrane. The young appears to bite through the latter and it comes out head first, aiding its emergence with the first pair of legs. As more of the body is freed the other legs in succession push away the egg membrane. In more than one case there was evidently great difficulty in discarding the membrane, which was eventually done by catching it against obstructions.

* *Boll. d. Soc. Entom. Ital.*, 1880, p. 46.

The comparison of nearly equal electrical resistances. By G. F. C. SEARLE, Sc.D., F.R.S., University Lecturer in Experimental Physics, Fellow of Peterhouse.

[*Read* 24 November 1913.]

§ 1. *Introduction.* The ratio of the resistances of two very nearly equal coils can be determined most accurately by finding the very small difference between their resistances. The method, which, perhaps, is most widely known, is that of Carey Foster. In that method the difference is expressed as the resistance of a measured length of the graduated wire of the Carey Foster bridge. It is therefore necessary to know the resistance of each centimetre of this wire and to have a table of calibration corrections, since it is impossible to procure an absolutely uniform wire. Even if the wire were initially uniform, it would by use become non-uniform through the wear caused by the contact of the sliding contact piece.

In the bridge designed by Dr J. A. Fleming and used for many years at the Cavendish Laboratory by Dr R. T. Glazebrook and others in the comparison of resistance coils with the standards of the British Association, the wire is about 1 metre in length and has a resistance of about 1/20 ohm. Thus to measure a difference of resistance of 1/200,000 ohm a movement of the contact piece of 1/10 mm. has to be observed and measured.

In recent years the use of Carey Foster's method has been abandoned for resistance comparisons of the highest precision and a method of shunting is now employed. This involves the use of a resistance box capable of furnishing high resistances, and, in strictness, the coils in this box should be compared with a standard resistance. But the resistance boxes now supplied by any good instrument maker are so accurately adjusted that it is quite unnecessary to calibrate them if they are only to be used as shunts. The great advantage of the method of shunting is that instead of dealing with the resistance of one or two millimetres of the wire of the bridge—a length which could not be easily read to less than $\frac{1}{10}$ mm.—we have to deal with shunting resistances measured by many hundreds or thousands of ohms, these resistances not differing from their nominal values by as much as one part in a thousand, if the resistance box has been well adjusted.

The method of shunting has been in use for some years at the National Physical Laboratory and in other standardising laboratories, but it has hardly made its way into the laboratory courses intended for elementary students of physics. It has, however, so

many advantages both as regards accuracy and as regards the instruction of students that it may be useful to other teachers to give an account of the method as employed in my practical class at the Cavendish Laboratory.

§ 2. *General theory.* The theory of the method is as follows: Let C, D (Figs. 1, 2) be two nearly equal resistance coils; in practice they would not differ by as much as one part in 1000.

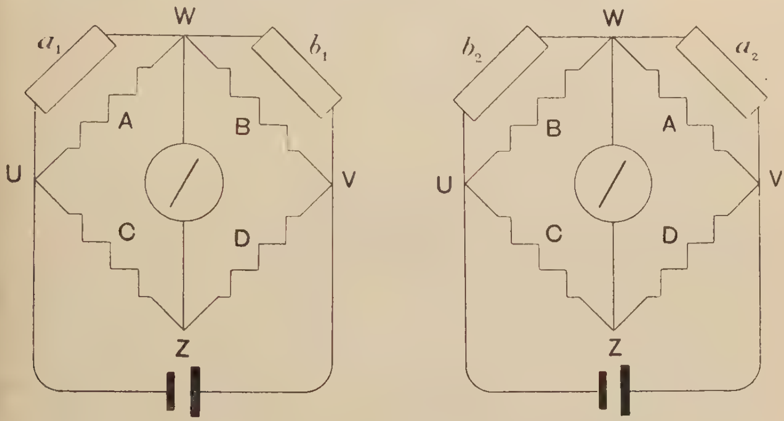


Fig. 1. Fig. 2.

The two coils A, B, which are to be compared, are first connected with the coils C, D to form the four arms of a Wheatstone's bridge, as in Fig. 1, the exact balance being obtained by shunting A with a high resistance a_1 and B with a high resistance b_1. The coils A and B are then interchanged, so that they are now arranged as in Fig. 2, and the balance is obtained by shunting A with a_2 and B with b_2.

It will not, as a rule, be necessary to shunt both A and B at the same time*, so that two of the four resistances a_1, a_2, b_1, b_2 will be infinite. But it will be convenient to consider the mathematics of the problem without this restriction.

Which of the two coils A and B will require shunting in either of the two arrangements will depend upon the relative values of A, B, C, D. The four possible cases are as follows:

A shunted in both arrangements.

B shunted in both arrangements.

A shunted in the first arrangement, B shunted in the second arrangement.

B shunted in the first arrangement, A shunted in the second arrangement.

* If no very high resistances are available, it may be necessary to apply shunts to both A and B in order to secure a satisfactory balance.

Let A_1, A_2 be the effective resistances of A when it is shunted by a_1, a_2 and B_1, B_2 the effective resistances of B when shunted by b_1, b_2. Then

$$\frac{1}{A_1}=\frac{1}{A}+\frac{1}{a_1},\qquad \frac{1}{A_2}=\frac{1}{A}+\frac{1}{a_2} \quad\text{..............(1)},$$

$$\frac{1}{B_1}=\frac{1}{B}+\frac{1}{b_1},\qquad \frac{1}{B_2}=\frac{1}{B}+\frac{1}{b_2} \quad\text{..............(2)}.$$

Since

$$\frac{C}{A_1}=\frac{D}{B_1}\quad\text{and}\quad \frac{D}{A_2}=\frac{C}{B_2} \quad\text{..................(3)},$$

when the bridge is balanced in the two cases, we find, by eliminating C and D,

$$\frac{1}{A_1A_2}=\frac{1}{B_1B_2} \quad\text{........................(4)*}.$$

Taking the square root of each side of (4), we have

$$\sqrt{\frac{1}{A_1A_2}}=\sqrt{\frac{1}{B_1B_2}} \quad\text{....................(5)}.$$

The left side of (5) is the geometric mean of $1/A_1$ and $1/A_2$. When A_1 and A_2 are nearly equal, this is very nearly the same as the arithmetic mean. Thus, if

$$\frac{1}{A_1}=\frac{1}{A_0}+\frac{1}{\alpha},\qquad \frac{1}{A_2}=\frac{1}{A_0}-\frac{1}{\alpha},$$

so that $1/A_0$ is the arithmetic mean of $1/A_1$ and $1/A_2$, we have

$$\sqrt{\frac{1}{A_1A_2}}=\sqrt{\frac{1}{A_0^2}-\frac{1}{\alpha^2}}=\frac{1}{A_0}\left(1-\frac{1}{2}\frac{A_0^2}{\alpha^2}-\frac{1}{8}\frac{A_0^4}{\alpha^4}-\dots\right).$$

Hence, if A_1 and A_2 are so nearly equal that $A_0^2/2\alpha^2$ is negligible compared with unity, we may use the arithmetic mean $1/A_0$ instead of the geometric mean. For example, if A_0/α is $1/1000$, $A_0^2/2\alpha^2$ is only $1/2{,}000{,}000$, which is negligible in all but the most precise work. In that case, however, the resistances would, probably, have been so well adjusted that A_0/α is less than $1/1000$. The same remarks, of course, apply to B_1 and B_2. Replacing the geometric means in (5) by the arithmetic means, we have

$$\frac{1}{2}\left(\frac{1}{A_1}+\frac{1}{A_2}\right)=\frac{1}{2}\left(\frac{1}{B_1}+\frac{1}{B_2}\right)$$

* Equation (4) is a quadratic for $1/A$ in terms of $1/B$. If we solve it, we find

$$\frac{1}{A}=\sqrt{\left(\frac{1}{B}+\frac{1}{b_1}\right)\left(\frac{1}{B}+\frac{1}{b_2}\right)+\frac{1}{4}\left(\frac{1}{a_1}-\frac{1}{a_2}\right)^2}-\frac{1}{2}\left(\frac{1}{a_1}+\frac{1}{a_2}\right)$$

which can be used in any case where a_1, a_2, b_1, b_2 are not very large compared with B.

or, by (1) and (2),

$$\frac{1}{A}+\frac{1}{2}\left(\frac{1}{a_1}+\frac{1}{a_2}\right)=\frac{1}{B}+\frac{1}{2}\left(\frac{1}{b_1}+\frac{1}{b_2}\right) \quad\text{............(6).}$$

From this equation the difference between $1/A$ and $1/B$ is found, when the shunts a_1, a_2, b_1, b_2 are known.

We also have, from (6),

$$\frac{A-B}{AB}=\frac{1}{B}-\frac{1}{A}=\frac{1}{2}\left(\frac{1}{a_1}+\frac{1}{a_2}-\frac{1}{b_1}-\frac{1}{b_2}\right).$$

Hence
$$A-B=\frac{AB}{2}\left(\frac{1}{a_1}+\frac{1}{a_2}-\frac{1}{b_1}-\frac{1}{b_2}\right) \quad\text{...............(7).}$$

When A and B are very nearly equal, it will be sufficient to use A^2 or B^2 instead of AB on the right side, or merely to use the nominal values of A and B on that side.

§ 3. *Practical details.* The measurements are easily made when suitable connecting pieces are used. In laboratories where serious comparisons of resistance are made, mercury cups formed in massive blocks of copper would be used in making the connexions. But mercury cups are out of place in a crowded practical class where the students have only a limited time in which to do the experiments; from the demonstrator's point of view the most important thing is that there should be nothing which can "go wrong."

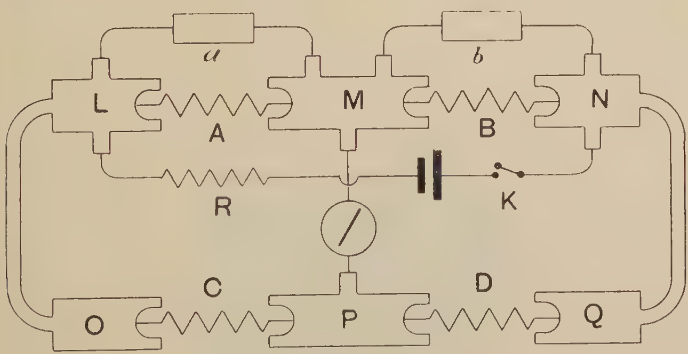


Fig. 3.

In Fig. 3, L, M, N, O, P, Q represent six strips of stout copper with forked ends for clamping under the terminals of the coils. The pieces L, O are connected by about 15 cm. of stout copper wire and the pieces N, Q are connected in a similar manner*. Short wires are soldered to L, M, N, P, as shown in Fig. 3, and by

* Copper wire is preferable to any stiffer connexion, since its use allows L and N to be moved without straining the terminals of the coils C and D.

these wires connexions are made to the battery, the galvanometer and the resistance boxes supplying the shunts, coupling screws being used. A more convenient plan is to fit the copper pieces with proper terminal screws. The four coils A, B, C, D are inserted in the gaps as shown diagrammatically in Fig. 3.

The resistance of the part of the copper connector OL, which lies between the coil C and the point where the battery wire is attached to L, together with the resistance of the part of P, which lies between C and the point where the galvanometer wire is attached to P, counts as part of C itself and is therefore eliminated in the equations. Similar remarks apply to the copper connectors which are joined to the coil D. The effect of the finite resistances of the connectors joined to A and B is discussed in § 9 and is shown to be negligible in practice.

To prevent undue heating of the coils, a sufficiently great resistance R should be placed in series with the battery. The galvanometer should be permanently connected to M and P, and a tapping key K should be placed in the *battery* circuit. In this way errors due to thermoelectric effects are avoided. The inductances of the coils are too small to give rise to trouble on making or breaking the battery connexion.

In order that, after a balance has been obtained, it should not be upset by changes in the resistances of the coils due to rise of temperature brought about by the passage of the current, it is necessary that the coil A should be similar to the coil B and that the coil C should be similar to the coil D. When the bridge is arranged as in Figs. 1 and 2, this similarity secures such constancy of the ratios A/B and C/D that the passage of the current does not upset the balance when once it has been obtained.

Fig. 4.

The "Sub-standards" of resistance (Fig. 4) supplied by Mr R. W. Paul have proved very suitable for the experiment. These coils are wound with wire of small temperature coefficient and are well ventilated so that they carry comparatively large currents without serious rise of temperature. Using four of these

sub-standards for A, B, C, D, in Figs. 1 and 2, the students at the Cavendish Laboratory are able to make reasonably good comparisons in a room where there is much vibration due to moving machinery, the galvanometer being a table instrument having a pointer moving over a divided scale.

The results given below are not intended to illustrate the power of the method when used under favourable conditions; they are intended rather to show how well the method works under unfavourable conditions.

§ 4. *Practical example.* The following results obtained by G. F. C. Searle and A. L. Hughes will illustrate the working of the method. Four "sub-standards" A, B, C, D, each of nominally one ohm resistance, were employed.

Position 1.

Coil A not shunted. Hence $a_1=\infty$ ohms and $1/a_1=0$ ohm^{-1}.
Coil B shunted with 2800 ohms. Hence $1/b_1=0{\cdot}000357$ ohm^{-1}.

Position 2.

Coil A not shunted. Hence $a_2=\infty$ ohms and $1/a_2=0$ ohm^{-1}.
Coil B shunted with 4800 ohms. Hence $1/b_2=0{\cdot}000208$ ohm^{-1}.

Hence, using (6),

$$\frac{1}{A}+\frac{1}{2}\left(\frac{1}{a_1}+\frac{1}{a_2}\right)=\frac{1}{B}+\frac{1}{2}\left(\frac{1}{b_1}+\frac{1}{b_2}\right),$$

we have

$$\frac{1}{A}=\frac{1}{B}+\frac{1}{2}(0{\cdot}000357+0{\cdot}000208)=\frac{1}{B}+0{\cdot}000282 \text{ ohm}^{-1},$$

or

$$B-A=AB\times 0{\cdot}000282 \text{ ohm}.$$

The difference between A and B is so small, and each is so nearly one ohm, that we may put $AB=1$ ohm^2 and thus obtain

$$B-A=0{\cdot}000282 \text{ ohm}.$$

§ 5. *Intercomparison of three coils.* A useful test of the accuracy of the method is obtained if three coils A, B, C are used. First A and B are compared as described in §§ 2, 3, using C and D as the auxiliary coils whose ratio is eliminated. Then A is compared with C, using B and D as the auxiliary coils. Finally B is compared with C, using A and D as the auxiliary coils. In this way the values of the three differences

$$A-B, \qquad A-C, \qquad B-C$$

are found. The accuracy of the work may be tested by comparing the value of $B-C$ found directly with that found from the two differences $A-B$ and $A-C$. The shunts on C may be denoted by c_1, c_2.

§ 6. *Practical example.* The intercomparison of three coils is illustrated by the following results obtained by G. F. C. Searle and A. L. Hughes. Sub-standards each of nominally one ohm resistance were used.

Comparison of A and B.

$$a_1=\infty,\quad 1/a_1=0,\quad b_1=2800,\quad 1/b_1=0{\cdot}000357$$
$$a_2=\infty,\quad 1/a_2=0,\quad b_2=4800,\quad 1/b_2=0{\cdot}000208.$$

Hence $$\frac{1}{A}=\frac{1}{B}+\frac{1}{2}(0{\cdot}000357+0{\cdot}000208)=\frac{1}{B}+0{\cdot}000282$$

and $$A-B=-AB\times 0{\cdot}000282=-0{\cdot}000282 \text{ ohm}.$$

Comparison of A and C.

$$a_1=\infty,\quad 1/a_1=0,\quad c_1=4000,\quad 1/c_1=0{\cdot}000250$$
$$a_2=785,\quad 1/a_2=0{\cdot}001274,\quad c_2=\infty,\quad 1/c_2=0.$$

Hence $$\frac{1}{A}+\frac{1}{2}\times 0{\cdot}001274=\frac{1}{C}+\frac{1}{2}\times 0{\cdot}000250$$

and $$A-C=AC\times 0{\cdot}000512=0{\cdot}000512 \text{ ohm}.$$

Comparison of B and C.

$$b_1=2700,\quad 1/b_1=0{\cdot}000370,\quad c_1=\infty,\quad 1/c_1=0$$
$$b_2=760,\quad 1/b_2=0{\cdot}001316,\quad c_2=\infty,\quad 1/c_2=0.$$

Hence $$\frac{1}{B}+\frac{1}{2}(0{\cdot}000370+0{\cdot}001316)=\frac{1}{C}$$

and $$B-C=BC\times 0{\cdot}000843=0{\cdot}000843 \text{ ohm}.$$

The value of $B-C$ deduced from the two differences $A-B$ and $A-C$ is

$$B-C=(A-C)-(A-B)=0{\cdot}000512+0{\cdot}000282=0{\cdot}000794 \text{ ohm}.$$

Thus the two values of $B-C$ only differ by $0{\cdot}000049$ ohm.

§ 7. *Intercomparison of four coils.* When the student has sufficient time he may determine *directly*, by the method of § 2, each of the six differences

$$A-B,\qquad A-C,\qquad A-D,\qquad B-C,\qquad B-D,\qquad C-D.$$

When this has been done, it will be found that the six differences are not quite consistent. Thus, in the example recorded in § 6, it was found by direct comparison of B and C that $B-C=0{\cdot}000843$ ohm but that

$$(A-C)-(A-B)=0{\cdot}000794 \text{ ohm}.$$

We have, then, to decide how to combine the six results so as to obtain the most probable values of the three differences $A-B$, $A-C$, $A-D$, it being supposed that each of the six observed differences has been found with the same care.

The method employed to obtain the desired result is the method of least squares.

Let us put

$$A - B = x, \qquad A - C = y, \qquad A - D = z.$$

Then

$$B - C = y - x, \qquad B - D = z - x, \qquad C - D = z - y.$$

Then the six observed differences $A - B$, $A - C$, &c. give us *six* equations for the determination of the *three* quantities x, y, z. These equations are, however, not quite consistent and we employ the method of least squares to reduce the six equations to three, which when solved will give us the most probable values of x, y, z.

Denoting the six differences by Δ_1, Δ_2, ... Δ_6, we have the following six equations

$$\begin{array}{lr} x = \Delta_1 & -x + y \qquad = \Delta_4 \\ y = \Delta_2 & -x \qquad + z = \Delta_5 \\ z = \Delta_3 & -y + z = \Delta_6 \end{array}$$

The method of least squares directs us to multiply each one of these equations by the coefficient of x in it and then to add the six equations together to form a single equation. In our case the coefficients of x taken in order are 1, 0, 0, -1, -1, 0. A second equation is formed by multiplying each one of the six equations by the coefficient of y in it and then adding the six equations together. A third equation is formed in like manner by adding together the six equations after each has been multiplied by the coefficient of z in it. When this is done in our case, we obtain the following three equations:

$$\begin{aligned} 3x - y - z &= \Delta_1 - \Delta_4 - \Delta_5 = \eta_1, \\ -x + 3y - z &= \Delta_2 + \Delta_4 - \Delta_6 = \eta_2, \\ -x - y + 3z &= \Delta_3 + \Delta_5 + \Delta_6 = \eta_3. \end{aligned}$$

The values of x, y, z—say, X, Y, Z—derived from these last three equations are the most probable values. We obtain

$$\begin{aligned} X &= \tfrac{1}{4}(2\eta_1 + \eta_2 + \eta_3), \\ Y &= \tfrac{1}{4}(\eta_1 + 2\eta_2 + \eta_3), \\ Z &= \tfrac{1}{4}(\eta_1 + \eta_2 + 2\eta_3). \end{aligned}$$

We can now determine the most probable values of A, B, C, D in terms of M, the mean value of these four quantities. For

$$M = \tfrac{1}{4}(A + B + C + D) = A - \tfrac{1}{4}(X + Y + Z).$$

Thus

$$A = M + \tfrac{1}{4}(X + Y + Z) = M + \tfrac{1}{4}(\eta_1 + \eta_2 + \eta_3),$$

$$B = A - X, \qquad C = A - Y, \qquad D = A - Z.$$

§ 8. *Practical example.* The following results were obtained by G. F. C. Searle and A. L. Hughes, using four sub-standards each nominally of one ohm resistance.

The six direct determinations of differences gave the values

$$\Delta_1 = A - B = -282\times 10^{-6}, \qquad \Delta_2 = A - C = 512\times 10^{-6} \text{ ohms},$$
$$\Delta_3 = A - D = 458\times 10^{-6}, \qquad \Delta_4 = B - C = 843\times 10^{-6} \text{ ohms},$$
$$\Delta_5 = B - D = 732\times 10^{-6}, \qquad \Delta_6 = C - D = -65\times 10^{-6} \text{ ohms}.$$

Hence
$$\eta_1 = \Delta_1 - \Delta_4 - \Delta_5 = -1857\times 10^{-6} \text{ ohms},$$
$$\eta_2 = \Delta_2 + \Delta_4 - \Delta_6 = 1420\times 10^{-6} \text{ ohms},$$
$$\eta_3 = \Delta_3 + \Delta_5 + \Delta_6 = 1125\times 10^{-6} \text{ ohms}.$$

Then
$$X = \tfrac{1}{4}(2\eta_1 + \eta_2 + \eta_3) = -292\times 10^{-6} \text{ ohms},$$
$$Y = \tfrac{1}{4}(\eta_1 + 2\eta_2 + \eta_3) = 527\times 10^{-6} \text{ ohms},$$
$$Z = \tfrac{1}{4}(\eta_1 + \eta_2 + 2\eta_3) = 453\times 10^{-6} \text{ ohms}.$$

We can now find A, B, C, D in terms of M, the mean value of the four resistances. Thus

$$A = M + \tfrac{1}{4}(\eta_1 + \eta_2 + \eta_3) = M + 172\times 10^{-6} \text{ ohms},$$
$$B = A - X = M + 464\times 10^{-6} \text{ ohms},$$
$$C = A - Y = M - 355\times 10^{-6} \text{ ohms},$$
$$D = A - Z = M - 281\times 10^{-6} \text{ ohms}.$$

If we assume that M is accurately one ohm, we have the values

$$A = 1{\cdot}000172, \qquad B = 1{\cdot}000464, \qquad C = 0{\cdot}999645, \qquad D = 0{\cdot}999719 \text{ ohms}.$$

The discrepancies between the observed and the calculated differences are shown in the table. The differences are given in millionths of an ohm.

	$A-B$	$A-C$	$A-D$	$B-C$	$B-D$	$C-D$
Observed	−282	512	458	843	732	−65
Calculated	−292	527	453	819	745	−74

The greatest discrepancy only amounts to 24 millionths of an ohm.

§ 9. *Effect of finite resistance of connectors.* If we treat the copper connectors as linear conductors, we can easily modify the equations so as to take account of the resistances of the various parts of the connectors *. When the connectors are treated as

* Methods of dealing with non-linear conductors are given in my paper "On resistances with current and potential terminals," *The Electrician*, March 31, April 7, 14, 21, 1911. The paper is also published separately by *The Electrician*.

linear conductors the arrangement may be represented diagrammatically by Fig. 5. The resistances of the parts of the connectors are represented by p, q, r, s, h, k, as shown in Fig. 5, and these resistances are very small compared with A or B.

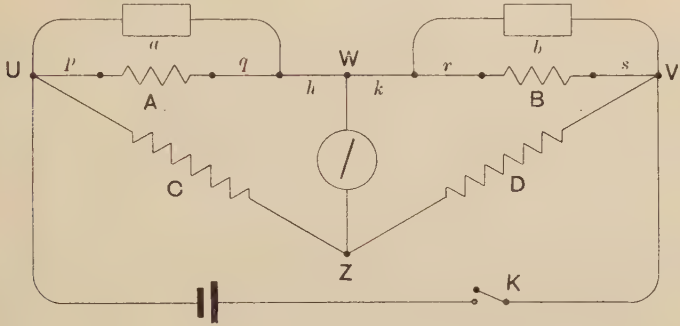


Fig. 5.

When A is in the left gap with a shunt a_1, while B is in the right gap with a shunt b_1, the resistances of the compound conductors between U and W and between V and W are

$$A_1' = h + \frac{(A+p+q)\,a_1}{A+p+q+a_1}, \qquad B_1' = k + \frac{(B+r+s)\,b_1}{B+r+s+b_1}.$$

When the coils A and B are interchanged and the new shunts are a_2 and b_2, the corresponding resistances are

$$A_2' = k + \frac{(A+r+s)\,a_2}{A+r+s+a_2}, \qquad B_2' = h + \frac{(B+p+q)\,b_2}{B+p+q+b_2}.$$

Since
$$\frac{A_1'}{C} = \frac{B_1'}{D} \quad \text{and} \quad \frac{A_2'}{D} = \frac{B_2'}{C}$$

we have
$$\sqrt{A_1'A_2'} = \sqrt{B_1'B_2'} \quad \text{.....................(8).}$$

On replacing the geometric means in (8) by the arithmetic means, the quantity $\frac{1}{2}(h+k)$ cancels and we have

$$\frac{1}{2}\left\{\frac{(A+p+q)\,a_1}{A+p+q+a_1} + \frac{(A+r+s)\,a_2}{A+r+s+a_2}\right\}$$
$$= \frac{1}{2}\left\{\frac{(B+r+s)\,b_1}{B+r+s+b_1} + \frac{(B+p+q)\,b_2}{B+p+q+b_2}\right\}$$

or
$$\frac{1}{2}\left\{\frac{A+p+q}{1+(A+p+q)/a_1} + \frac{A+r+s}{1+(A+r+s)/a_2}\right\}$$
$$= \frac{1}{2}\left\{\frac{B+r+s}{1+(B+r+s)/b_1} + \frac{B+p+q}{1+(B+p+q)/b_2}\right\}.$$

Expanding each of the denominators as far as the second term, we have

$$\tfrac{1}{2}\{A+p+q-(A+p+q)^2/a_1+A+r+s-(A+r+s)^2/a_2\}$$
$$=\tfrac{1}{2}\{B+r+s-(B+r+s)^2/b_1+B+p+q-(B+p+q)^2/b_2\}.$$

Since p, q, r, s are very small compared with A or B, we may neglect $2Ap/a_1$, $2pq/a_1$, p^2/a_1 and similar terms. We then obtain

$$A-\frac{A^2}{2}\left(\frac{1}{a_1}+\frac{1}{a_2}\right)=B-\frac{B^2}{2}\left(\frac{1}{b_1}+\frac{1}{b_2}\right).$$

Here we may write AB for A^2 and for B^2, and then we find

$$A-B=\frac{AB}{2}\left(\frac{1}{a_1}+\frac{1}{a_2}-\frac{1}{b_1}-\frac{1}{b_2}\right)$$

which is identical with equation (7) of § 2.

The Distribution of the Stars in relation to Spectral Type. By Professor A. S. EDDINGTON, Trinity College.

[*Read* 24 November 1913]

It is well known that the concentration of stars to the galactic plane is not shown equally by the different spectral classes. Type B is the most condensed, and the others follow in the order A, F, G, K, M, i.e. the sequence coincides with the usually accepted order of evolution. Formerly it seemed probable that this result was due to a progression in the average distance of these classes of stars, for, on the hypothesis that the stellar system is of oblate form, the greater the distance the greater will be the concentration to be expected. This explanation fitted in well with certain direct evidence as to the luminosities of the different spectral types. Recent determinations by Boss and Campbell of the average distances of the stars of different spectral types negative this explanation in a most decided manner. It appears, for instance, that the M stars are on the average more remote and more luminous than Type A. We have to return to the view that there is a real difference in the distribution of the spectral types. Apparently the stars have been formed mainly in the galactic plane; the earliest type with their small velocities have not strayed far from it; the latest type with their large velocities have had time to become much more uniformly dispersed.

There is an outstanding question of great difficulty. In parallax investigations it is found that the M stars are the *faintest* of all the types; in statistical discussions of proper motions, etc., they are found to be the *brightest* except Type B. Similar difficulties occur with the other types. Russell has put forward the theory that Type M consists of two divisions, one being the very earliest and the other the latest stage in evolution. Against this it may be urged that both divisions of Type M are characterised by very high velocities in space; this seems to indicate a close relation between them. Further, as far as statistical investigations are concerned, Russell's theory inverts the usually accepted order of stellar evolution. The result arrived at in the previous paragraph would thus have to be reversed,—the stars as formed are fairly uniformly dispersed and have high velocities; afterwards they lose their velocities and become concentrated to the galactic plane. This is not so intelligible as the previous conclusion.

PROCEEDINGS

OF THE

Cambridge Philosophical Society.

The oxygen content of the river Cam before and after receiving the Cambridge sewage effluent. By J. E. PURVIS, M.A., Corpus Christi College, and E. H. BLACK, M.B. (Edin.).

[*Read* 23 February 1914.]

The 8th Report (1912) of the Royal Commission on Sewage Disposal, Vol. I., deals with the standards to be applied to sewage and sewage effluents discharging into rivers and streams, and the tests which, in the opinion of the Commissioners, should be used in determining those standards. They discuss also the connection between the physical and the chemical conditions of streams receiving sewage liquids, the various tests which are now employed, such as the amount of ammoniacal nitrogen, the amount of oxygen absorbed from permanganate of potassium in four hours, and the amount of dissolved oxygen taken up in five days. Finally they selected the amount of dissolved oxygen which is consumed in five days at 18° C. as the basis of a standard.

It is obvious that there are local and seasonal variations in the conditions of rivers and streams receiving sewage or sewage effluents; but they conclude that if 100,000 c.c. of a river water do not normally take up more than 0·4 gram of dissolved oxygen in five days, the river will be free from signs of pollution; and that if the river gives a higher figure than this, it will show signs of pollution, except perhaps in very cold weather. They therefore decided that this figure ought not to be exceeded by the mixture of the rivers and the polluting streams discharging into them. The experiments were carried out at 65° F. (18·3° C.), for when the five days' test was carried out at the various temperatures of different seasons, varying results were obtained. They also adopted the normal dry weather flow of the river.

The quality of the river or stream as the receiver of sewage or an effluent is a very important factor to be considered in the disposal of sewage or sewage effluents; and the Commissioners have classified rivers into groups so that, for example:

Very clean	take up	0·1 gram	of dissolved oxygen in 100,000 parts of the stream in 5 days.
Clean	,,	0·2 ,,	of dissolved oxygen in 100,000 parts of the stream in 5 days.
Fairly clean	,,	0·3 ,,	of dissolved oxygen in 100,000 parts of the stream in 5 days.
Doubtful	,,	0·5 ,,	of dissolved oxygen in 100,000 parts of the stream in 5 days.
Bad	,,	1·0 ,,	of dissolved oxygen in 100,000 parts of the stream in 5 days.

They consider that, under ordinary conditions, the average quality of the diluting water, for the purpose of arriving at a standard, should be represented by 0·2, that is, by a "clean river."

But the most important factor is the degree of dilution afforded by a river receiving the discharge; because there are numerous instances in which the degree of dilution is sufficient to dispose of the sewage by natural agencies without cost or injury to the community.

Considering, however, the various methods of sewage treatment, as, for example, tank treatment with or without chemical precipitation, or artificially constructed filters or sewage farms, the Commissioners recommend that, in those cases where a complete system of sewage disposal is necessary, the sewage effluent shall not contain more than 3 grams of suspended matter per 100,000, and that, including its suspended matters, it should not take up more than 2 grams of dissolved oxygen in five days at 60° F. (18·3° C.).

The Report also discusses the cases where, owing to the relatively small volume of the river, a more stringent standard is necessary; and, on the other hand, conditions of dilution which indicate that a relaxation of the normal standard may be allowed. For example, a claim for a relaxed standard may be considered when (1) the particular river water when mixed with sewage or sewage effluent does not take up more than 0·4 gram of dissolved oxygen per 100,000 in five days, and (2) when it can be shown that the river will receive no further pollution until it has recovered itself so far as not to take up in five days an amount of dissolved oxygen much in excess of that which it took up before receiving the first discharge.

In view of this Report, the authors have studied the condition of the sewage effluent poured into the Cam from the

Sewage Farm on Milton Road, and also the Cam itself above and below the effluent outfall, to see how far the conditions of the river and the effluent are comparable with the standards suggested by the Commissioners.

A very extensive research would be necessary for a complete survey. It would mean a daily examination of the river and the effluent, and perhaps twice a day. But a fairly comprehensive view may be obtained by obtaining an analysis once a week, during the summer and winter months. The condition of the river can be investigated in the dry and wet seasons; and the dilution, as well as the varying pollution, should give a fair indication of the general conditions of the river and the effluent.

In connection with the condition of the Cam above and below the sewage effluent outfall, reference may be made to a paper by Purvis and Rayner*. In the investigation it was shown that the chemical purification, as distinct from the bacterial purification, was moderately good, as determined by the estimation of the two ammonias and the amount of oxygen absorbed from potassium permanganate in four hours. At two miles below the outfall, the river showed a definite amount of purification notwithstanding the fact that at $\frac{3}{4}$ of a mile below there was some contamination from another source. They also proved that above this contaminating influence, at half a mile below the outfall, the chemical purification was fairly good.

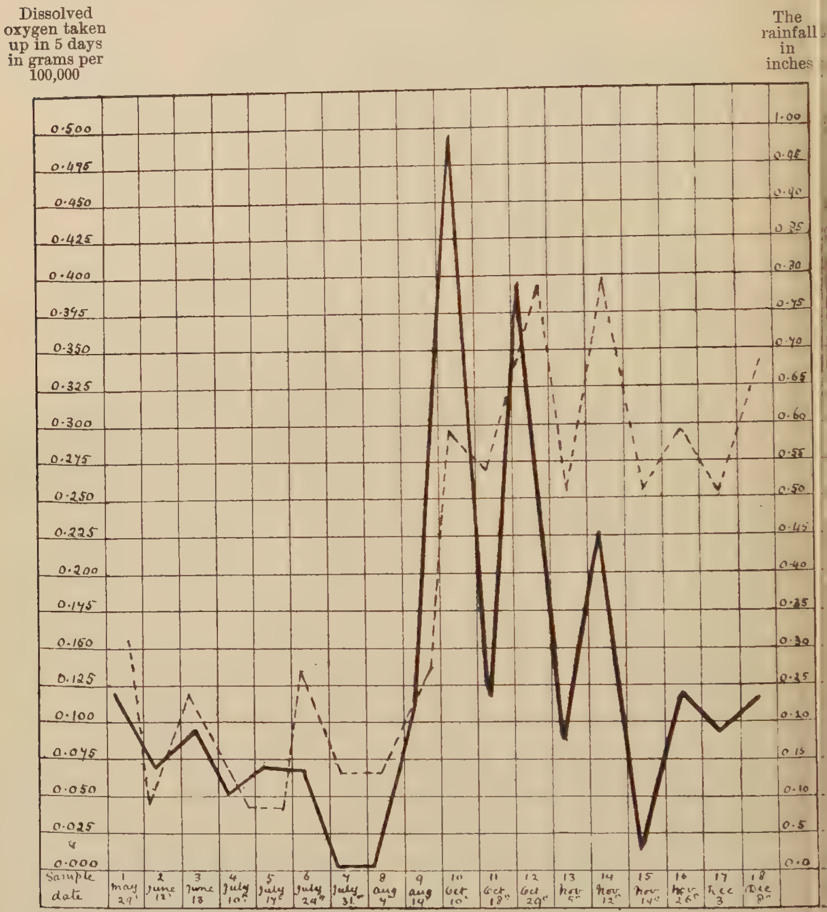
The method of analysis of the present investigation was that used by Letts and Blake†. The process is simple, and the details can be obtained in these publications. The results of the various determinations are given in the tables (pp. 363—368).

The more important facts which arise from a comparison of these analyses are the following: (*a*) The solids in suspension in the effluent were, on several occasions in the summer months, above the standard of 3 grams per 100,000; and on these occasions an offensive smell was noticed after five days' incubation. On four occasions during the winter months the suspended solids were also above the standard; but, on the other hand, there was no smell after five days' incubation. On examining these solids microscopically it was found that they chiefly consisted of zooglœa masses of a filamentous bacillus which had developed and had grown on the inside of the drain pipes and inspection chambers. They were not fæcal substances which produced an unpleasant smell like those noticed in the summer. The appearance of the

* *Journ. Roy. Sanitary Inst.*, 1913, vol. XXXIV. p. 479.

† *Proc. Roy. Dub. Soc.*, vol. IX. (N.S.) pt. iv. No. 33. Also the 5th Report, Royal Commission on Sewage Disposal, Appendix 6, p. 221, and the 8th Report, vol. II. Appendix.

masses of bacteria when floating or suspended in water resembled flakes of shredded paper or pieces of cotton wool in size from a three-penny piece to a two-shilling piece. They were constricted at one end, the point of attachment of the growth; the



– – – – dissolved oxygen absorbed in five days in grams per 100,000.
▬▬▬ rainfall in inches during the four days previous to that on which the sample was taken.

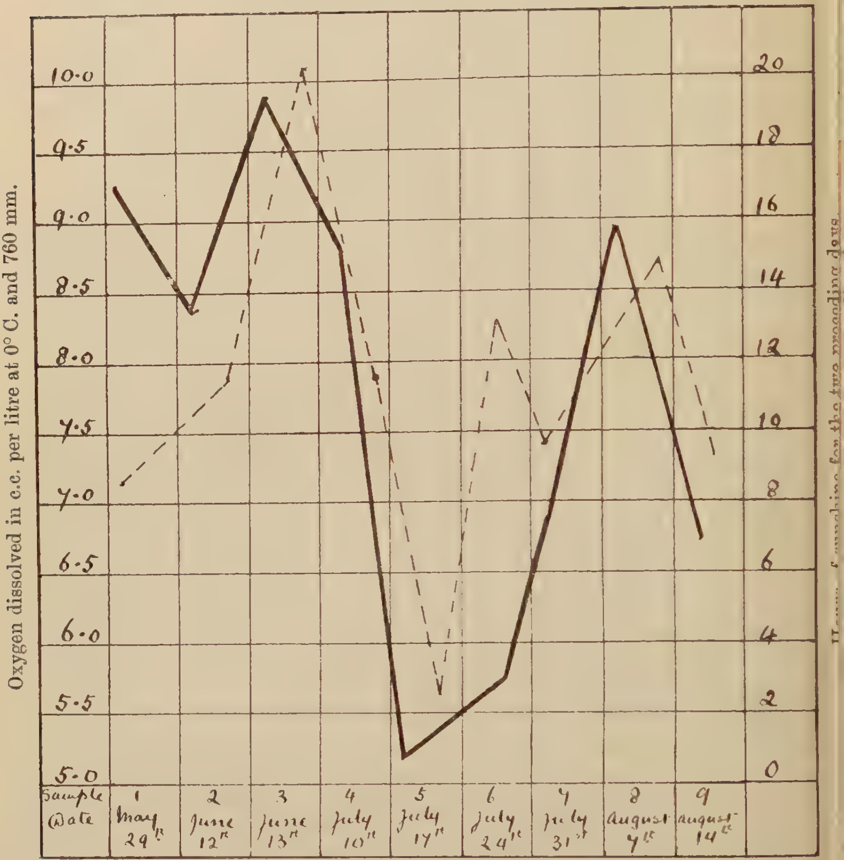
Fig. 1. Curves showing the degree of pollution as indicated by the rise and fall of the amount of oxygen absorbed in five days by the river Cam 150 feet above the effluent outfall, and its relation to the rainfall.

other end and body of the mass was spread out fan-wise, and when held up to the light resembled frost on a window-pane. Microscopically they consisted of long chains of rod-shaped

bacteria, resembling closely the anthracoid group. Each rod was attached to the next, and no free members were seen; they had the appearance of spore formation in the centre of each rod; they were more abundant in the effluent in cold weather. It is proposed to study them in more detail later. (*b*) The amount of oxygen absorbed by the river above the outfall was always above the standard of a clean river water (0·2 gram) in the winter months from October to December inclusive, whereas in the summer months, from May to August inclusive, it was always below the standard. It cannot be said, therefore, that the river itself always satisfies the standard of the Royal Commission. (*c*) Only on two occasions (August 14 and October 10) was the amount of oxygen taken up by the effluent below the standard of the Commissioners. (*d*) On the other hand, the purification which takes place when the effluent mixes with the river water is fairly rapid; and, although the mixture of the effluent and river at 50 feet below the outfall gives a figure which is sometimes above and sometimes below the standard of 0·4 gram of oxygen absorbed in five days at 18° C., yet the river $\frac{1}{4}$ mile down as regards its cleanliness compares fairly well with the standards of a diluting water suggested by the Commissioners. For example, five of the analyses would grade the river as "clean," ten as "fairly clean" and three as "doubtful" at $\frac{1}{4}$ mile below the effluent outfall.

Three factors at least may be suggested to explain the difference in the results of the summer and winter months. They are (1) the rainfall, (2) the number of hours of sunshine and their influence upon aquatic vegetation, and (3) the temperature. The curves in Fig. 1 show that as the rainfall increases, the amount of oxygen absorbed in five days also increases; and, it will be noticed, that this takes place at the beginning of the rainy season in October. An increase in the rainfall is accompanied by an increase in the pollution of the river, and a corresponding increase in the oxygen absorbed, and this is well shown by the two curves rising and falling together. The second factor is the presence of aquatic vegetation influenced by the number of hours of sunshine; and the curves of Fig. 2 illustrate the variation in the amount of dissolved oxygen which rises and falls with the number of hours of sunshine in the summer months. It is well known that aquatic plants give off more oxygen under the influence of the sun than in its absence, and this fact explains the increase in the dissolved oxygen. Such an explanation is confirmed by the curves of Fig. 3 for the winter months, where there is no regularity like that indicated by the curves in Fig. 2. Although there is a decrease in the number of hours of sunshine, and an almost entire absence of aquatic plants,

which had died or been removed, the increasing amount of dissolved oxygen observed from November 12 to December 8 (see Fig. 3) is explained by the diminution in the amount of pollution which follows the decrease of the rainfall between those

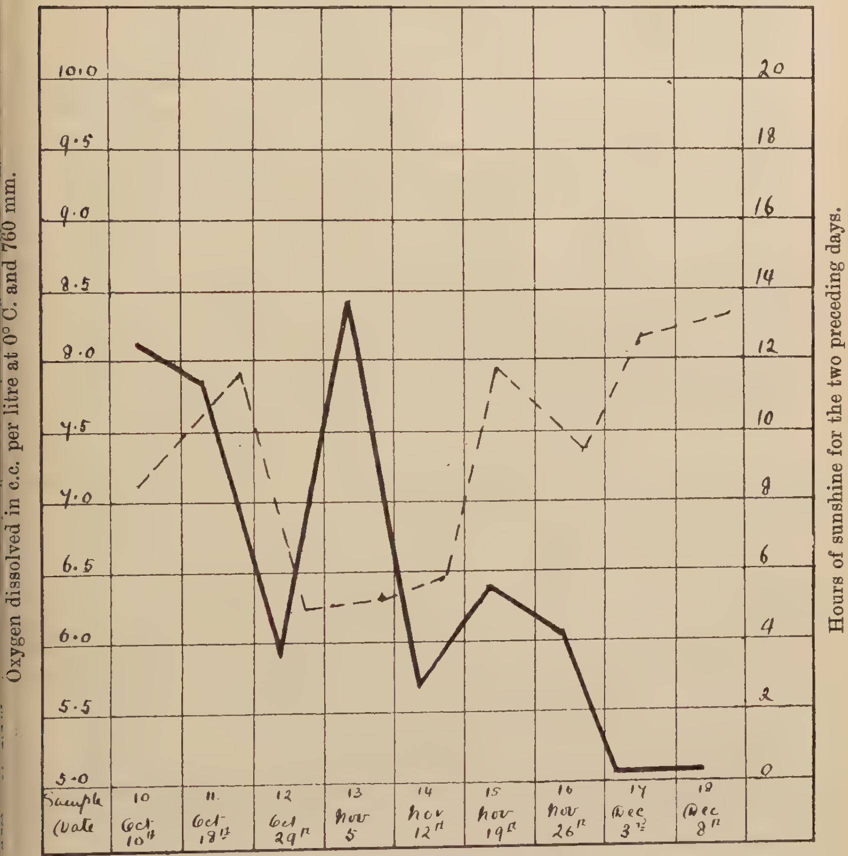


- - - - dissolved oxygen in c.c. per litre at 0° C. and 760 mm.
——— hours of sunshine for the two days preceding that on which the sample was taken.

Fig. 2. Curves showing the connection between the number of hours of sunshine and the amount of oxygen dissolved in the Cam in c.c. per litre, 150 feet above the effluent outfall.

two dates, together with the fall in the temperature of the water from 20° C. on May 29 to 5° C. on December 8. The determinations by Winkler of the number of c.c. of oxygen held in solution in one litre of water are quoted in the 8th Report of

the Royal Commission on Sewage Disposal, Vol. II. Appendix, p. iv. From this reference it will be seen that the temperature exhibits a controlling influence on the amount of oxygen dissolved. For example, at 5° C., 8·9 c.c. of oxygen are held by 1 litre of water,



- - - - dissolved oxygen in c.c. per litre at 0° C. and 760 mm.
——— hours of sunshine for the two days preceding that on which the sample was taken.

Fig. 3. Curves showing the relationship between the diminution in the pollution and the gradual increase in the amount of oxygen in the river between Nov. 12th to Dec. 8th, and the gradual decrease in the temperature of the water whereby more oxygen was dissolved from the atmosphere than in the warmer months. The fall of the oxygen figure on Oct. 29th was caused by an increased pollution of the river after rain.

and 6·4 c.c. at 20° C. The Commissioners consider that at about 16° C. a clean river water may be taken as containing 7 c.c. of oxygen per litre; but the authors found, at 16° C. (60° F.) and

767 mm., on June 13, for example, 10·086 c.c. of dissolved oxygen in the Cam water above the effluent outfall. On reference to Fig. 2 this corresponds to the maximum amount of sunshine for the two preceding days.

To sum up this investigation as it affects the disposal of the sewage effluent into the Cam, it is evident that (1) the seasonal variations had an important influence on the composition of the river as the receiver of the effluent, for the oxygen absorbed figure was below the standard in the summer months and above the standard in the winter months; (2) the suspended solids in the effluent were above the standard on several occasions, particularly in the summer months, when they were fæcal solids and not masses of bacteria; (3) the oxygen absorbed in five days by the effluent satisfied the standard of the Commissioners only twice in eighteen times; (4) with the increased pollution of the river as the diluting medium during the winter months, although the sewage effluent was of much better quality than that discharged into the stream during the summer months, the oxygen absorbed figures of the mixture of the polluting discharge and the river 50 feet below the effluent outfall exceeded the standard oxygen absorbed figure of 0·4, at the rate of 89·2 per cent. of the samples taken from October to December, as compared with 25 per cent. of those collected from May to August; (5) on the other hand, there is the important fact that the recovery or self-purification of the river is fairly rapid as shown by the figures for the oxygen absorbed at $\frac{1}{4}$ mile down the river below the effluent outfall. Such purification is brought about partly by the rapid absorption of the dissolved oxygen from the air and which is being continually replenished, partly by the oxygen given off by aquatic plants under the influence of sunlight, and to some extent by the nitrates, when present in the effluent, which are produced from the oxidation of the sewage as it passes through the filter beds; and these influences come into action as oxidisers of the dissolved organic matters. It should also be remembered that the river receives no further sewage pollution till it reaches Clayhithe, $1\frac{3}{4}$ miles below the effluent, where there is some contamination; but after that there is no pollution till the river reaches Stretham, $8\frac{1}{2}$ miles down.

However, accepting the effluents as fairly representative, and without considering the rapid purification when they are mixed with the river, an increase in the volume of the river so that the ratio of the effluent and the river should be about 1 to 20 in the summer, and about 1 to 25 in the winter, it is probable that the effluent discharged into the stream would satisfy the standards of the Royal Commission. It will be seen from the tables that the dilution of the effluent varied between 1 of

the effluent to 12 and 20 of the river water. It is obviously impossible to increase and maintain a larger volume of the river, even if it were desirable from other considerations; and, it cannot yet be decided whether it will be necessary to spend more money on a larger area of filter beds, until it is clearer that the additional settling tanks in course of construction and the beds now in use are giving their best results. For the present, it is more important to aim at the production of an effluent, which shall contain fewer suspended solids and consume less oxygen in five days. This can be accomplished by a more complete separation of the solids in the settling tanks, and the filtration of all the liquid through the beds. The various sections of the filter beds should always be kept in good condition by periodic scarifying, ploughing and resting, in order to break up the surface of the soil, and thoroughly ærate the subsoil and the gravelly sand below.

In connection with this investigation it is desirable to remember that the disposal and purification of the sewage effluent is concerned with "the harm caused by allowing unpurified, or "imperfectly purified, sewage to flow into streams, thereby causing "the de-æration of the water of the river, and consequent injury "to fish; the putrefaction of organic matter in the river to such "an extent as to cause nuisance; the production of sewage fungus "and other objectionable growths; the deposition of suspended "matter, and its accumulation in the river bed or behind weirs; "the discharge into the river of substances, in solution or "suspension, which are poisonous to fish or to live stock drinking "from the stream; the discoloration of the river; and the discharge into the river of micro-organisms of intestinal derivation, "some of which are of a kind liable, under certain circumstances, "to give rise to disease" (5th Report of the Royal Commission on Sewage Disposal, p. 217). It has not yet been very closely concerned with the bacterial purification as distinct from the chemical purification. Like all effluents, the effluent from the Cambridge sewage farm is polluted with all kinds of bacteria, and is therefore potentially dangerous. It has been shown, for example, by Purvis and Rayner (*loc. cit.*) that the *Bacillus coli*, an intestinal bacillus found in the sewage effluent, can be traced down the Cam for at least four miles below the effluent outfall. Whether it will be necessary to sterilise sewage effluents before they are discharged into streams is a question which has not yet received adequate attention; but if a river, which is the receiver of sewage pollutions, is the source of supply of water for drinking purposes, it should undergo an elaborate system of bacterial purification. The researches of Dr Houston, the Director of the Laboratories of the London Metropolitan Water Board, are of the greatest value in this direction.

Description of the Flora, etc., in the river Cam.

During the summer months, an abundance of aquatic vegetation grows and flourishes in the river Cam. A long ribbon-shaped weed, the *Sparganium* sp., grows most profusely everywhere across the whole bed of the river for miles above and below Cambridge. Where there are deposits of soft mud, especially below the sewage effluent outfall, *Zannichellia palustris* grows well, as does an imported weed *Elodea canadensis.* Confervæ are also abundant, and grow well on the soft mud below the effluent outfall.

From October to December the vegetation had almost entirely disappeared or it had been removed from the river bed. On several occasions shoals of live fish were seen both immediately above and below the outfall; and at no time from May to December were any dead fish seen below the effluent outfall.

Mud Deposits from the Sewage Effluent in the river Cam.

Deposits of black mud were found in patches below the effluent outfall. On December 8, before collecting the sample of river water at $\frac{1}{4}$ mile below the outfall, two barges passing along the river stirred up a quantity of filamentous bacteria and sludge from the bottom. It could be traced along the whole $\frac{1}{4}$ mile below the outfall. The total solids in the sample taken at $\frac{1}{4}$ mile below were estimated to be 9·4 grams per 100,000; and at $\frac{1}{8}$ mile below the cord to which the thermometer was attached in midstream became coated with a gelatinous deposit of masses of the same bacteria which had apparently been stirred up by the passing barges. Similar filamentous bacteria have been shortly described above as having been found in the effluent itself.

We are indebted to Mr Lynch, the Curator of the Cambridge Botanic Gardens, for identifying the aquatic flora, and for supplying the meteorological data.

Date 1913	Hour of collecting	Location of the sample	Temperature of the water °C.	Barometric pressure in mm.	Dilution	Feet below the surface	Total suspended solids in grams per 100,000	Oxygen dissolved in c.c. per litre at 0° C. and 760 mm. At the commencement	Oxygen dissolved in c.c. per litre at 0° C. and 760 mm. After five days' incubation at 18° C.	Oxygen absorbed in five days in grams per 100,000	Putrefaction on incubation at 18° C.
May 29	1 p.m.	River 150 feet above the effluent outfall	20	760	...	1·5	...	7·183	6·208	0·142	...
,,	,,	Sewage effluent	20	,,	...	...	3·2	0·807	nil	9·903	H_2S present
,,	,,	Mixture of effluent and river water 50 feet below effluent outfall	20	,,	1 to 15	1·5	...	5·557	2·822	0·392	...
,,	,,	River $\frac{1}{4}$ mile below the effluent outfall	20	,,	...	1·5	...	7·057	5·180	0·338	...
June 12	10.30 a.m.	River 150 feet above the effluent outfall	15	764	...	1·5	...	7·776	7·479	0·041	...
,,	,,	Sewage effluent	15	,,	...	...	2·9	0·230	nil	6·451	H_2S present
,,	,,	Mixture of effluent and river water 50 feet below effluent outfall	15	,,	1 to 16	1·5	...	7·559	3·465	0·586	...
,,	,,	River $\frac{1}{4}$ mile below the effluent outfall	15	,,	...	1·5	...	8·651	4·934	0·561	...
June 13	10 a.m.	River 150 feet above the effluent outfall	16	767	...	1·5	...	10·086	9·268	0·117	...
,,	,,	Sewage effluent	16	,,	...	...	2·8	2·727	nil	4·300	nil
,,	,,	Mixture of effluent and river water 50 feet below effluent outfall	16	,,	1 to 16	1·5	...	9·796	7·912	0·270	...
,,	,,	River $\frac{1}{4}$ mile below the effluent outfall	16	,,	...	1·5	..	10·057	7·437	0·375	...

Date 1913	Hour of collecting	Location of the sample	Temperature of the water ° C.	Barometric pressure in mm.	Dilution	Feet below the surface	Total suspended solids in grams per 100,000	Oxygen dissolved in c.c. per litre at 0° C. and 760 mm. At the commencement	Oxygen dissolved in c.c. per litre at 0° C. and 760 mm. After five days' incubation at 18° C.	Oxygen absorbed in five days in grams per 100,000	Putrefaction on incubation at 18° C.
July 10	10 a.m.	River 150 feet above the effluent outfall	16	758	...	1·5	...	7·793	7·496	0·042	...
"	"	Sewage effluent	...	Sample Bottle lost in sewer				...	...	...	...
"	"	Mixture of effluent and river water 50 feet below effluent outfall	16	758	1 to 15	1·5	...	7·311	5·849	0·209	...
"	"	River $\frac{1}{4}$ mile below the effluent outfall	16	"	...	1·5	...	8·542	7·487	0·151	...
July 17	10 a.m.	River 150 feet above the effluent outfall	17	762	...	1·5	...	5·693	5·397	0·041	...
"	"	Sewage effluent	17	"	...	...	3·0	1·098	nil	7·066	nil
"	"	Mixture of effluent and river water 50 feet below effluent outfall	17	"	1 to 15	1·5	...	5·556	2·942	0·377	...
"	"	River $\frac{1}{4}$ mile below the effluent outfall	17	"	...	1·5	...	6·818	4·428	0·342	...
July 24	1 p.m.	River 150 feet above the effluent outfall	15	766	...	1·5	...	8·364	7·474	0·127	...
"	"	Sewage effluent	15	"	...	...	3·3	1·386	nil	4·547	H_2S present
"	"	Mixture of effluent and river water 50 feet below effluent outfall	15	"	1 to 17	1·5	...	8·188	(spoilt 7·018	spoilt) 0·167	...
"	"	River $\frac{1}{4}$ mile below the effluent outfall	15	"	...	1·5	...	8·949	6·553	0·342	...

Date 1913	Hour of collecting	Location of the sample	Temperature of the water ° C.	Barometric pressure in mm.	Dilution	Feet below the surface	Total suspended solids in grams per 100,000	c.c. per litre at 0° C. and 760 mm.		Oxygen absorbed in five days in grams per 100,000	Putrefaction on incubation at 18° C.
								At the commencement	After five days' incubation at 18° C.		
July 31	10.30 a.m.	River 150 feet above the effluent outfall	17	762	...	1·5	...	7·479	6·886	0·085	...
,,	,,	Sewage effluent	17	,,	...	...	1·7	1·959	nil	2·919	nil
,,	,,	Mixture of effluent and river water 50 feet below effluent outfall	17	,,	1 to 12	1·5	...	7·019	5·557	0·209	...
,,	,,	River $\frac{1}{4}$ mile below the effluent outfall	17	,,	...	1·5	...	8·120	6·792	0·190	...
Aug. 7	12 m.	River 150 feet above the effluent outfall	16	762	...	3	...	8·661	8·067	0·085	...
,,	,,	Sewage effluent	16	,,	...	...	6·2	2·251	nil	5·220	H_2S present
,,	,,	Mixture of effluent and river water 50 feet below effluent outfall	16	,,	1 to 14	3	...	7·311	0·876	0·922	...
,,	,,	River $\frac{1}{4}$ mile below the effluent outfall	16	,,	...	3	...	8·942	6·818	0·304	...
Aug. 14	10.30 a.m.	River 150 feet above the effluent outfall	17	763	...	3	...	7·479	6·589	0·127	...
,,	,,	Sewage effluent	17	,,	...	...	2·5	1·959	nil	0·617	nil
,,	,,	Mixture of effluent and river water 50 feet below effluent outfall	17	,,	1 to 17	3	...	6·727	4·972	0·251	...
,,	,,	River $\frac{1}{4}$ mile below the effluent outfall	17	,,	...	3	...	8·385	7·323	0·152	...

Date 1913	Hour of collecting	Location of the sample	Temperature of the water °C.	Barometric pressure in mm.	Dilution	Feet below the surface	Total suspended solids in grams per 100,000	Oxygen dissolved in c.c. per litre at 0° C. and 760 mm.		Oxygen absorbed in five days in grams per 100,000	Putrefaction on incubation at 18° C.
								At the commencement	After five days' incubation at 18° C.		
Oct. 10	12 m.	River 150 feet above the effluent outfall	12	764	...	3	...	7·179	5·101	0·297	...
,,	,,	Sewage effluent	12	,,	...	...	10·0	1·389	nil	1·804	nil
,,	,,	Mixture of effluent and river water 50 feet below effluent outfall	12	,,	1 to 17	3	...	6·142	3·802	0·335	...
,,	,,	River ¼ mile below the effluent outfall	12	,,	...	3	...	7·379	5·743	0·234	...
Oct. 18	12 m.	River 150 feet above the effluent outfall	12	764	...	3	...	7·852	5·991	0·266	...
,,	,,	Sewage effluent	12	,,	...	...	4·0	3·799	nil	5·680	nil
,,	,,	Mixture of effluent and river water 50 feet below effluent outfall	12	,,	1 to 17	3	...	8·515	5·264	0·466	...
,,	,,	River ¼ mile below the effluent outfall	12	,,	...	3	...	8·306	7·106	0·172	...
Oct. 29	10 a.m.	River 150 feet above the effluent outfall	12	744	...	3	...	6·292	3·649	0·378	...
,,	,,	Sewage effluent	12	,,	...	...	1·0	2·824	nil	3·001	nil
,,	,,	Mixture of effluent and river water 50 feet below effluent outfall	12	,,	1 to 17	3	...	5·263	1·769	0·500	...
,,	,,	River ¼ mile below the	12	,,	...	3	...	6·813	4·042	0·397	...

Date 1913	Hour of collecting	Location of the sample	Temperature of the water ° C.	Barometric pressure in mm.	Dilution	Feet below the surface	Total suspended solids in grams per 100,000	c.c. per litre at 0° C. and 760 mm. — At the commencement	After five days' incubation at 18° C.	Oxygen absorbed in five days in grams per 100,000	Putrefaction on incubation at 18° C.
Nov. 5	12 m.	River 150 feet above the effluent outfall	10	752	...	3	...	6·300	4·539	0·252	...
,,	,,	Sewage effluent	10	,,	...	...	6·1	2·238	nil	3·045	nil
,,	,,	Mixture of effluent and river water 50 feet below effluent outfall	10	,,	1 to 17	3	...	4·468	1·184	0·470	...
,,	,,	River $\frac{1}{4}$ mile below the effluent outfall	10	,,	...	3	...	7·046	4·315	0·391	...
Nov. 12	11 a.m.	River 150 feet above the effluent outfall	9	744	...	3	...	6·913	4·231	0·384	...
,,	,,	Sewage effluent	9	,,	...	...	1·3	1·961	nil	2·484	nil
,,	,,	Mixture of effluent and river water 50 feet below effluent outfall	9	,,	1 to 17	3	...	5·559	nil	0·796	...
,,	,,	River $\frac{1}{4}$ mile below the effluent outfall	9	,,	...	3	...	6·496	2·371	0·591	...
Nov. 19	10.30 a.m.	River 150 feet above the effluent outfall	8	771	...	3	...	7·776	5·995	0·255	...
,,	,,	Sewage effluent	8	,,	...	...	2·6	2·824	nil	2·752	nil
,,	,,	Mixture of effluent and river water 50 feet below effluent outfall	8	,,	1 to 15	3	...	6·705	1·022	0·814	...
,,	,,	River $\frac{1}{4}$ mile below the effluent outfall	8	,,	...	3	...	8·176	5·723	0·351	...

Date 1913	Hour of collecting	Location of the sample	Temperature of the water °C.	Barometric pressure in mm.	Dilution	Feet below the surface	Total suspended solids in grams per 100,000	Oxygen dissolved in c.c. per litre at 0° C. and 760 mm.		Oxygen absorbed in five days in grams per 100,000	Putrefaction on incubation at 18° C.
								At the commencement	After five days' incubation at 18° C.		
Nov. 26	11 a.m.	River 150 feet above the effluent outfall	8	769	...	3	...	7·479	5·406	0·297	...
,,	,,	Sewage effluent	8	,,	...	...	0·292	3·112	nil	2·592	nil
,,	,,	Mixture of effluent and river water 50 feet below effluent outfall	8	,,	1 to 20	3	...	7·842	3·002	0·693	...
,,	,,	River ¼ mile below the effluent outfall	8	,,	...	3	...	8·722	6·248	0·354	...
Dec. 3	11.30 a.m.	River 150 feet above the effluent outfall	9	755	...	3	...	8·073	6·297	0·254	...
,,	,,	Sewage effluent	9	,,	...	...	7·0	3·112	nil	2·873	nil
,,	,,	Mixture of effluent and river water 50 feet below effluent outfall	9	,,	1 to 17	3	...	7·544	4·422	0·447	...
,,	,,	River ¼ mile below the effluent outfall	9	,,	...	...	...	8·722	5·702	0·432	...
Dec. 8	11.30 a.m.	River 150 feet above the effluent outfall	5	767	...	3	...	8·373	5·995	0·340	...
,,	,,	Sewage effluent	5	,,	...	...	2·1	2·531	nil	2·334	nil
,,	,,	Mixture of effluent and river water 50 feet below effluent outfall	5	,,	1 to 15	3	...	7·550	3·579	0·596	...
,,	,,	River ¼ mile below the	5	,,	...	3	9·4	8·293	4·359	0·563	...

On Root Development in Stratiotes aloides *L. with special reference to the occurrence of Amitosis in an embryonic tissue.*

By Agnes Arber, D.Sc. (Lond.), F.L.S., formerly Fellow of Newnham College. (Communicated by Dr Arber.)

(Plates VIII and IX.)

[*Read* 9 February 1914.]

I. *Introduction.*

On August 11th, 1910, I collected a few plants of the Water Soldier, *Stratiotes aloides* L., at Roslyn Pits, Ely, with the intention of examining them in connexion with a general study of water plants, on which I have been engaged for some time. On cutting sections of the stems, I noticed certain peculiarities in the young adventitious roots embedded in the tissues of the axis, the chief of which was the occurrence of apparent amitosis. These plants had merely been preserved in methylated spirit, with no idea of using them for cytological purposes, but in the following year (May 30th, 1911) a number of further examples from Roslyn Pits were dissected and fixed on the spot in Flemming's strong solution, acetic-alcohol, and methylated spirit. In 1912, for the sake of having a control from some other locality, two plants were obtained from Perry's Hardy Plant Farm, Enfield. Hand sections were stained with methyl green in 1 °/₀ acetic acid, Ehrlich's acid haematoxylin diluted with an equal volume of potash water, borax carmine, etc., while for microtome sections, Flemming's triple stain and Heidenhain's iron-alum haematoxylin were used. The hand sections, especially those stained with methyl green and mounted in dilute glycerine, were found on the whole to be the most favourable for the particular purpose.

I have pleasure in expressing my indebtedness to the Committee of the Balfour Laboratory, where this work has been carried out, and also to my friend, Mr E. Aveling Green, who kept records for me of the rate of growth of the roots in the case of some plants of the Water Soldier cultivated in a pond in his garden.

Before describing the observations on the nuclei which form the main subject of the present paper, I wish to draw attention to a few points concerning the general structure and development of the adventitious roots of *Stratiotes aloides*.

II. *The Structure and Development of the Adventitious Roots of* Stratiotes aloides *L.*

The rosette of aloe-like leaves, which characterises the Water Soldier, arises from an abbreviated stem, which is represented in radial longitudinal section in Pl. VIII, Fig. 1. A series of adventitious roots is shown, becoming progressively younger towards the stem apex. They arise at the outer limit of the central vascular region of the axis.

Van Tieghem and Douliot* describe the young root as enclosed externally in a digestive sac arising from the stem endodermis, which is followed internally by a root-cap derived from the pericycle. I cannot, however, confirm this description, as it appears to me quite impossible to demonstrate that the digestive sac is cortical and the root-cap stelar in origin, since no distinct endodermis and pericycle can be seen in the stem, and there is also no visible distinction between root-cap and digestive sac in the root. My observations agree with those of Miss D. G. Scott†, who also failed to distinguish a root-cap and digestive sac, and who reports that the endodermis of the stem, if present, could not be determined. In the present paper I shall use the term "root-cap" for the *entire covering* of the root apex (the outer part of which functions as a "digestive sac" in passing through the stem), without regard to the distinction drawn by Van Tieghem and Douliot.

The piliferous layer is marked out extremely early. In the youngest roots, which have not yet emerged from the stem tissues, it is visible as a columnar layer rich in contents and with large nuclei. The same is true of the apical region of the long roots. Near the root-tip, while it is still enclosed in the root-cap the future root-hair cells are already marked out by their very large size and relatively gigantic nuclei (Pl. VIII, Fig. 3). Before these cells begin to protrude outwards to form hairs they become considerably enlarged on the inner side, displacing the cells of the layer internal to them. The cells of this layer divide more frequently than the rest of the cortex, with the result that the base of the root-hair cell becomes enclosed in what may be described as a jacket of small cells (*j*, Pl. VIII, Figs. 3 and 4).

In the mature root the cortex is sharply separated into two regions, an inner region in which the cells are radially arranged and an outer region in which the cells, which are larger, are

* Van Tieghem, Ph. and Douliot, H., "Recherches comparatives sur l'origine des membres endogènes dans les plantes vasculaires," *Ann. des Sci. nat.*, 7 sér. Bot. T. 8, pp. 337, 338 and Pl. 36, Figs. 557—560, 1888.

† Scott, D. G., "The Apical Meristems of the Roots of Certain Aquatic Monocotyledons," *New Phyt.*, vol. v. p. 119, 1906.

irregularly placed. The inner region is differentiated into a lacunar part which may be called the middle cortex, and a compact part to which the name of inner cortex may be confined. The lacunar zone is characterised by large air spaces separated by radial plates of cells, as a rule only one cell wide in the tangential direction. These radial plates are continuous with the radial files of cells which make up the inner cortex. The origin of the lacunæ is of some interest. The whole inner region of the cortex must be visualised as consisting of radially arranged plates, one cell wide, which in the early stages are so placed as to leave no spaces between. The cells composing the plates divide very rapidly, and a number of new cell-walls are formed, all in planes at right angles to the long axis of the root. The result is that each plate elongates in the direction of growth of the root, but, owing to the rapidity of its cell-divisions, the plates grow in length faster than the rest of the root, and are thus forced into undulations, since they become too long to retain their normal vertical position. The possibility of their taking up this sinuous form is due to the fact that the root enlarges in diameter and thus allows room for the separation of the plates. It will readily be seen that a series of plates, side by side, elongating independently, and at the same time prevented from stretching to their full length, will naturally become detached from one another at certain points, leaving spaces between. The result of these processes is that the middle cortex, as seen in transverse section, consists of radial plates of cells, like the spokes of a wheel, separated by lacunæ, whereas in tangential section the plates are found to meet their neighbours at intervals so as to form a network (Pl. VIII, Fig. 2).

The structure of the mature root of *Stratiotes aloides* has been described by Van Tieghem and Douliot*. These authors must have selected an unusually small specimen for study, for they describe the root as pentarch, whereas I have found as many as eight protoxylem elements and about eight metaxylem vessels alternating with eight phloem groups, each consisting of one to three sieve-tubes with their accompanying companion cells. Lignification is extremely slight; the metaxylem elements bear delicate scalariform thickenings.

In young roots the embryonic vessels are represented by files of large elements with correspondingly large nuclei (Pl. IX, Fig. 6). The young sieve-tubes are narrower in lumen than the young vessels. They consist, at an early stage, of segmented tubes, poor in contents and without nuclei. The segments are shorter than in the case of the young vessels, and the partition walls are horizontal instead of oblique. The accompanying cells are not typical companion cells, since they are much shorter than the

* *l.c.* p. 337.

adjacent segments of the sieve-tube. Their horizontal walls bear no relation to those of the sieve-tube, showing that they have not been derived from the same mother-cell.

Like those of so many water plants, the roots of *Stratiotes* in their young stages are green in colour. The chlorophyll grains occur chiefly in the cortex, especially in its inner region, and also very richly in the root-tip, both in the root-cap and internal tissues. Starch occurs abundantly in the inner cortex. A small quantity also occurs in the central cylinder, especially in the developing vessels and sieve-tubes.

III. *The Nuclei.*

(i) *Observations.*

The young adventitious roots of *Stratiotes aloides*, while still enclosed in the stem tissue or just emerging from it, show two very marked cytological peculiarities—firstly, that the cells, especially those of the root-cap and cortex, not infrequently contain more than one nucleus, and secondly, that the nuclei themselves, both in the cortex and the stele, are often bilobed. Multinucleate cells also sometimes occur in the adjacent tissues of the parent stem. Pl. VIII, Fig. 5 shows one of the most extreme cases I have observed, as regards number of nuclei. Here the outer cells of the root-cap of a young root, and also certain cells of the stem cortex through which it was dissolving its way, are characterised by numerous nuclei, one cell of the root-cap containing at least 12. This case is however of minor interest, since the tissues in question may well be held to be in a decadent condition but in the examples figured in Plate IX, the cells concerned belong to the normal tissues of the leaf, root-cortex and root-stele, which are still undergoing development. In Pl. IX, Fig. 7 *a*, cells belonging to the root cortex and containing more than one nucleus are shown, while Pl. IX, Figs. 6 and 9 *a*—*e* represent lobed nuclei and binucleate cells occurring in the xylem parenchyma and other tissues of the central cylinder. Lobed nuclei are notably frequent in the cells immediately surrounding the vessels; in Pl. IX, Fig. 6 three cases will be seen in which these xylem parenchyma elements were binucleate, while in one of these cells (x) each member of the pair of nuclei was itself bilobed. Lobed nuclei and cells with more than one nucleus are not confined to the root and the adjacent stem tissue, but are also to be found, though comparatively rarely, in the meristematic apical region of the stem and in the leaf (Pl. IX Figs. 8 *a* and *b*). In the latter organ they occur more frequently towards the base, where growth is presumably taking place, than in the upper part where the tissues are mature. It is, however only in the young root that these nuclear peculiarities become a really conspicuous feature.

Various instances of lobed nuclei have been described in the higher plants, especially among the Monocotyledons, but the case of *Stratiotes* differs from all those previously recorded in two points—firstly, that *the lobing is of a markedly regular and uniform type*, and secondly, that *it occurs, not only in the root-cap, which may well be regarded as a somewhat abnormal tissue, but also in the developing cortex and stele.*

The appearance of a lobed nucleus will be more clearly understood by reference to Pl. IX, Figs. 6—9, than from description. It is better shown in Pl. IX, Figs. 7—9, which were drawn from hand sections, than in Pl. IX, Fig. 6, for which microtome sections were employed. In studying amitosis it is important to view the nucleus as a whole, but in microtome sections the knife is apt to mutilate it, and there is also more danger of distortion, owing to the necessary preliminary treatment and the heating in the paraffin. The result is that hand sections, though so little used in general cytology, become of special value in this particular case. A comparative study of the lobed nuclei, as seen in microtome and hand sections, shows that in almost every instance there is originally an indentation on one side only, the nucleus retaining its convex form on the opposite side and presenting the general appearance of a so-called "resting" nucleus (Pl. IX, Fig. 9 *e*). The two lobes appear, at early stages, to be unequal in size, the one which contains the nucleolus being the larger. At later stages the two lobes seem to become equalised, and the nucleus ultimately has the appearance of being almost bisected. There is generally a nucleolus in each lobe, due possibly to the division of the original single one, while sometimes a third occurs in the median plane (cf. the cell marked x in Pl. IX, Fig. 9 a). The nuclei in the cells marked y, y', x and z in Pl. IX, Fig. 6 show different stages in the lobing of the nucleus, and similar stages can be followed in Pl. IX, Fig. 9. Occasionally two nuclei are seen lying closely side by side as if one of the lobed nuclei had just separated completely into two (Pl. IX, Fig. 9 c).

(ii) *Interpretation.*

The first questions which arise, in considering the observations recorded above, are whether the phenomenon which I have described as "lobing" of the nucleus is natural or artificially induced, and, if it is natural, whether it is normal or pathological. I think we may conclude that it is natural, since I have observed it in material fixed in methylated spirit, acetic alcohol, and Flemming's strong solution. It has however been suggested to me that it may be an abnormality, possibly due to the poisoning of the roots during life by some substance present on the water, such as marsh gas. This is, of course, conceivable, but it seems to me unlikely. It is

true that Miss Kemp* has shown that nuclei of abnormal shape may be produced experimentally by poisoning young roots, but the results are far from being so uniform and regular as those just described for *Stratiotes.* The plants in which I have observed the lobing of the nuclei were obtained from two different localities in three different years; it is scarcely likely that identical toxic effects would occur independently in three sets of material, which, in each case, appeared to be quite healthy. It should also be remembered that the young adventitious roots, in which the multinucleate cells and lobed nuclei were observed, were still more or less completely embedded in the stem tissues of the parent, and thus presumably protected to some extent from adverse external conditions.

The presence, in the young roots of *Stratiotes*, of nuclei bilobed in various degrees, and also of certain cells containing more than one nucleus, seems to indicate that the lobing culminates in complete bisection. I believe that this is the case, and that amitosis takes place; I am inclined to go further and to think that these amitoses may be followed directly, or after an interval, by cell-wall formation, and that *amitosis thus actually plays a part, supplementary to karyokinesis, in the development of the embryonic root of* Stratiotes†. It is naturally almost impossible to prove that cell-walls are formed in connexion with these direct nuclear divisions, but I have more than once seen appearances decidedly suggestive of such an occurrence. The fact also that, in the young roots, bilobed nuclei are much more frequent than multinucleate cells, and, again, that the mature roots are not characterised either by bilobed nuclei, or by a number of multinucleate cells corresponding with the numerous bilobed nuclei seen in the younger stages, is difficult to explain unless wall formation has occurred between daughter nuclei formed by direct division, for there is no evidence that any nuclei are resorbed.

It is, in the nature of the case, very difficult, if not impossible, to offer a convincing proof of the contention that amitosis plays an active part in the growth of the young roots of *Stratiotes*, and I have hence allowed more than four years to elapse since I made my first observations on the subject, as I felt reluctant to put forward such a heretical opinion in any haste. I am aware that cytologists may prefer to regard the occurrence of these lobed nuclei as a mere meaningless anomaly. However, each time that

* Kemp, H. P., "On the Question of the Occurrence of 'Heterotypical Reduction' in Somatic Cells," *Ann. Bot.*, vol. XXIV. p. 775, 1910.

† I have observed a nucleus dividing by karyokinesis in a section of a root-stele in which lobed nuclei also occurred.

I have returned to the subject, my original impressions have been strengthened, and I think it is perhaps now advisable to publish a preliminary account of my conclusions in the hope that they may receive confirmation or correction from other workers.

It has been suggested to me that, even if the facts are as I suppose, it is not necessary to regard this form of nuclear division as genuine amitosis, but that it may be interpreted as a masked form of karyokinesis, due to incomplete separation of the chromosomes. I think this view is somewhat strained, and would be difficult to accept in any case, but in regard to *Stratiotes* it is certainly untenable. It could only hold good if the chromosomes were few and large, whereas in this plant they are small and numerous. This point can readily be observed in the case of the nuclei dividing by normal karyokinesis, which are frequently to be found in the root-tips.

The remarkable difference in size between the ordinary vegetative nuclei and those of the young root-hair cells and vessels (cf. Pl. VIII, Figs. 3 and 4, and Pl. IX, Fig. 6) suggests that the nuclei of *Stratiotes aloides* are unusually plastic,—thus partaking in the general plasticity which is so marked a feature of water plants, and which has probably been a primary factor in determining the possibility of any particular group or species adopting the aquatic habit. Assuming that amitosis does actually occur in the young roots, we may, I think, interpret it as a special adaptation to the unusual requirements of the species. It is well known that the young plants of the Water Soldier, produced at the ends of stolons arising from the parent rosette, pass the winter at the bottom of the water and rise to the surface in the spring or early summer. Roots are not needed so long as the plant is submerged, but, when it rises to begin its floating phase, a quantity of remarkably long roots are produced with great rapidity. The use of these very long roots is probably to maintain the equilibrium of the rosette. I noticed, in the case of two plants which I cultivated, that the loss of their roots, through the depredations of water-snails, deprived them of all power of keeping upright in the water, so that they were generally to be found floating on their sides. The young plant rises to the surface in the form of a rosette, not, as in the case of the related *Hydrocharis*, in the form of a compact winter-bud; being, as it were, full-fledged, it requires its roots at once. That the growth of the roots is unusually rapid is proved by some measurements which Mr Aveling Green has kindly taken for me. He kept records, during part of July and August 1911, of the growth of eleven roots belonging to three plants of *Stratiotes aloides* cultivated in a pond in his garden, and several times observed an increase of over 2 inches in 24 hours; on one occasion, even $2\frac{7}{8}$ inches was

required. The tentative suggestion which I wish to bring forward is that amitosis has been adopted in the young roots of *Stratiotes* as a means of very rapid nuclear multiplication*, which supplements karyokinesis and thus renders possible a period of extremely rapid growth. It may, further, in this case be associated with the somewhat peculiar conditions under which the young roots develope. Owing to the corm-like form of the short main axis of the plant, the adventitious roots arise at some little depth from the surface and have to force their way for an appreciable distance through solid cortical tissue (Pl. VIII. Fig. 1), which must act as a temporary check upon their expansion. Luxuriant growth-activity is a well known characteristic of water plants, but under the confined circumstances in which the adventitious roots of *Stratiotes aloides* are initiated, this energy of development does not seem to find an adequate outlet in actual increase in size. It is perhaps conceivable that it may be temporarily diverted into other channels, and find its expression in amitosis.

IV. *On the Significance of Amitosis.*

Amitosis or direct nuclear division, seems to be generally regarded at the present time as a degeneration process, at least where it occurs among the higher plants. This, which we may describe as the orthodox view, has been championed by Strasburger†, who held that karyokinesis and "fragmentation" were two entirely different processes, the former taking place under the influence of the surrounding protoplasm, and the latter occurring when the influence of the protoplasm was on the wane, so that the nucleus "seinen eigenen Gestaltungstrieben folgen kann‡." He stated that he knew no case in which cell division followed amitosis, which he regarded as a phenomenon of senility. The same view has been taken by Zimmermann§ and other writers. Johow‖, on the contrary, who was the first to point out that amitosis is a wide-spread phenomenon among Monocotyledons, protests against the use of the word "fragmentation" on account of its pathological implication. It was Johow who drew attention to the pith cells of *Tradescantia* which are now so widely used for teaching purposes to illustrate amitotic nuclear division, and he

* Cf. Shibata's work on amitosis in mycorhizal tubercles, referred to in the next section of the present paper.

† Strasburger, E., "Einige Bemerkungen über vielkernige Zellen und über die Embryogenie von Lupinus," *Bot. Zeit.* 1880. p. 845 etc. (See also *Ibid.*, "Die Ontogenie der Zelle seit 1875," *Progressus Rei Bot.*, Bd. I. Heft I., p. 22 etc., 1907.)

‡ *l.c.* p. 852.

§ Zimmermann, A., "Die Morphologie und Physiologie des pflanzlichen Zellkernes," p. 49, Jena, 1896.

‖ Johow, F., "Untersuchungen über die Zellkerne in den Secretbehältern und Parenchymzellen der höheren Monocotylen," *Inaug. Dissert.*, Bonn, 1880.

remarks on the fact that these cells, though advanced in age, still retain their living, streaming protoplasm, and include chlorophyll and starch.

Some very remarkable results bearing on the meaning of amitosis have been obtained in connexion with the study of mycorhiza. Werner* showed that in the case of certain cells of infected roots of *Listera* and *Orchis* there is a kind of fragmentation which is not a dying condition, but a special adaptation in an actively working nucleus. Shibata+ also, who studied the mycorhizal tubercles of *Podocarpus*, demonstrated that in the infected cells, which are digesting the fungus, the nuclei divide repeatedly by amitosis. This is not a death phenomenon, but must be regarded as a rapid means of nuclear multiplication. After the digestion of the fungus is ended, normal karyokinetic figures can often be seen in the multinucleate tubercle cells, showing that the nuclei, after repeated amitotic divisions, still retain the power of dividing by mitosis. I am not aware that these results of Shibata's have actually received confirmation from more recent workers, but, if they are correct, they are of fundamental importance, since it is scarcely possible to reconcile them with the theory of the permanence of the chromosomes—a theory which already shows symptoms of crystallising into a dogma.

The amitosis in the cortex and stele of *Stratiotes aloides*, described in the present paper, seems to be unique among recorded cases in respect of the immature condition of the tissues in which it has been observed. It lends support to the view that amitosis is by no means always a senile phenomenon—a view which has, in recent years, been upheld by certain zoological writers‡. This opinion has hitherto received little acceptation on the botanical side, perhaps because the attention of cytologists has been, of late, so closely riveted upon karyokinesis and, more particularly, meiosis, that other phases in the life of the nucleus have suffered comparative neglect.

* Magnus, W., "Studien an der endotrophen *Mycorrhiza* von *Neottia Nidus avis* L." *Jahrb. f. wiss. Bot.*, vol. xxxv. p. 205, 1900.

+ Shibata, K., "Cytologische Studien über die endotrophen Mykorrhizen," *Jahrb. f. wiss. Bot.*, vol. xxxvii. p. 643, 1902.

‡ See for instance Child, C. M., "Studies on the relation between Amitosis and Mitosis," *Biol. Bull.*, Woods Holl, Mass., vols. 12 and 13, 1906 and 1907; Glaser, O. C., "A statistical study of Mitosis and Amitosis in the Entoderm of *Fasciolaria tulipa* var. *distans*," *Biol. Bull.*, Woods Holl, Mass., vol. 14, p. 219, 1908; Walker, C. E., *The Essentials of Cytology*, London, 1907, p. 30; Foot, K. and Strobell, E. C., "Amitosis in the Ovary of *Protenor belfragei* and a Study of the Chromatin Nucleolus," *Archiv f. Zellforschung*, Bd. vii. p. 190, 1912.

V. *Summary.*

In the present paper an account is given of certain features in the general development and the cytology of the adventitious roots of *Stratiotes aloides* L., which may be briefly summarised as follows:—

A. *Anatomical Results.*

(i) The apex of the young adventitious root is clothed in a uniform cap of tissue, in which no distinction can be recognised between a pericyclic root-cap and an endodermal digestive sac. In this respect the results agree with those of D. G. Scott, and are opposed to those of Van Tieghem and Douliot.

(ii) The origin of the lacunæ of the middle cortex is shown to be due to differences in the rate of growth of the different tissue regions of the root.

B. *Cytological Results.*

(i) The nuclei of the young vessels and of the young root hairs are shown to be relatively of great size—a feature which possibly indicates unusual plasticity in the nuclei of this plant.

(ii) In the stem and leaf, bilobed nuclei and cells with more than one nucleus are shown to occur, but this peculiarity is much more important and conspicuous in the young adventitious roots where it occurs in the root-cap, cortex and stele. These observations have been made upon plants collected in 1910, 1911 and 1912 from two different localities. It is suggested that *amitosis supplements karyokinesis in the early development of the adventitious roots.* The behaviour of the nuclei is considered in relation to the life-history of the species, and the paper concludes with a brief discussion of the significance of amitosis.

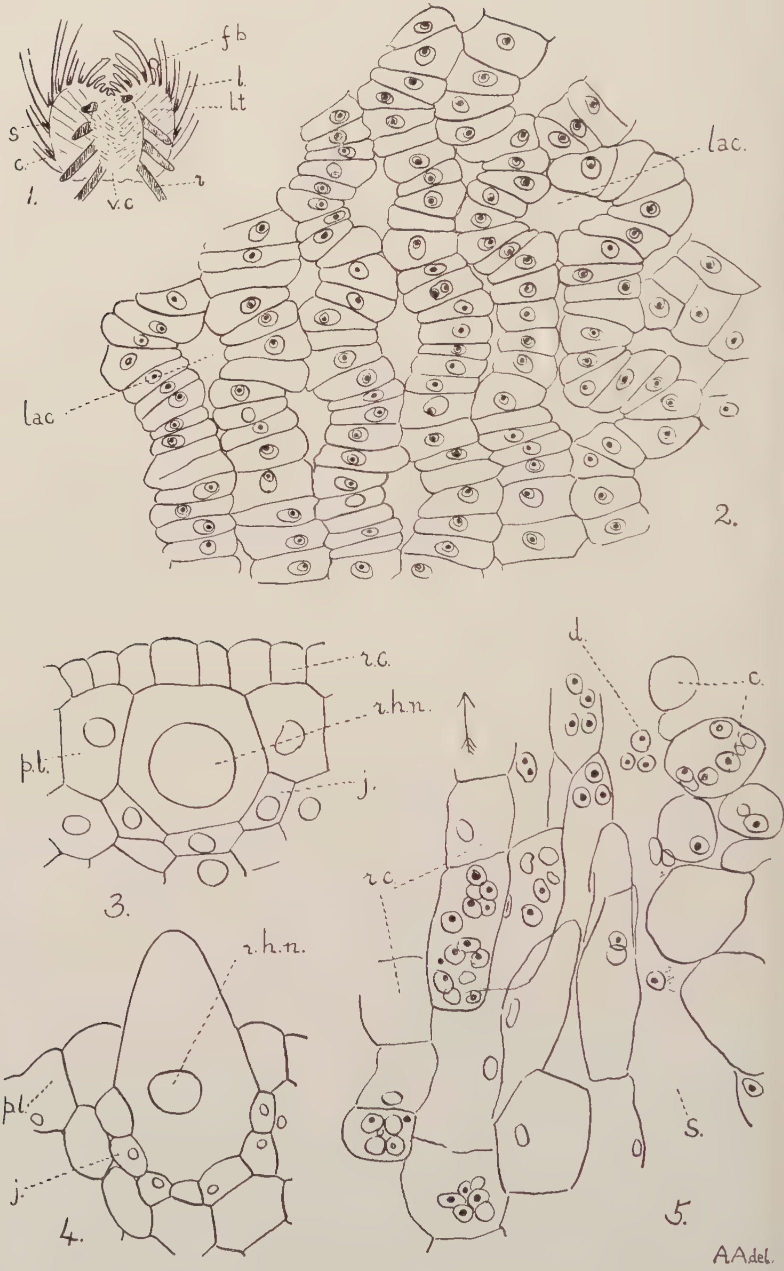
Explanation of Plates.

Plate VIII.

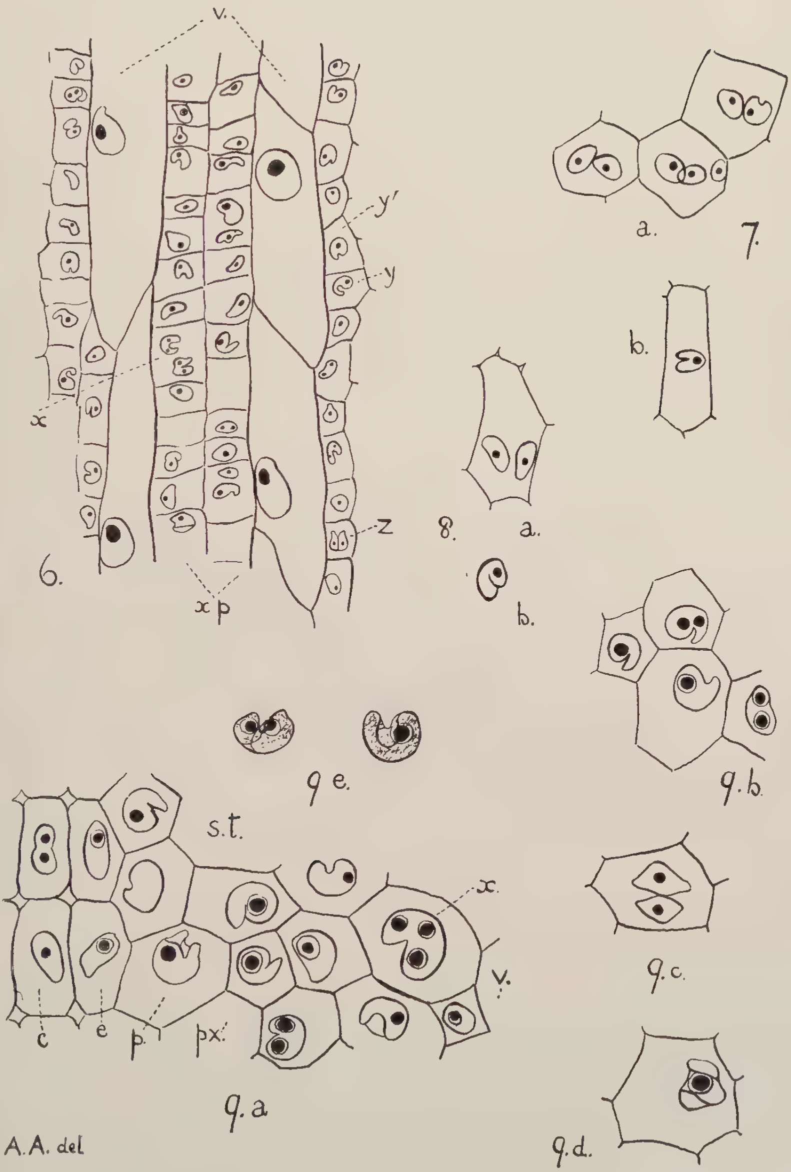
Stratiotes aloides L.

Fig. 1. Semi-diagrammatic sketch of a stem, as it appears in August, bisected longitudinally. (*v.c.* = vascular central region of stem; *c.* = stem cortex; *l.t.* = leaf trace; *l.* = leaf; *f.b.* = young stolon; *s.* = squamula intravaginalis; *r.* = adventitious root.) (Nat. size.)

Fig. 2. Tangential section through the middle cortex of a young root to show the origin of the lacunæ (*lac.*). (× 318.)



fb
l.
lt
s
c.
1.
v.c
r
lac.
lac
2.
r.c.
r.h.n.
p.l.
j.
3.
r.h.n.
p.l.
j.
4.
d.
c.
r.c.
s.
5.
A.A del.



A. A. del

Fig. 3. Edge of a transverse section through a root near the apex to show the relatively large size of the nucleus (*r.h.n.*) in a cell which is destined to form a root hair. One layer of dying root-cap tissue (*r.c.*) remains outside the piliferous layer (*p.l.*). The base of the root-hair cell is enclosed in a jacket of small cells (*j.*). (× 318.)

Fig. 4. Edge of a transverse section between 1 and 2 cms. from the apex of a root for comparison with Fig. 3 (on a smaller scale). No root-cap tissue is present, and one cell of the piliferous layer (*p.l.*) has begun to grow out into a root hair. The jacket of cells (*j.*) at the base of the root hair has become more conspicuous. (× 198.)

Fig. 5. Small part of a longitudinal section of an adventitious root embedded in stem tissue, to show multinucleate cells in the root-cap (*r.c.*) and in the stem cortex (*c.*). The arrow indicates the direction of the root-apex. *S.* = space between root-cap and stem tissue which contains nuclei and other remains of disintegrating cells (*d.*). (× 318.)

Plate IX.

Stratiotes aloides L.

Fig. 6. Part of the stele from a longitudinal section of a young adventitious root of a plant collected August 11th, 1910, passing through two embryonic vessels (*v.*) and their associated parenchyma cells (*x.p.*), many of which have lobed nuclei. In the cells marked *y*, *y'*, *x* and *z*, different stages of amitosis can be observed, while the cell marked *x* is also an example of a binucleate cell. Drawn from three successive microtome sections. (× 318.)

Fig. 7 *a*. Cells of the outer cortex containing more than one nucleus, from a transverse section of a young adventitious root. (× 318.)

Fig. 7 *b*. A cell of the outer cortex containing a lobed nucleus from a longitudinal section of an adventitious root. (× 318.)

Figs. 8 *a* and *b*. A binucleate cell and a lobed nucleus from a longitudinal section of the base of a leaf. (× 318.)

Figs. 9 *a*—*d*. Parts of the stele from a transverse hand section of a young adventitious root of a plant collected August 11th, 1910, showing lobed nuclei in the pericycle and stelar parenchyma, and, in 9 *c*, a case of actual division of the nucleus in a xylem parenchyma cell. (× 900.)

(*c.* = cortex; *e.* = endodermis; *p.* = pericycle; *px.* = ? protoxylem; *v.* = metaxylem vessel; *s.t.* = ? sieve tube.)

Fig. 9 *e*. Two lobed nuclei from a transverse section of the stele of a young root, showing the lobing in greater detail. (× 900.)

Amitosis in the Parenchyma of Water-Plants. By R. C. McLean, B.Sc., Lecturer in Botany at University College, Reading. (Communicated by Professor Seward.)

[*Read* 9 February 1914.]

It is desired to record the observation that the amitotic or direct process of nuclear division commonly occurs in the cortical parenchyma of aquatic angiosperms.

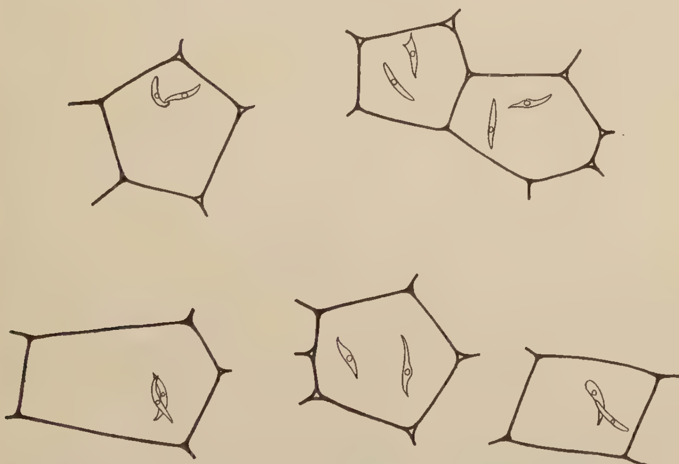
The phenomenon was first noticed in *Myriophyllum proserpinacoides* and afterwards in *Hippuris vulgaris.* This suggested that it might be characteristic of aquatics, and several other species, both Dicotyledons and Monocotyledons, were investigated from this point of view, with the result that a wider distribution of the phenomenon was discovered than had been presupposed to be the case. The transverse section of the stem-internode of *Myriophyllum* exactly resembles a wheel in its general outline. The hub is formed by the stele, which consists of a central mass of pith, around which lie a small number (six or seven) of vascular bundles which are simply collateral, neither xylem nor phloëm being strongly developed, while around all this lies a well-marked endodermis. The cortex consists of three parts, an inner zone, enclosing the stele, an outer zone immediately under the epidermis, forming the rim of the wheel, and an intermediate zone which consists of long strands of parenchyma—the spokes of the wheel, which separate the large air-lacunae from one another. It is in the innermost zone, that immediately surrounding the stele, that amitosis is most easily observed, although it has been seen in the outer cortical zone and in the trabeculae between them as well, only more seldom. In these latter cases the nuclei show the common spheroidal form. In *Hippuris* the stele is central as in *Myriophyllum,* but the cortex shows only one zone, consisting of a wide zone of reticulate trabeculae surrounding numerous large air-lacunae. Of all the plants so far examined, *Hippuris* shows the phenomenon more clearly and more widely spread in the tissues than any other.

In order to see the nuclei well it is best to use fairly thick sections, transverse or longitudinal, which include at least the thickness of one whole layer of parenchyma cells. These may then be observed unstained in glycerine, or stained carefully with carbol-fuchsin, acetic acid, methyl-green, or other direct acting nuclear stain. If the chloroplasts take the stain it will be difficult to distinguish the nuclei among them.

The general distribution of amitosis in the tissues follows the general distribution of growth. Cells showing it are commoner in

young stems than in older ones; they are much more frequent in sections taken close to a node than in those taken about the middle of the internode. They are also more frequent in the inner zones of the cortex, and the frequency diminishes towards the periphery of the stem.

The appearances presented are somewhat peculiar. Irregularity of outline is a well-known characteristic of nuclei in amitosis and these are no exception to the rule. The prevailing form, however, is an elongated spindle-shape, often twisted until it appears sigmoid. Sometimes the nuclear outline is amoeboid, the nucleus appearing to send out pseudopodia, its own diameter or more in length. These pseudopodia are distinctly acute and taper off insensibly into the cytoplasm. They are not mere lobes.



Paired nuclei in cortical cells of *Hippuris vulgaris*. In all cases the two nuclei lay in the same focal plane. × 240.

So common is the sigmoid form, resembling in outline the diatom *Pleurosigma*, that even when stages of actual amitosis are not found the existence of amitosis may be inferred from the nuclear form in the tissue under observation. Sometimes the length of these nuclei is as much as ten or twelve times their diameter.

It must be noted in conjunction with the last remark that cell-division does not follow nuclear division for some time, so that the sigmoid forms above referred to are almost always to be found associated together in pairs in the same cell. Rarely three may be met with in one cell, and not infrequently the nuclei in each pair may be twisted round one another, although not in any way united, recalling the appearance presented by the alga *Raphidium* which both resembles these nuclei in form and in the way which

they twist round one another. Apparently the separation of the nuclei from one another after division is very slow. Large and conspicuous nucleoli are always present, either one or, occasionally, two in each nucleus. The nucleolus sometimes causes a bulging-out of one side of the fusiform nuclei.

Stages in the actual separation of the two daughter-nuclei may be observed. No constriction is formed, but the process proceeds like the longitudinal fission in the Flagellata, from end to end, by gradual separation of the two daughter-nuclei. Amitosis is the only form of nuclear division which has been recognized in the tissues investigated, and from its exceeding frequency in the constituent cells it may be inferred that it is the only form occurring there.

Besides the two plants—*Myriophyllum* and *Hippuris*—mentioned above the following plants show the same phenomena in their cortical tissues.

Dicotyledons	*Monocotyledons*
Trapa bifida.	Elodea canadensis.
Jussieuia sp.	Potamogeton lucens.
(Hippuris).	Limnocharis sp.
(Myriophyllum).	Aponogeton sp.

All the above are aquatics, but two land plants have also been noted as showing resemblances to the aquatics in the above respects. These are *Dionæa muscipula* and *Polypodium ireoides.* The first is of course a marsh plant, but the second is an epiphyte, and as far removed from an aquatic as may well be. This suggests that the phenomena of amitosis in plants may well be much more widespread than has hitherto been supposed, and opens up a new field for thought in regard to its theoretical importance in cytology. It is generally admitted that amitosis represents only a fragmentation rather than a qualitative division of the nuclear substance, and that the complex phenomena of mitosis are adapted to the segregation of the histogenetic characters resident in that substance. Mitosis should therefore be the characteristic form of nuclear division in tissues which are undergoing ontogenetic growth. If, however, growth continues in a tissue which has already become fully differentiated, it is hard to see what further need there is for mitosis to take place. Amitosis *may* therefore be the constant form of nuclear division between sister-cells in all fully differentiated tissues which remain alive and continue to grow in bulk, although this does not preclude the possibility of its occurrence in meristematic tissues as well.

The History of the occurrence of Azolla in the British Isles and in Europe generally. By A. S. MARSH, B.A., Trinity College. (Communicated by Professor Seward.)

[*Read* 9 February 1914.]

In the middle of October 1913 a species of Azolla was found in Jesus Ditch, Cambridge, by Mr H. Jeffreys of St John's College. Mr Moss called my attention to the fact, and at his suggestion and with his frequent kind assistance I have identified the species and collected a few notes on the distribution of plants of this genus in Europe generally and the British Isles in particular.

The Cambridge plant I found to be *Azolla filiculoides* Lam. It was growing among the *Lemna*, but two or three large patches, several metres broad, bore Azolla almost pure, the dull brownish colour of the plant as seen in large masses showing up markedly against the bright green of the duckweed. When first found the plants seemed to be without reproductive organs, but on November 2nd it was bearing micro- and macro-sporocarps in some quantity. On November 26th, after several sharp frosts, the Azolla was growing vigorously, still with sporocarps, and had spread over larger areas, at the eastern end of the ditch becoming the dominant species of the aquatic vegetation. At the present time (February 9th) it is very abundant, but very red in colour and broken up into small pieces.

As to means of introduction of this fern into Cambridge we are completely ignorant. The nearest of the previously recorded stations is the Norfolk Broads area, while the obvious suggestion, that we are dealing with a Botanic Garden escape, is untenable, since there was before this discovery no Azolla except *A. caroliniana* being grown at the Cambridge Botanic Garden.

Azolla, according to Baker*, is a genus with five species inhabiting the tropics and warm temperate regions of both hemispheres. Of these species two have been introduced into Europe, and both occur in the British Isles. These two are *A. caroliniana*, which occurs native in America from Lake Ontario to Brazil, and *A. filiculoides*, from South America†.

The characters of these two species have been well summed up in two recent papers on the occurrence of *A. filiculoides* in

* Baker, *Fern Allies*, p. 137, London, 1887.

† The distributions are as given in Coste, *Flore de France*, III. pp. 702, 703, Paris, 1906, and Ascherson u. Graebner, *Synopsis der mitteleuropäischen Flora*, I. p. 114, Leipzig, 1896.

Europe, and from the accounts of these authors (viz. Bernard* and Béguinot and Traverso†), from Baker‡ and from von Martius§, the following details of the principal differences between the species are taken.

Azolla filiculoides (Lamarck, *Encyclopédie Méthodique: Botanique*, T. I. p. 343 and plate 863, 1783). The plants are in dense tufted masses, the ends of the shoots being porrect and often protruding, not lying flat on the surface of the water as in the other species. The whole shoot is much larger and thicker, the branching is more compound and the branches are closer together. The upper lobes of the leaves have a broad distinct margin and bear numerous unicellular trichomes on their upper surfaces. The reproductive organs show the most distinctive characters. The *glochidia* or hooked hairs which are attached to the *massulæ* or microspore masses have non-septate stalks. The macrospore wall is furnished with large, deep, circular pits.

Azolla caroliniana (Willdenow, *Species Plantarum*, v. p. 541, 1810). The plants are much smaller with much less dense branching. They lie flat on the surface of the water. The roots are not as numerous or as conspicuous as in *A. filiculoides*. The margin of the upper leaf lobe is not as broad as in the other species, and the trichomes of the upper surface are said to be bicellular, though I have not been able to observe this character satisfactorily. The *glochidia* have 3—5 transverse septa in the stalk, and the macrospore wall is not pitted but merely finely granulate.

The history of the genus in Europe began in 1872, when *A. caroliniana* was introduced into continental botanic gardens, whence it soon escaped into neighbouring ditches and ponds, and multiplied enormously. In 1878 De Bary described it as a "new water-pest" in Kassel, and in 1885 it was very abundant at Leyden and Boskoop in Holland‖. It was also found at Bonn, Giessen¶ and Strassburg in 1885, and in Berlin in 1887**. In Bohemia it was found by Čelakovský near Pilsen in 1895, and it had spread much earlier into England (1883), France (1879) and Italy (1886)††.

* Bernard, *Recueil des Trav. Bot. Néerland.*, I. pp. 1—14, 1904, quoted in the *Report of the Botanical Exchange Club for* 1912, p. 186.

† Béguinot e Traverso, "*Azolla filiculoides* Lam. nuovo inquilino della flora italiana," *Bull. Soc. Bot. Ital.*, pp. 143—151, 1906. ‡ Baker, *loc. cit.*

§ von Martius, *Flora Brasiliensis*, vol. I. part II. p. 657, plate 82, Leipzig, 1884.

‖ Kittel, *Gartenflora*, 1885.

¶ Dosch u. Scriba, *Excursionsflora Hessen*, 3te Auflage, p. 24.

** Luerssen, *Farnpflanzen*, p. 598.

†† This account is taken chiefly from Ascherson and Graebner, *loc. cit.*, but see also Saccardo, *Cronologia della Flora Italiana*, Padova, 1909; *Ibid.*, "De diffusione Azollæ carolinianæ per Europam," *Hedwigia*, 1892, p. 217; Béguinot and Traverso, *loc. cit.*, where many additional references are given.

In England *A. caroliniana* was first obtained at Pindon (Middlesex), and an account of this is published in *Science Gossip* for 1883. It has been recently reported from various spots in the Thames valley, between Oxford and London, but it must be remembered that until Ostenfeld* pointed out the fact in 1912 (from specimens found in 1911) it was not realised that we had any Azolla other than *A. caroliniana*. For instance, Druce† (1908) gives only one species, *A. caroliniana*. The following records for the British Isles have been published, though, until the material has been re-examined in the light of Ostenfeld's discovery, they must be considered records for the genus rather than for the species. Azolla described as *A. caroliniana* has been found at Hayes Place (Kent), Oxford, Sonning, Henley, Enfield, Sunbury and Suleham‡. Of these I have been able to examine material from Sunbury and Enfield kindly sent by Mr C. E. Britton. The Sunbury plant is *A. filiculoides*, the Enfield specimen *A. caroliniana*. Another Azolla from Enfield was sent by Mr Holloway, but this was *A. filiculoides*. The Norfolk Azolla, which is good *A. filiculoides*, has also been several times referred to as *A. caroliniana*. I have seen *A. caroliniana* from one other British locality, viz. Godalming, where it was found in 1913.

The species is described by Ascherson and Graebner (1896) as fruiting only very rarely, they knowing of only one case of fruit being produced in Europe—a record from Bordeaux. No fruiting material has been found in the British Isles, although fruiting *A. filiculoides* has more than once been described under the wrong specific name.

Azolla filiculoides was introduced into Europe in 1880 by Roze§, who naïvely remarks, "Le climat de Bordeaux paraît, du reste, assez bien convenir à ces deux espèces américaines, car quelques poignées de la première [*A. caroliniana*] en 1879, et de la seconde [*A. filiculoides*] en 1880, jetées çà et là dans les fossés des marais de cette ville, ont donné naissance à une légion innombrable de ces plantes, qui ont envahi presque tous les fossés, mares et étangs du département de la Gironde."

It spread over many parts of France and then into other countries. In 1896 Ascherson and Graebner knew of it only in western and northern France. In 1900 it had reached Italy||. In the British Isles *A. filiculoides* was first noticed as a distinct

* Ostenfeld, "Floristic Results of the International Excursion," *New Phyt.* XI. p. 127, 1912.

† Druce, *List of British Plants*, p. 88, Oxford, 1908.

‡ *Reports of the Botanical Exchange Club*, 1910, p. 609; 1911, p. 56; 1912, pp. 186, 220; *Journal of Botany*, XL. p. 113, 1902; XLVIII. p. 332, 1910.

§ Roze, "Contribution à l'étude de la fécondation chez les Azolla," *Bull. de la Soc. bot. de France*, XXX. p. 198, 1883.

|| Saccardo, *Cronologia, loc. cit.*; Béguinot e Traverso, *loc. cit.*

species by Ostenfeld * in 1911, who found it at Woodbastwick, Norfolk, and at Queenstown Junction, Co. Cork. It was, however, present in this country before that time. The Sunbury record of *A. caroliniana* in 1910 † should certainly be ascribed to the other species, while the Azolla was noticed in the Norfolk Broads ‡ before Ostenfeld's identification.

I have also seen fruiting specimens of *A. filiculoides* found in 1912 at Almondsbury, West Gloucestershire, and kindly sent me by Miss I. M. Roper. The same species now occurs at Reading, where it is peculiar in being without the endophytic blue-green alga, *Anabœna*, which usually inhabits the cavity of the upper leaf-lobe.

At present *A. filiculoides* seems to be growing in importance as a constituent of British vegetation, for, as the result of the floods of 1912, it has been distributed over large areas in Norfolk. It is described as occupying a definite position as a member of the association of *Typha angustifolia*, especially in South Walsham and Ranworth Broads §.

A. filiculoides fruits quite readily in Europe. Both the specimens found by Ostenfeld were fruiting, the Almondsbury and the Sunbury plants were in fruit, and I obtained fruit last autumn, not only from Cambridge but, by the kindness of Mr W. E. Palmer, of St John's College (the author of the article in *Nature*), also from Norfolk. Ascherson and Graebner ‖ also describe it as a freely fruiting species.

In conclusion, I should like to suggest that it is of some importance to keep a look-out for Azollas in the British Isles as, in the event of their becoming important factors in our vegetation, as full a knowledge as possible of their early history in the country would be of great interest and value.

* Ostenfeld, *loc. cit.*; *Report of the Bot. Exch. Club for* 1912, pp. 220, 301.

† *Rep. Bot. Exch. Club for* 1910, p. 609; *Journ. Bot.* XLVIII. p. 332, 1910.

‡ *Rep. Bot. Exch. Club for* 1910 *and* 1911, *loc. cit.*

§ Palmer, "Azolla in Norfolk," *Nature*, XCII. p. 233, 1913. The plant is wrongly named *A. caroliniana*, but I have seen fruiting specimens, which prove it to be *A. filiculoides*.

‖ *Loc. cit.*, p. 115.

A Simplification of the Logic of Relations.

By N. Wiener, Ph.D. (Communicated by Mr G. H. Hardy.)

[*Read* 23 February 1914.]

Two axioms, known as the axioms of reducibility, are stated on page 174 of the first volume of the *Principia Mathematica* of Whitehead and Russell. One of these, ∗12·1, is essential to the treatment of identity, descriptions, classes, and relations: the other, ∗12·11, is involved only in the theory of relations. ∗12·11 is applied directly only in

∗20·701·702·703 and ∗21·12·13·151·3·701·702·703.

It states that, given any propositional function ϕ of two variable individuals, there is another propositional function of two variable individuals, involving no apparent variables, and having the same truth-value as ϕ for the same arguments, or in symbols:

$$\vdash : (\exists f) : \phi(x, y) . \equiv . f!(x, y).$$

In ∗20 and ∗21·701·702·703 all that is done with ∗12·11 is to extend it to cases where the arguments of ϕ and f are classes and relations: ∗12·11 is essential to the development of the calculus of relations only owing to its application in ∗21·12·13·151·3. Here it is needed to make the transition between the definition of a binary relation and its uses. This is due to the fact that a binary relation itself is not defined, but only propositions about it, and ∗12·11 is needed to assure us that these propositions about it behave as if there were a real object with which they concern themselves. The authors of the *Principia* wish to treat a binary relation as the extension of a propositional function of two variables: that is, when they speak about the relation between x and y when $\phi(x, y)$, they mean to speak of any propositional function which holds of those values of x and y, and only those values, of which ϕ holds. Now, as it leads one into vicious-circle paradoxes to speak directly of "any propositional function which holds of those values of x and y, and those only, of which ϕ holds," they first define a proposition concerning the relation between x and y when $\phi(x, y)$ as a proposition concerning *a propositional function involving no apparent variables* which holds of x and y when and only when $\phi(x, y)$. Then they need to use ∗12·11 to assure us that, whatever ϕ may be, there always is some such propositional function. Now, if we can discover a propositional function ψ of one variable so correlated with ϕ that its extension

is determined uniquely by that of ϕ, and vice versa—if, to put it in symbols, when ψ' bears to ϕ' the same relation that ψ bears to ϕ, $\vdash :. \phi'(x, y) . \equiv_{x,y} . \phi(x, y) : \equiv : \psi'\alpha . \equiv_\alpha . \psi\alpha$—, we can entirely avoid the use of $*12{\cdot}11$, and interpret any proposition concerning the extension of ϕ as if it concerned the extension of ψ; for the existence of the extension of a propositional function of one variable is assured to us by $*12{\cdot}1$, quite as that of one of two variables is by $*12{\cdot}11$. Now, is such a ψ the propositional function

$$(\exists x, y) . \phi(x, y) . \alpha = \iota\text{‘}(\iota\text{‘}\iota\text{‘}x \cup \iota\text{‘}\Lambda) \cup \iota\text{‘}\iota\text{‘}\iota\text{‘}y.$$

For it is clear that for each ordered pair of values of x and y there is one and only one value of α, and vice versa. On the one hand, as $\iota\text{‘}(\iota\text{‘}\iota\text{‘}x \cup \iota\text{‘}\Lambda)$ is determined uniquely by x, and $\iota\text{‘}\iota\text{‘}\iota\text{‘}y$ is determined uniquely by y, $\iota\text{‘}(\iota\text{‘}\iota\text{‘}x \cup \iota\text{‘}\Lambda) \cup \iota\text{‘}\iota\text{‘}\iota\text{‘}y$ is determined uniquely by x and y. On the other hand, if

$$\iota\text{‘}(\iota\text{‘}\iota\text{‘}x \cup \iota\text{‘}\Lambda) \cup \iota\text{‘}\iota\text{‘}\iota\text{‘}y = \iota\text{‘}(\iota\text{‘}\iota\text{‘}z \cup \iota\text{‘}\Lambda) \cup \iota\text{‘}\iota\text{‘}\iota\text{‘}w,$$

either $\iota\text{‘}\iota\text{‘}y = \iota\text{‘}\iota\text{‘}z \cup \iota\text{‘}\Lambda$ or $\iota\text{‘}\iota\text{‘}y = \iota\text{‘}\iota\text{‘}w$. The former supposition is clearly impossible, for, as $\iota\text{‘}z \neq \Lambda$, $\iota\text{‘}\iota\text{‘}z \cup \iota\text{‘}\Lambda$ is not a unit class. From the latter alternative we conclude immediately that $y = w$. Similarly, $x = z$.

Therefore, when x and y are of the same type, we can make the following definition:

$$\hat{x}\hat{y}\phi(x, y) = \hat{\alpha}\{(\exists x, y) . \phi(x, y) . \alpha = \iota\text{‘}(\iota\text{‘}\iota\text{‘}x \cup \iota\text{‘}\Lambda) \cup \iota\text{‘}\iota\text{‘}\iota\text{‘}y\} \quad \text{Df.*}$$

It will be seen that in this definition of $\hat{x}\hat{y}\phi(x, y)$ it is essential that the x and the y should be of the same type, for if they are not $\iota\text{‘}(\iota\text{‘}\iota\text{‘}x \cup \iota\text{‘}\Lambda)$ and $\iota\text{‘}\iota\text{‘}\iota\text{‘}y$ will not be, and $\iota\text{‘}(\iota\text{‘}\iota\text{‘}x \cup \iota\text{‘}\Lambda) \cup \iota\text{‘}\iota\text{‘}\iota\text{‘}y$ will be meaningless. To overcome this limitation, and secure typical ambiguity for domain and converse domain of $\hat{x}\hat{y}\phi(x, y)$ separately, we make the following definitions:

$$\hat{\alpha}\hat{y}\phi(\alpha, y) = \hat{\kappa}\{(\exists \alpha, y) . \phi(\alpha, y) . \kappa = \iota\text{‘}(\iota\text{‘}\iota\text{‘}\alpha \cup \iota\text{‘}\Lambda) \cup \iota\text{‘}\iota\text{‘}(\iota\text{‘}\iota\text{‘}y \cup \iota\text{‘}\Lambda)\} \quad \text{Df.}$$

$$\hat{\kappa}\hat{y}\phi(\kappa, y) = \hat{\mu}\{(\exists \kappa, y) . \phi(\kappa, y) . \mu = \iota\text{‘}(\iota\text{‘}\iota\text{‘}\kappa \cup \iota\text{‘}\Lambda) \cup \iota\text{‘}\iota\text{‘}[\iota\text{‘}(\iota\text{‘}\iota\text{‘}y \cup \iota\text{‘}\Lambda) \cup \iota\text{‘}\Lambda]\} \quad \text{Df.}$$

etc.

$$\hat{x}\hat{\beta}\phi(x, \beta) = \hat{\kappa}\{(\exists x, \beta) . \phi(x, \beta) . \kappa = \iota\text{‘}[\iota\text{‘}(\iota\text{‘}\iota\text{‘}x \cup \iota\text{‘}\Lambda) \cup \iota\text{‘}\Lambda] \cup \iota\text{‘}\iota\text{‘}\iota\text{‘}\beta\} \quad \text{Df.}$$

$$\hat{x}\hat{\lambda}\phi(x, \lambda) = \hat{\mu}\{(\exists x, \lambda) . \mu = \iota\text{‘}\{\iota\text{‘}[\iota\text{‘}(\iota\text{‘}\iota\text{‘}x \cup \iota\text{‘}\Lambda) \cup \iota\text{‘}\Lambda] \cup \iota\text{‘}\Lambda\} \cup \iota\text{‘}\iota\text{‘}\iota\text{‘}\lambda\} \quad \text{Df.}$$

etc.

* This may seem circular as ι is a relation, defined in the *Principia* as $\overrightarrow{I}$, but it really is not circular, for $\iota\text{‘}x$ may be defined directly as the class, $\hat{y}(y = x)$.

Though these definitions may seem to conflict with one another, they really do not conflict, for where one of them is applicable, the others are meaningless, since they define relations between objects of different types. Moreover, it is easy to see that our definitions are so chosen that

$$\vdash : \hat{\mu}\hat{\nu}\phi(\mu, \nu) = \hat{\varpi}\hat{\rho}\psi(\varpi, \rho) \,.\supset.\, t`\mathrm{D}`\hat{\mu}\hat{\nu}\phi(\mu, \nu)$$
$$= t`\mathrm{D}`\hat{\varpi}\hat{\rho}\psi(\varpi, \rho) \,.\, t`\text{Ɑ}`\hat{\mu}\hat{\nu}\phi(\mu, \nu) = t`\text{Ɑ}`\hat{\varpi}\hat{\rho}\psi(\varpi, \rho).$$

This is important, as we might easily have defined relations so that they might have several domains or converse domains of different types. This is why we did not define $\hat{\alpha}\hat{y}\phi(\alpha, y)$ simply as

$$\hat{\kappa}\{(\exists\alpha, y) \,.\, \phi(\alpha, y) \,.\, \kappa = \iota`(\iota`\iota`\alpha \cup \iota`\Lambda) \cup \iota`\iota`\iota`\iota`y\},$$

for this would also represent

$$\hat{\alpha}\hat{\beta}\{(\exists y) \,.\, \phi(\alpha, y) \,.\, \beta = \iota`y\}.$$

It will be seen that what we have done is practically to revert to Schröder's treatment of a relation as a class of ordered couples. The complicated apparatus of $\iota`$s and $\Lambda`$s of which we have made use is simply and solely devised for the purpose of constructing a class which shall depend only on an ordered pair of values of x and y, and which shall correspond to only one such pair. The particular method selected of doing this is largely a matter of choice: for example, I might have substituted V, or any other constant class not a unit class, and existing in every type of classes, in every place I have written Λ.

Our changed definition of $\hat{x}\hat{y}\phi(x, y)$ renders it necessary to give new definitions of several other symbols fundamental to the theory of relations. I give the following table of such definitions:

$$\mathrm{Rel} = \hat{\kappa}\{\kappa \subset \hat{x}\hat{y}(x = x \,.\, y = y)\} \qquad \text{Df.}$$
$$xRy \,.=.\, \hat{z}\hat{w}\{z = x \,.\, w = y\} \subset R \,.\, R \,\epsilon\, \mathrm{Rel} \qquad \text{Df.}$$
$$\phi R \,.=.\, (\exists\alpha) \,.\, \alpha = R \,.\, \alpha \,\epsilon\, \mathrm{Rel} \,.\, \phi\alpha \qquad \text{Df.*}$$
$$(R) \,.\, \phi R :=: \alpha \,\epsilon\, \mathrm{Rel} \,.\supset_\alpha.\, \phi\alpha \qquad \text{Df.}$$
$$(\exists R) \,.\, \phi R :=.\, (\exists\alpha) \,.\, \alpha \,\epsilon\, \mathrm{Rel} \,.\, \phi\alpha \qquad \text{Df.}$$

The first two and the last two of these definitions replace ∗21·03·02 and ∗21·07·071 respectively. From these definitions and the laws

* We shall understand in this way any propositional functions containing capital letters in the positions proper to their arguments. Thus $\sim\phi R$ shall be understood as

$$(\exists\alpha) \,.\, \alpha = R \,.\, \alpha \,\epsilon\, \mathrm{Rel} \,.\, \sim\phi\alpha,$$

and not as

$$\alpha = R \,.\, \alpha \,\epsilon\, \mathrm{Rel} \,.\supset_\alpha.\, \sim\phi\alpha.$$

We make this definition as well as the two following ones because a propositional function of a class of the sort we have defined as a relation may significantly take as arguments classes of the same type which are not relations, and we wish to define propositional functions of relations in such a manner as to require that their arguments be relations.

of the calculus of classes it is an exceedingly simple matter to deduce any of the propositions of $*21$ which are not explicitly used for the purpose of deriving the properties of relations from the particular definition of relations given there, and from this it is easy to prove that the formal properties of the objects I call relations are essentially the same as those of the relations of the *Principia.*

But it is obvious that since they are also classes, our relations will possess some formal properties not possessed by those of the *Principia.* I give in conclusion a table of some of the more interesting of these:

$$\vdash . R \cup S = R \mathbin{\dot\cup} S$$
$$\vdash . R \cap S = R \mathbin{\dot\cap} S$$
$$\vdash : R \subset S . \equiv . R \mathbin{\dot\subset} S$$
$$\vdash . R - S . \equiv . R \mathbin{\dot-} S$$
$$\vdash . \dot{V} \subset V$$
$$\vdash . \Lambda = \dot\Lambda$$
$$\vdash . \mathrm{Rel} \subset \mathrm{Cls}$$
$$\vdash : Rp\kappa . \equiv . R\dot{p}\kappa$$
$$\vdash : Rs\kappa . \equiv . R\dot{s}\kappa$$
$$\vdash . \alpha + \beta \mathrm{\ sm\ } s\text{‘}\alpha \uparrow \beta$$
$$\vdash . \alpha \times \beta \mathrm{\ sm\ } \alpha \uparrow \beta$$

A Double-Four Mechanism. By G. T. Bennett, M.A., Emmanuel College.

(Plate X.)

[*Read* 24 November 1913.]

§ 1. A plane mechanism of eight pieces may suitably be called a *double-four* when each of four pieces 1, 2, 3, 4 is linked to all the four pieces 1′, 2′, 3′, 4′ except to 1′, 2′, 3′, 4′ respectively. A schematic form of diagram may conveniently be drawn as in Fig. 1, differing not greatly in appearance from a double-four of straight lines arranged in chequer fashion. Each of the eight

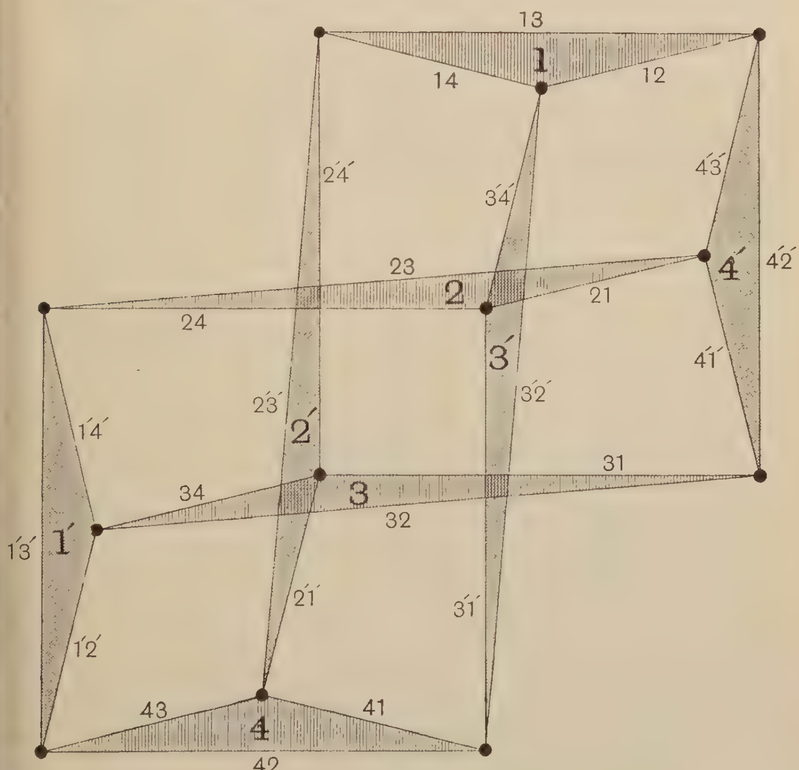


Fig. 1.

pieces is represented by a triangular plate, whose vertices are linked by pins to three other plates. The notation to be used here may be described as follows. The vertex of plate 1 which is linked to plate 2′ will be called 12′, and the vertex of 2′ which is linked to 1 will be called 2′1; so that points 12′ and 2′1 coincide

when the plates are assembled as a mechanism. The side of plate 1 opposite to vertex 12′ will be denoted by 12. These rules of notation are to be followed for all the twenty-four vertices and all the twenty-four sides. (The numerals assigned to any vertex have always one accented, and those assigned to any side have both accented or neither.)

If the pins at points 12′, 21′, 34′, 43′ are removed, the whole divides into separate mechanisms. One of these consists of plates 1, 3′, 2, 4′ consecutively linked at the vertices of a deformable quadrilateral which may be denoted 13′24′. Its sides are 12, 3′4′, 21, 4′3′, and the free vertices of the four triangles form a quadrangle 12′, 3′4, 21′, 4′3. The other mechanism consists of plates 2′, 4, 1′, 3 consecutively linked at the vertices of a deformable quadrilateral 2′41′3 with sides 2′1′, 43, 1′2′, 34; the free vertices of the triangles forming a quadrangle 2′1, 43′, 1′2, 34′. The two quadrangles are congruent, and coincide when the mechanism is reassembled. Such a division into two parts may be effected in three different ways; and there are thus three such pairs of quadrilateral linkages. Mechanisms of the type described are singular in possessing one degree of freedom; for the connectivity gives, normally, fourfold stiffness. They were first discussed by Kempe ("Conjugate four-piece linkages," *Proc. Lond. Math. Soc.* 1878, Vol. IX. pp. 133—147), who gave five different species, and afterwards by Darboux ("Recherches sur un système articulé," *Bulletin des Sciences Math.* 2 série, t. III. 1879, pp. 151—192), who carried out an exhaustive analysis and completed the catalogue by the addition of a species in which the three pairs of quadrilateral linkages are all contra-parallelograms (here called isograms). He shows that the material of the mechanism depends upon six parameters only, and that the three pairs of isograms are similar in pairs; but, for the rest, leaves the figure dependent for its precise description upon thirteen simultaneous equations in complex variables. It is this mechanism which is to be further discussed here. As a consequence of some investigations the following cardinal properties offer themselves. (α) The centre of similitude for each pair of isograms is the same. (β) The feet of the perpendiculars from this centre upon the axes of symmetry of any two pairs of isograms are the vertices of an isogram. (γ) This isogram is similar to the third pair of isograms of the mechanism, and with the same centre of similitude. These results being found, a simple means arises of constructing the mechanism *ab initio*. As it has been somewhat elusive, and as its geometry is a little uncommon, kinematically, there seems sufficient reason for a short study such as is here presented.

§ 2. An auxiliary diagram (Fig. 2), not itself a mechanism, may be described first. It consists of an axis of symmetry x,

four points A, B, C, D on one side of x, and A', B', C', D' their images in x, on the other. The figure contains six isograms, such as $ABA'B'$. Each gives a ratio, such as AB/AB' (less than unity), for the lengths of the pairs of equal sides. The figure depends for its shape on only six parameters, so that, if the six ratios are kept constant, normal expectation indicates an invariable form for the figure, its size alone remaining variable. A porism, however,

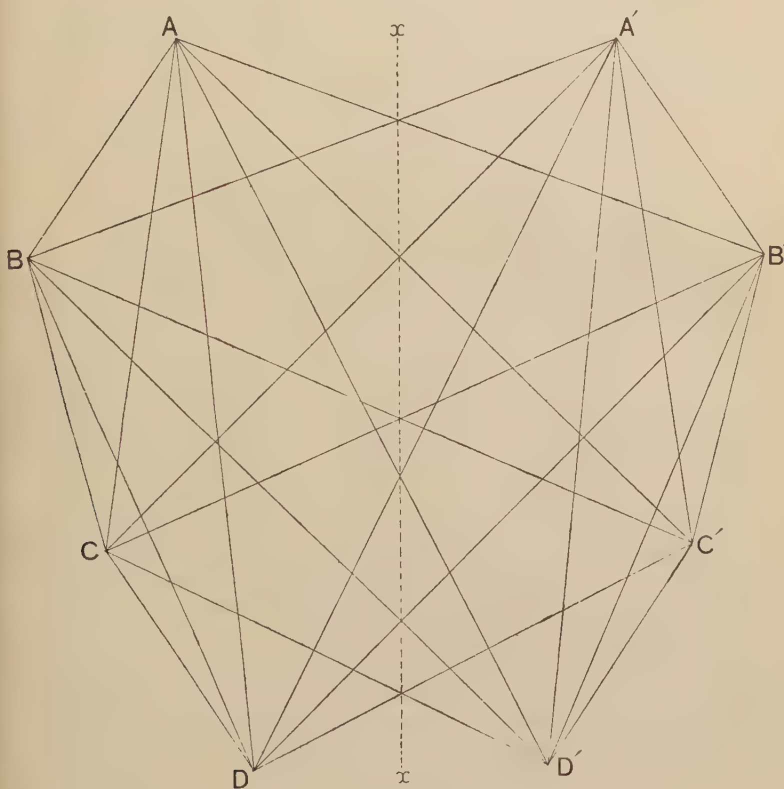


Fig. 2.

upsets this presumption. The six ratios are subject to a relation which may be found thus:—

Taking any point P it is possible (owing to the collinearity of the middle points of AA', BB', CC', DD') to find constant multipliers a, b, c, d such that the equation

$$a(PA^2+PA'^2)+b(PB^2+PB'^2)+c(PC^2+PC'^2)$$
$$+d(PD^2+PD'^2)=0 \ldots\ldots \text{(i)}$$

is identically true for all positions of P. Taking P at A,

$$a \, . \, AA'^2 + b\,(AB^2 + AB'^2) + c\,(AC^2 + AC'^2) + d\,(AD^2 + AD'^2) = 0 \quad \text{......(ii)},$$

and putting $\qquad (AB/AB')^2 = (k_{12} - 1)/(k_{12} + 1)$,

so that $\qquad k_{12} = (AB^2 + AB'^2)/AA' \, . \, BB' \quad \text{..............(iii)},$

(ii) becomes

$$a \, . \, AA' + b \, . \, BB' \, . \, k_{12} + c \, . \, CC' \, . \, k_{13} + d \, . \, DD' \, . \, k_{14} = 0 \quad \text{...(iv)}.$$

By taking P in turn at B, C and D three other such equations are obtained, and hence by elimination

$$\begin{vmatrix} 1 & k_{12} & k_{13} & k_{14} \\ k_{21} & 1 & k_{23} & k_{24} \\ k_{31} & k_{32} & 1 & k_{34} \\ k_{41} & k_{42} & k_{43} & 1 \end{vmatrix} = 0 \quad \text{..................(v)}.$$

The six ratios, such as AB/AB', remain all constant, therefore, if any five are kept constant; and the figure may therefore assume a single infinity of different shapes while each of the six isograms retains a constant value for the ratio of its sides.

The variation of shape may be obtained from any arbitrary initial form by inverting the figure of eight points from a varying point O on the axis of symmetry x. If the radius of inversion is R, the lengths AB and AB' become, after inversion,

$$R^2 . \, AB/OA \, . \, OB \quad \text{and} \quad R^2 . \, AB'/OA \, . \, OB',$$

the ratio of which is AB/AB', the same as before. Inversion leaves each of the six isograms with an unaltered value for the ratio of its sides. Specially, when O passes to infinity on x, and the circle becomes a line normal to x, inversion becomes reflexion in this normal and produces an image of the original figure.

It will be convenient to suppose that the arbitrary size of the figure is kept always such that the product $AA' \, . \, BB' \, . \, CC' \, . \, DD'$ remains constant. This is secured by making R^4 proportional to $OA \, . \, OB \, . \, OC \, . \, OD$. An equivalent form of the constant is

$$(AB'^2 - AB^2)(CD'^2 - CD^2),$$

and hence $AB.CD$ is constant and also $AB'.CD$, $AB.C'D$ and $AB'.C'D$. The product of any two sides taken one from each of a companion pair of isograms remains constant.

§ 3. From the figure of § 2, which may be called a *symmetrogram*, may now be constructed the isogram double-four. The geometry is best expressed in terms of vector multiplication; the product of two vectors being equal to the product of two others if the product of their lengths and the sum of their angles are in each case the same

Let the lines joining D to the six points A, B, C, A', B', C' be taken as vectors, and let all the vector-products of these be taken in pairs, omitting only the products $DA.DA'$, $DB.DB'$, $DC.DC'$. The extremities of these twelve vectors give a figure of twelve points. The vector $DA.DB'$ gives a point $12'$, $DA'.DB$ gives a point $21'$, and so on, in accordance with the tabular scheme

	$1'$	$2'$	$3'$	$4'$
1	—	AB'	$C'A$	$B'C'$
2	$A'B$	—	BC'	$C'A'$
3	CA'	$B'C$	—	$A'B'$
4	BC	CA	AB	—

where, on joining D to the extremities of any line of the symmetrogram entered in the table, the vector-product is to have its extremity named by the numerals of the same row and column. (It may be understood that an arbitrary unit vector divides all the products in common, maintaining the vector dimensions correctly and giving an arbitrary scale and orientation to the resultant figure.)

Consider the triangle formed by the points $41'$, $42'$, $43'$. Its sides 41, 42, 43, as vectors, are given by the differences of the vector-products $DB.DC$, $DC.DA$, $DA.DB$, and are therefore equal to $DA.BC$, $DB.CA$, $DC.AB$ (with zero sum). The lengths of these vectors have appeared as constants in § 2, and hence the triangle $41'$, $42'$, $43'$ has sides of constant length. Similar results hold for each of eight triangles 1, 2, 3, 4, $1'$, $2'$, $3'$, $4'$. The sides of each triangle have lengths equal to those of the vector-products of pairs of sides of an associated quadrangle of the symmetrogram; the four points consisting of the three points named in the row or column of the above table, together with D. A mechanism is therefore obtained, of the double-four type, with a notation corresponding to that of Fig. 1. Further, the three pairs of deformable quadrilaterals are all isograms. One is derived from the isogram $ABA'B'$ by multiplying it, from D as centre, by the vector DC; and the companion isogram is got by multiplying by DC'. These are both similar to the isogram $ABA'B'$; the ratio of their linear dimensions and the angle of inclination of their axes being given by the ratio and inclination of the vectors DC and DC'. A list of the lengths of the twelve pairs of equal lines of the mechanism figure is as follows:

$$\left.\begin{array}{ll} 23 = |DA'.BC'| & 14 = |DA.BC| \\ 31 = |DB'.CA'| & 24 = |DB.CA| \\ 12 = |DC'.AB'| & 34 = |DC.AB| \\ 2'3' = |DA.BC'| & 1'4' = |DA'.BC| \\ 3'1' = |DB.CA'| & 2'4' = |DB'.CA| \\ 1'2' = |DC.AB'| & 3'4' = |DC'.AB| \end{array}\right\} \quad \text{.........(vi).}$$

Six relations should connect these twelve lengths. It appears immediately that

$$23.14 = 2'3'.1'4', \quad 31.24 = 3'1'.2'4', \quad 12.34 = 1'2'.3'4' \quad \text{......(vii)},$$

giving three relations. Further (using lengths and not vectors)

$$23^2 - 2'3'^2 = (DA'^2 - DA^2)\,BC'^2 = \quad AA'.DD'.BC'^2,$$
$$14^2 - 1'4'^2 = (DA^2 - DA'^2)\,BC^2 = -AA'.DD'.BC^2,$$

and hence

$$(23^2 + 14^2) - (2'3'^2 + 1'4'^2) = AA'.BB'.CC'.DD' \quad \text{......(viii)},$$

the constant of § 2, giving two further relations. The sixth relation is supplied by the determinantal equation (v) on putting for k_{12} its value in terms of the sides given by (iii), and by $AB/AB' = 34/1'2' = 3'4'/12$ jointly; and similarly for the other elements of the determinant.

The angles of the two sets of plates are also simply related. The angle of plate 1 opposite to the side 12 may be denoted by $\widehat{12}$, and similarly for the rest. Then at the point $12'$ the sum (or difference) of the angles $\widehat{12}$ and $\widehat{2'1'}$ is equal to the sum (or difference) of the angles of two of the isograms; and this same sum (or difference) occurs for the angles at the points $21'$, $34'$, $43'$. Similar results hold for the rest of the angles. The angles of the plates 1, 2, 3, 4 thus serve to determine those of the other set of plates; and one method of derivation may be put thus:—Let angles α, β, γ be taken such that

$$2\alpha = \widehat{14} + \widehat{23} + \widehat{32} + \widehat{41},$$
$$2\beta = \widehat{13} + \widehat{24} + \widehat{31} + \widehat{42},$$
$$2\gamma = \widehat{12} + \widehat{21} + \widehat{34} + \widehat{43}.$$

Then on subtracting from α, β, γ the angles of any one plate (those namely which occur in α, β, γ respectively) the angles of a plate of the second set are obtained.

§ 4. A deforming isogram, starting from any arbitrary form, may pass through a cycle of fresh forms and revert to its original

form, the relative rotation of adjacent sides being four right angles. Among the forms occur two in which the vertices and sides are all in one straight line; and the original form itself appears in all four times, on two pairs of occasions which alternate with the rectilinear forms. This simultaneous cyclic performance of all the six isograms of the mechanism may be followed by observing the effect on the symmetrogram of inverting from the travelling point O. As regards the isogram $ABA'B'$, it inverts into collinear points when O crosses the circumference of the circumcircle; and it inverts into an isogram similar to $ABA'B'$ when O is at either diagonal point N or N', or at the centre of the circle, or at infinity. Moreover the two isograms $1'32'4$ and $24'13'$, similar to $ABA'B'$, will have parallel axes when O crosses the circumference of the circle circumscribing $CDC'D'$; and when O passes the centre of this circle the axes are inclined at the same angle as originally (with O at infinity).

As regards the kinematics of the instantaneous movement, any two of the eight pieces have a centre of relative rotation, and the three centres associated with any three pieces, taken in pairs, must be collinear. The figure necessarily possesses the requisite collinearities, and they may be readily accounted for. Let the instantaneous centre for plates 1 and 2 be denoted by (12), and similarly for all others. Of centres such as (12′) there are twelve, these being permanent centres given by the connecting pins themselves. Of centres such as (12) and (1′2′) there are altogether twelve. The centre (12) is collinear with (13′) and (23′), and is also collinear with (14′) and (24′); and similarly for all such others. For any pair of equal sides of an isogram, that is, the plates they carry have as instantaneous centre the (diagonal) point of intersection of the other two equal sides.

Of sets of three plates there occur three different types, of which 123′, 1′2′3′ and 144′ may be taken as representative. For the first, the collinearity of (12), (13′) and (23′) has already been noticed. For the second, the collinearity of (1′2′), (2′3′), (3′1′) may be seen thus. The vector from D to (1′2′), a diagonal point of the isogram $1'32'4$, is given by the product-vector $DC\,.\,DN$, where N is the intersection of AB and x; (2′3′) is given similarly by $DA\,.\,DL$ and (3′1′) by $DB\,.\,DM$. These are the three products of pairs of vectors drawn from a point D to the three pairs of vertices of a complete quadrilateral, formed by the sides of the triangle ABC and the line x; and hence the extremities of the three product-vectors are collinear.

There remain only the points (11′), (22′), (33′), (44′) to be considered. The last should be collinear with three pairs such as (14) and (14′), and also with three pairs such as (1′4) and (1′4′).

It may be found without difficulty that a vector from D equal to the vector function

$$(DA'.DB'.DC' - DA.DB.DC)/(AA' + BB' + CC')$$

gives a point (44′) satisfying all six conditions. Exchange of A and A' in this formula gives (11′), and similarly for (22′) and (33′). The twenty-eight instantaneous centres are thus all accounted for, and their collinearity in sets of three.

§ 5. The double-four mechanism described so far is not the only one derivable from the symmetrogram of § 2. The point D has, specially, been used as a centre for vector-products, and the mechanism thus associated with D may be named (D). The use of D' in place of D gives a mechanism (D') which is merely the image of (D) in x. But three fresh pairs of mechanisms (A) and (A'), (B) and (B'), (C) and (C') complete a set of eight, consisting of four distinct mechanisms (A), (B), (C), (D), each accompanied by its image. These four may now be compared.

Among the eight triangular plates of which (C) is composed there occurs one whose vertices are the extremities of product-vectors $CA.CB$, $CB.CD$, $CD.CA$. The sides of this triangle, as vectors, are given by $CD.BA$, $CA.DB$, $CB.AD$; and this triangle is identical in dimensions with the plate 4; and similarly for all the rest. The mechanism (C) is composed of the same material as (D), but it is differently put together. One of the isograms of (C) is obtained by multiplying the isogram $ABA'B'$ from C by CD: and one of the isograms of (D) is got by multiplying the same isogram $ABA'B'$ from D by DC. These isograms are therefore congruent, with parallel sides, and a half-turn rotation would bring them into coincidence. The triangles on corresponding sides, moreover, are both congruent and homothetic. The other pair of congruent isograms of (D) and (C) are got by multiplying the isogram $ABA'B'$ by DC' from D and CD' from C. Compare with each the isogram of (C') got by the multiplier $C'D$. The (C) and (C') isograms, with the triangles on their sides, are images in x; and the (D) and (C') isograms have the triangles on their sides congruent and homothetic, and would themselves come to coincidence by a half-turn. There results the following method of converting the mechanism (D) into (C), namely:—(i) Separate (D) into the two isogram mechanisms 13′24′ and 2′41′3. (ii) Exchange two vertices of each plate 1, 3′, 2, 4′ by giving it a half-turn about the middle point of the side of the isogram on which it stands. (iii) Exchange two vertices of each plate 2′, 4, 1′, 3 in the same way. (iv) Turn either of these new four-piece mechanisms upside-down; i.e. give it a half-turn about some line in its own plane. (v) Unite the free vertices of 1, 3′, 2, 4′ with those of 1′, 3, 2′, 4 respectively. The double-four (C) thus formed

has for its tetrads of plates 1, 2, 3, 4 paired with (and so not linked to) $2'$, $1'$, $4'$, $3'$ respectively. The double-fours (A) and (B), which may be similarly obtained from (D), have 1, 2, 3, 4 associated, the one with $4'$, $3'$, $2'$, $1'$ and the other with $3'$, $4'$, $1'$, $2'$. A double exchange of numerals suffices to convert the list of connections used for (D) into those necessary for (A), (B) or (C). Thus, in passing from (D) to (A), any vertex of 1, 2, 3, 4 is replaced by some fresh vertex in making attachment to any the same vertex of $1'$, $2'$, $3'$, $4'$; and the rule is to exchange the numerals 1 with 4 and 2 with 3. Since, *ex. gr.*, in (D) $12'$ is attached to $2'1$ (according to the original notation itself), so for (A) $43'$ is attached to $2'1$. The rule holds for all twelve pins. It may be observed that, taking the aggregate of all four mechanisms, any particular plate 1, 2, 3 or 4 is connected in turn with all the twelve vertices of the plates $1'$, $2'$, $3'$, $4'$, and conversely. The two tetrads remain distinct throughout, and connections occur only between members of opposite tetrads; but the different pairings which distinguish the double-fours are peculiar to each mechanism in turn. The mechanism given in Fig. 3 is the mechanism (D) derived from the symmetrogram of Fig. 2.

§ 6. Some of the special forms of the isogram double-four mechanism (D) may be briefly noticed, arising from special forms of the symmetrogram from which it is derived.

(i) If A, B, C, D are concyclic, the triangular plate 4 becomes a straight bar.

(ii) If A, B, C, D are concyclic, and also A', B', C, D, then both plates 3 and 4 become bars.

(iii) If A, B', C', D and A', B, C', D and A', B', C, D are concyclic, then plates 1, 2, 3 are bars. In this case the symmetrogram is obtainable from the figure of any triangle ABC, with A', B', C' as the feet of its perpendiculars, and D as orthocentre, by inverting from any point on the polar circle.

(iv) If A, B, C, D are on a circle with diameter x, then A', B', C', D' are also on the circle. All eight pieces are then bars, and the axes of the three pairs of isograms are concurrent. The figure occurs in a paper of the author's (*Lond. Math. Soc.* 1911, Ser. 2, Vol. x. p. 333). The eight points of the symmetrogram of Fig. 2 are approximately concyclic; and as a consequence the plates of Fig. 3 take an elongated form differing not greatly from bars.

(v) If D and D' coincide, the double-four is symmetric about x, and pairs of its plates are images in x.

(vi) If D and D' are coincident in case (iv) the eight-bar

mechanism becomes symmetrical. This is the figure given, apparently, by Darboux (*loc. cit.* p. 174).

(vii) If D and D' are at infinity on x, the product-vector extremities may (as a limit) be taken at the middle points of the sides of the isograms $ABA'B'$, $BCB'C'$, $CAC'A'$. The mechanism is the same as in case (v), but at a different stage of its deformation. Since the plate given by the middle points of triangle ABC is similar to ABC and of half the size, and similarly for others, it follows that the material for such a mechanism may be supplied by the eight triangles whose vertices are A or A', B or B', C or C' in the symmetrogram.

(viii) If C and C' coincide with D and D', the isogram $1'32'4$ of (D) becomes evanescent in size, and the pieces $1'$, $2'$, 3, 4 become bars linked together at a common extremity.

In all these cases peculiarities affecting the mechanisms (A), (B), (C) will accompany those of (D).

NOTE. Fig. 1 has served a purely schematic purpose in the foregoing treatment of the isogram double-four: its three pairs of quadrilaterals are drawn as parallelograms instead of isograms. But, as a consequence, it is itself a double-four mechanism of another species, and is included in his list by Darboux (*loc. cit.* p. 164). He identifies it with a last case of Kempe; but with some confusion, for the latter has a connectivity different from that of the double-four type. The freedom of the mechanism is a simple consequence of the parallelogram construction. It may be noticed that if the pins are removed, the set of plates 1, 2, 3, 4 may, without rotation, be brought together so that pairs of equal sides coincide, the vertices of the four triangles forming a quadrangle; and similarly for $1'$ $2'$ $3'$ $4'$. So that the material for the mechanism may be supplied by the eight triangles given by two arbitrary quadrangles. (In Fig. 1 these quadrangles are made, unessentially, and for simplicity, two equal parallelograms.)

But this mechanism, though ostensibly a double-four, should in strictness be classified as spurious or improper; for the pairs of pieces that are not linked have centres of relative rotation which are not variable but permanent, and for which the missing pins of connection may be supplied. The centre for 1 and $1'$ is such that it completes the figure of the first quadrangle when associated with the vertices of 1; and with the vertices of $1'$ it forms the figure of the second quadrangle; and similarly for the other three pairs. The completed mechanism consists then of four rigid quadrangles linked by their points to four other quadrangles;

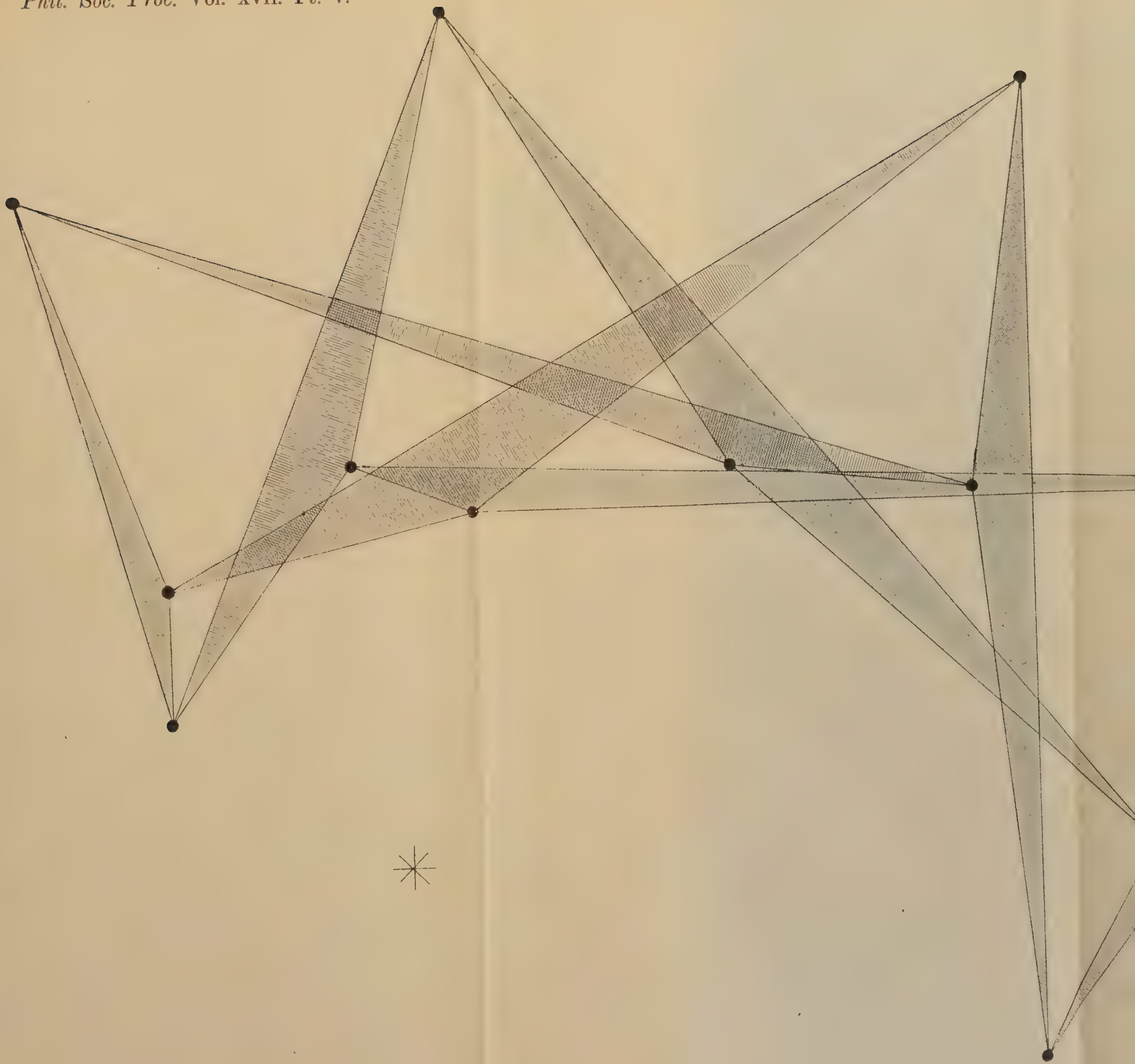


Fig. 3. The ✻ marks the centre of similitude of the three pairs of isograms.

each separate set of four being congruent and homothetic. Such a parallelogram link-work may be generalized, obviously, to include any number of pieces linked to any number of others. A lattice-work, if made of two separately equal sets of arbitrarily curved slats, would afford a representative example.

The parallelogram case and the isogram case above examined stand somewhat as companions among the species of double-four. For all other species the angles of the plates are equal or supplementary at every pin; and of these the most interesting has been specially treated by Fontené (*Nouvelles Annales de Math.* 1903, pp. 529—549, 1904, pp. 8—29).

On the Nature of the Internal Work done during the Evaporation of a Liquid. By R. D. KLEEMAN, D.Sc. (Adelaide), B.A., Emmanuel College.

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When a molecule passes from the liquid into the gaseous state it absorbs energy in overcoming the attraction of the molecules of the liquid, and in changing its internal energy. No energy is absorbed or evolved which is due to a change in the kinetic energy of translation. This can be shown to follow from the observed fact that the temperature indicated by a thermometer is independent of the nature of the material of the bulb, and consequently of the attraction it exerts upon the surrounding substance. The velocity of translation of a molecule in a substance when it passes through a point where the forces due to the surrounding molecules neutralize one another, is then independent of the density of the substance*. The average distribution of the molecules in the substance corresponds to each molecule being situated at a point possessing the property mentioned. It follows then at once that in separating the molecules of the substance by an infinite distance from one another energy can only be expended in the way described. Thus the internal heat of evaporation L of a molecule when the saturated vapour behaves as a perfect gas may be written

$$L = U + (u - u_a) \text{ ....} (1),$$

where U denotes the work done against molecular attraction, and $u - u_a$ the change in internal molecular energy, where u_a denotes the internal energy of a molecule in the gaseous state. Therefore if the liquid undergoes a small change in density the corresponding changes in the potential energy of attraction and internal molecular energy are dU and du.

The quantity du has usually not been considered by most investigators of the properties of matter, or has been assumed to be small in comparison with dU. Thus for example the equation of state of van der Waals does not take it into account. It must however occur in the equation of state†. On the other hand, other investigators have supposed that du is of the same order of magnitude as dU‡. I have brought forward circumstantial evidence that du is small in comparison with dU§, but more direct evidence is

* *Phil. Mag.*, July 1912, pp. 101—118.
† *loc. cit.*, Sept. 1912, pp. 391—401.
‡ *loc. cit.*, Jan. 1912, p. 111.
§ *Proc. Camb. Phil. Soc.*, XVI. Pt. 6, pp. 540—559 (1912).

desirable. We shall see later that the point in question is of more than passing importance. Accordingly I endeavoured to obtain an expression for $u - u_a$ in terms of other quantities. An expression was obtained corresponding to matter in the critical state as follows.

It will be convenient to consider first the exact meaning of the quantities on the right-hand side of equation (1). When a molecule is ejected from the surface of a liquid and passes out of the sphere of influence of attraction of the liquid molecules, it may undergo a change in the configuration of its atoms during the journey. This we would in fact expect since each molecule in the liquid state is under the action of the forces of attraction of the surrounding molecules whose resultant effect is a more or less radial force, whose centre is in the molecule, tending to separate the atoms from one another, and from which the molecule is relieved when passing into the gaseous state. This change in configuration may modify the law of attraction between the molecules. When the molecule is out of the sphere of action of the molecules of the liquid, or in the perfectly gaseous state, it may not yet be in a state of internal equilibrium. And the adjustment of equilibrium may give rise to a displacement of energy some of which may be algebraically communicated to the surrounding molecules. Thus an evolution or absorption of heat may occur when the molecule undergoes bombardment by other molecules after it has passed out of the influence of the molecules of the liquid. This heat is evidently the quantity $u - u_a$. It will be obvious that the same reasoning applies if the molecule passes into the gaseous state in stages, due to the existence of a surface transition layer. During each stage a change in internal energy equal to du will take place. But Σdu may then not be exactly equal to the value that $u - u_a$ would have in the absence of the transition layer.

Next it will be necessary to consider briefly the equilibrium of a substance. When in equilibrium the external pressure and force of contraction due to molecular attraction, called the intrinsic pressure, is balanced by the pressure due to the motion of translation of the molecules. We may suppose, as in the kinetic theory of gases, that the motion of translation of the molecules takes place parallel to three lines at right angles to one another, one-third of the molecules moving parallel to each line. If n denote the number of molecules crossing a cm.^{2} per second of a plane at right angles to one of the lines, and p and P_n denote respectively the external and intrinsic pressure in dynes, I have shown* that

$$p + P_n = n 2{\cdot}534 \times 10^{-20} \sqrt{T_m} \quad \ldots\ldots\ldots\ldots\ldots(2),$$

* *Phil. Mag.*, July 1912, pp. 103—109.

a relation which is independent of the density of the substance, where T denotes the temperature and m the molecular weight of a molecule relative to that of hydrogen. Its deduction was based on the fact that the temperature indicated by a thermometer is independent of the nature of its bulb, and consequently of the molecular attraction it exerts on the surrounding molecules.

Now let us consider the behaviour under certain conditions of a substance which is contained in a cylinder as shown in figure 1, in which slides a piston A. The system is kept at constant temperature. Suppose that the substance is a liquid and that the piston is in contact with its surface. Also suppose that the material of the piston consists of solidified liquid, in which case there is no change in density of the liquid as we pass from it into the material of the piston. Now suppose that the piston is instantaneously removed from the liquid surface to some distance away. Then initially αn molecules will leave the liquid surface per second, namely those that are able to overcome the attraction of the liquid, where α is a fraction. The substance may also be in such a state that α is unity, in which case the liquid surface is bodily projected after the piston. The properties of the substance corresponding to different values of α will now be considered.

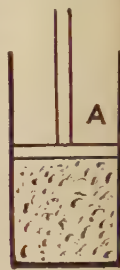

Fig. 1.

It will first be shown that if $\alpha = 1$ the substance cannot give rise to another phase of the substance which is in contact with it in equilibrium. For suppose that A and B in figure 2 are two

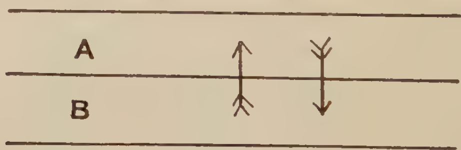

Fig. 2.

homogeneous slabs of the substance which are in equilibrium in contact with one another under the same external pressure, and that for the slab A the value of α is unity. It follows then that at any instant all the molecules in the boundary surface of the slab A move bodily into the slab B. And since there is equilibrium an equal number must move from the slab B into A. But this can only be the case if the density of the slab B is the same as that of A. It will be easy to see that this also holds if in the beginning we suppose that a transition layer exists at the boundary of the slabs. For this transition layer may be cut up into an infinite number of homogeneous slabs, each pair of which may be treated in succession in the same way, beginning from the A side.

If α is a fraction it can be shown that the substance can exist in two phases. Thus suppose that some of the substance is contained in the cylinder of figure 1 in contact with the piston, and let the piston be displaced in the same way as before. The surface of the substance will go on shedding molecules till the density of the vapour is such that the same number of molecules are received upon the surface per second as leave it. This number must be larger than αn, since the attraction of the vapour helps the molecules to get away from the liquid surface. But it must be smaller than n, for if equal to n it follows in the same way as before that both phases must have the same density, and hence there would be only one phase. The condition that another phase could exist in contact with the substance is therefore that α should be less than unity. When this condition is satisfied the phases are realizable in practice. Thus let α_1 denote the value of α for a quantity of the substance whose density is less than that just considered. Now the value of n increases with increase of density of the substance, while that of α decreases, since the attraction the kinetic energy of a molecule has to overcome increases with increase of density. If the two surfaces of the substances which are of different densities are placed in contact, the values of α_1, n_1 and αn are increased along the boundary, since the attraction of each substance helps the molecules of the other substance to get away from it. But the increase is greater in the case of the less dense substance than in the case of the other, on account of it being bounded by a substance of greater density than itself, and the opposite applies to the other substance. Thus it should be possible to find a density for the second substance which is less than that of the first, so that when the substances are placed in contact with one another the number of molecules passing from one into the other is the same, in which case they will be in equilibrium. Upon reflection it will be evident that the existence of a surface transition layer does not invalidate the foregoing conclusions.

The foregoing considerations show that when a substance is in the critical state $\alpha = 1$. Now a molecule could get away from the surface of a mass of substance and out of the influence of its molecules after displacement of the piston in the process described only when its kinetic energy is equal to the energy expended in overcoming the attraction of the substance. The kinetic energy of a molecule in the substance that could be expended in this way is that which it has when passing through a point at which the forces of the surrounding molecules neutralize one another. For we have seen that when a substance is converted at constant temperature into the perfectly gaseous state this kinetic energy remains constant, the heat being expended in overcoming external and internal

molecular forces. This kinetic energy therefore represents available energy, since it is evidently available for being converted into other forms of energy when the substance is in the perfectly gaseous state. It follows therefore that at the critical point $\frac{m_a V^2}{2} = U$, where V denotes the velocity of a molecule in the gaseous state and m_a its absolute mass. From the kinetic theory of gases we have $V^2 = \frac{3RT}{m}$, where m is the molecular weight of a molecule relative to that of a hydrogen molecule and $R = 8{\cdot}26 \times 10^7$. Hence at the critical point

$$U = \left(\frac{RTm_a}{m}\right) 1{\cdot}5 \quad\quad (3).$$

In a previous paper* I have shown that at the critical point

$$U + u - u_a = \left\{T\left(\frac{dp}{dT}\right)_v - p\right\} m_a v \quad\quad (4),$$

where v denotes the volume of a gram of substance. This equation was deduced from thermodynamics without introducing any assumptions. It was also shown that according to the facts the right-hand side of the equation may be written $m_a vp\, 6{\cdot}5$. Thus the equation may be written

$$U + u - u_a = \left(\frac{RTm_a}{m}\right) 1{\cdot}76 \quad\quad (5),$$

since according to Young and Thoma's law $pv = \frac{RT}{3{\cdot}7m}$ at the critical point. Equations (4) and (5) then give at once

$$u - u_a = \left(\frac{RTm_a}{m}\right) {\cdot}2 = {\cdot}13\, U \quad\quad (6).$$

It appears therefore that when a molecule passes from a substance into a less dense substance it absorbs heat in the latter substance on being bombarded by the surrounding molecules till it is internally in equilibrium. The amount of heat absorbed is about 10 % of the total heat absorbed or energy spent in the transference of the molecule. The change in internal energy of the molecule is thus small in comparison with other energy changes as we might expect.

As has been remarked before, it is of importance to obtain some definite information about the quantity $u - u_a$. Thus I have shown in a previous paper† that if $u - u_a$ is small in comparison with U the attraction between two molecules decreases

* *loc. cit.*, Sept. 1912, p. 395.

† *Proc. Camb. Phil. Soc.*, XVI. Pt. 6, pp. 540—559 (1912).

with increase of temperature, and that it follows from the Joule-Thomson effect that it varies approximately inversely as the temperature. This effect is more likely to be brought about, as pointed out in the paper quoted, by a relative displacement of the atoms of each molecule with change of temperature than by a true change in the forces of attraction. It follows therefore that the intrinsic pressure term in the equation of state is a function of the temperature as well as the volume of the substance. Also the variation of the viscosity of a gas with change of temperature is not due only to a change in the velocity of translation of the molecules but also to a variation of the forces of attraction between them.

A formula for the intrinsic pressure was also obtained which gives the quantity to the same degree of approximation that $U+u-u_a$ is equal to U.

Since $u-u_a$ is small in comparison with U it is possible to obtain some information about the law of molecular attraction from a study of the internal heats of evaporation of liquids. This I have carried out in previous papers assuming the foregoing. It was shown* that if no other assumption is made besides the one mentioned it is mathematically impossible to determine completely the law of molecular attraction from internal heat of evaporation data, in other words the law obtained should contain an arbitrary function of the temperature and distance of separation of the molecules. This arises from the fact that we do not know *a priori* how the attraction varies with the temperature, other conditions remaining the same. And this point cannot be determined from the data in question because we cannot say how much of the change in internal heat of evaporation with rise of temperature is due to a separation of the molecules of the liquid due to its decrease in density, and how much due to a change in the attraction of the molecules. But even if this point were determined the law would still contain an arbitrary function. For the internal heat of evaporation is the difference of two quantities, viz. the potential energies of attraction in the states of liquid and saturated vapour, and we evidently cannot recover a curve given the difference between certain ordinates. It is therefore of primary importance to obtain the law of attraction in the form involving an arbitrary function for we can then be sure that the part of the law outside the function is correct. On the other hand if we obtain a definite law by the help of assumptions, say assuming the form of the law, an agreement with the facts does not mean that our assumptions are correct. This follows from the mathematical theorem so often ignored by scientists that an infinite number of formulae can be found to express a set of facts equally well, each

* *Phil. Mag.*, Jan. 1911, pp. 83—102.

of which corresponds to a different hypothesis. It is evident therefore that there is only one way to obtain reliable information about the law of molecular attraction from internal heat of evaporation data. Any other way might lead to the discovery of excellent formulae for the internal heat of evaporation, but could by no means give any definite information about the law in question.

I have obtained* in the way described the law

$$\phi\left(\frac{T_c}{T}, \frac{x_c}{z}\right)\frac{\Sigma\sqrt{m_1}}{z^5},$$

where z denotes the distance of separation of the molecules which are at the temperature T, x_c that of the molecules in the liquid state at the critical temperature, and $\phi\left(\frac{T_c}{T}, \frac{x_c}{z}\right)$ is a function of these quantities left arbitrary. The quantity $\Sigma\sqrt{m_1}$ denotes the sum of the square roots of the atomic weights of the atoms of a molecule; and the attraction of "cohesion" of an atom is thus proportional to the square root of its atomic weight. Since I have shown in this paper that $u - u_a$ is small in comparison with U, the deduction of the law is placed on a perfectly sound basis. It is evident that each of the definite laws that have been obtained by introducing assumptions correspond to certain forms of the arbitrary function. This connection has been discussed at length†. It was also deduced‡ that the function ϕ does not vary much with z and roughly inversely as T from other data than those relating to the internal heat of evaporation.

Note. It will be convenient here to correct two misprints. Equation (27) on page 639 of the *Proc. Camb. Phil. Soc.*, vol. XVI. Pt. 7, should read $\eta = k_7 \dfrac{m^{\frac{5}{6}} p_c}{T_c^{\frac{1}{2}} \rho_c^{\frac{1}{3}}}$; and equation (2) on page 177, vol. XVII, Pt. 1, should read $T = M_2^2 \dfrac{(\Sigma c_a)^2}{(\Sigma c_v)^{\frac{4}{3}}}$.

* *loc. cit.* and papers quoted therein.

† *loc. cit.*, Oct. 1911, pp. 566—586.

‡ *loc. cit.*, May 1910, p. 807; and *Proc. Camb. Phil. Soc.*, XVI. Pt. 6, 1912, pp. 553—559.

The Work done in the Formation of a Surface Transition Layer of a Liquid Mixture of Substances. By R. D. KLEEMAN, D.Sc. (Adelaide), B.A., Emmanuel College.

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In a previous paper* I have shown that the surface tension which a pure substance would have if no surface transition layer were formed would be given by

$$\left\{\frac{1+\frac{c_2}{2}+\frac{c_3}{3}+\dots}{1+c_2+c_3+\dots}\right\} 6\lambda_2 = U m_a^{\frac{1}{3}} \rho^{\frac{2}{3}} \quad \text{..............(1)},$$

where U denotes the energy expended in overcoming molecular attraction on separating the molecules of a gram of the substance an infinite distance from one another, ρ denotes the density of the substance, and m_a the absolute molecular weight of a molecule. This equation is based on Laplace's definition of surface tension according to which it is the work done against molecular attraction per unit increase of surface on cutting a slab of a substance into two slabs and separating them an infinite distance from one another. Besides the assumption is made that the attraction between two molecules separated by a distance z is given by a law of the form $\frac{k}{z^r}$, where k and r are constants or functions of T, the temperature. The constants $c_2, c_3, c_4, \dots c_n$ in the expression

$$\left\{\frac{1+\frac{c_2}{2}+\frac{c_3}{3}+\dots}{1+c_2+c_3+\dots}\right\}$$

are then functions of r. Since the molecular attraction is approximately given by a law of the above form, and the foregoing expression is not sensitive to variations in the values of the constants $c_2, c_3, c_4, \dots c_n$, the value of the expression may be calculated with fair accuracy by means of a law of attraction which has been found to fit approximately the facts. In this manner I obtained ·876 for the value of the expression putting $r = 5$, and equation (1) accordingly became

$$\lambda_2 = \frac{U m_a^{\frac{1}{3}} \rho^{\frac{2}{3}}}{\cdot 876 \times 6} \quad \text{.......................(2)}.$$

* *Phil. Mag.*, Dec. 1912, pp. 876—885.

Before considering the application of this equation to mixtures it will be useful to reconsider its deduction.

The true law of molecular attraction is probably not given by a law of such simplicity as the foregoing. But there is nothing against supposing for purposes of calculation that the molecules are replaced by a set of molecules obeying such a law provided that the surface tension in question and the value of U remain unaltered. This can be realized since we have two variables at our disposal, namely k and r. Equation (1) is thus fundamentally correct. It then remains to find the *equivalent* law of molecular attraction of the form given. This need only be approximately realized for the reasons given. To see how the value of the expression depends on the value of r it was calculated for the case $r = 2$, which gave ·71. When $r = \infty$ it will be easily seen that it is equal to unity. The correct value thus lies between ·7 and 1. The attraction varies much more rapidly with the distance of separation of the molecules than corresponding to $r = 2$, r being approximately equal to 5*, or 4 to which corresponds van der Waals' equation of state. It is evident therefore that the approximately correct value of 5 for r should give within a few per cent. the correct value of the expression.

When we are dealing with a mixture of substances we may suppose as before that the molecules are replaced by a set of molecules equal to one another and which obey a law of attraction of the form given. This is evidently possible for the same reasons as stated above. The equivalent law of attraction will obviously be very approximately the same as for a pure substance, and the value of the expression discussed practically not altered. The molecular weight m_a occurs in connection with the number of molecules in the substance. In the case of a mixture it is therefore only necessary to substitute for m_a the average molecular weight. The quantity U refers in all cases to a gram of the substance.

When the ingredients of the mixture consist of liquids whose heats of evaporation in the pure state are known for temperatures at which their vapours approximately obey the laws of a perfect gas, the values of U may be approximately calculated corresponding to this region of temperatures. The heat of evaporation of a substance whose vapour approximately obeys the laws of a perfect gas is equal to the energy expended in separating the molecules an infinite distance from one another. This heat is expended in overcoming the attraction between the molecules and producing changes in their internal energy. I have shown† that the latter

* *loc. cit.*, May 1910, p. 807.
† Previous paper in this volume.

part of the heat expended is small in comparison with the former part, and the internal heat of evaporation may therefore be taken as approximately equal to the energy expended against molecular attraction. By definition U for a mixture is equal to the energy expended in overcoming the molecular attraction on separating the molecules of a gram of the mixture an infinite distance from one another, and hence approximately equal to the heat of mixture of the ingredients and their internal heats of evaporation in the pure state. If the ingredients of the mixture consist of the substances a and b in the proportion of n_a and n_b by weight, we have

$$U = \frac{L_a n_a + L_b n_b}{n_a + n_b} + H_m \quad \ldots\ldots\ldots\ldots\ldots\ldots (3),$$

where L_a and L_b denote the internal heats of evaporation of a gram of substance a and b respectively, and H_m the heat of mixture of the ingredients of a gram of the mixture. H_m is usually small in comparison with both L_a and L_b and may therefore be neglected.

I have calculated the surface tension λ_2 of a few mixtures of substances of which Ramsay and Aston* have measured the ordinary surface tension λ_1. The constituent substances are not associated in the pure state and according to the experimenters mentioned do not undergo any radical chemical change on mixing, that is, the resultant substance is more or less a pure mixture. The results I obtained are given in Table I. The values of U were calculated by means of equation (3) neglecting H_m, and using the internal heats of evaporation L_a and L_b given in the Table, which were interpolated from the values calculated by Mills†. The absolute mass of the hydrogen atom is taken as in the previous paper equal to $1{\cdot}61 \times 10^{-24}$ gram, the value obtained by Rutherford from experiments on the α particle. The difference $\lambda_2 - \lambda_1$ is the external work done in the formation of the transition layer.

It will be seen that the value of λ_2 for a mixture of C_6H_6 and CCl_4 is practically independent of the relative concentrations of the ingredients. This is probably intimately connected with the fact that this holds also for the values of λ_1. The results I obtained previously with pure substances are given in Table II for comparison, L_1 denoting the internal heat of evaporation. It will be seen that in the case of pure substances $\lambda_2 - \lambda_1$ is practically independent of the temperature. But for mixtures this does not hold, its value increases with increase of temperature. At low temperatures $\lambda_2 - \lambda_1$ is smaller for a mixture than for each of the ingredients in the pure state. The difference in the behaviour of mixtures and pure substances is no doubt due to adsorption effects.

* *Zeit. f. Phys. Chemie*, 15, 1894, p. 88.

† *Journ. of Phys. Chem.* VIII. p. 405 (1904).

TABLE I.

T	ρ	λ_1	$L_a \cdot (C_6H_6)$	$L_b \cdot (CCl_4)$	U	λ_2	$\lambda_2 - \lambda_1$
Mixture of 1 C_6H_6 to 1 CCl_4. Average $m_a = 116 \times 1{\cdot}61 \times 10^{-24}$ grm.							
289	1·2597	27·70	97·11	46·97	63·84	34·0	6·31
318·2	1·2095	23·50	91·8	44·39	60·27	31·3	7·75
351·2	1·1596	19·71	85·3	41·64	56·27	28·4	8·73
10 C_6H_6 to 17 CCl_4. $m_a = 125{\cdot}8 \times 1{\cdot}61 \times 10^{-24}$ grm.							
286·2	1·3509	27·66	97·75	47·24	58·86	33·75	6·09
319·6	1·2942	23·51	91·20	44·35	55·09	30·70	7·19
351·4	1·2411	19·52	85·00	41·60	51·56	27·94	8·42
2 C_6H_6 to 1 CCl_4. $m_a = 103{\cdot}3 \times 1{\cdot}61 \times 10^{-24}$ grm.							
283·8	1·1384	28·55	98·12	47·40	72·76	34·83	6·28
319·2	1·0877	23·99	91·80	44·39	68·09	31·64	7·65
351·2	1·0431	20·00	85·30	41·64	63·47	28·67	8·67
1 $CHCl_3$ to 1 CS_2. $m_a = 97{\cdot}7 \times 1{\cdot}61 \times 10^{-24}$ grm.							
282	1·4026	29·16	61·41	81·78	69·36	37·48	8·32
317·9	1·3406	24·49	57·30	76·79	64·88	34·02	9·53
334	1·3128	22·23	55·49	74·29	62·82	32·48	10·25

TABLE II.

Ethyl oxide $C_4H_{10}O$						Carbon tetrachloride CCl_4					
T	L_1	ρ	λ_2	λ_1	$\lambda_2-\lambda_1$	T	L_1	ρ	λ_2	λ_1	$\lambda_2-\lambda_1$
313	75·36	·6894	23·12	14·05	9·07	363	40·62	1·4554	26·18	17·60	8·68
323	73·01	·6764	22·12	12·94	9·18	373	39·68	1·4343	25·33	16·48	8·85
333	70·79	·6658	21·20	11·80	9·40	383	38·64	1·4124	24·41	15·41	9·00
343	68·35	·6532	20·27	10·72	9·55	393	37·63	1·3902	23·52	14·32	9·20
353	65·85	·6402	19·27	9·67	9·60	403	36·58	1·3680	22·62	13·27	9·35
363	63·31	·6250	18·19	8·63	9·56	413	35·56	1·3450	21·94	12·22	9·72
373	60·33	·6105	17·16	7·63	9·53	423	34·42	1·3215	20·81	11·21	9·60
Methyl formate $C_2H_4O_2$						Benzene C_6H_6					
303	107·5	·9598	38·35	23·09	15·26	353	85·62	·8415	29·87	20·28	9·59
313	103·9	·9447	36·64	21·56	15·08	363	83·74	·8041	28·97	19·16	9·81
323	99·51	·9294	34·74	20·05	14·69	373	81·98	·7927	28·06	18·02	10·04
333	95·59	·9133	32·98	18·58	14·40	383	80·05	·7809	27·15	16·86	10·29
343	92·16	·8968	31·94	17·55	13·89	393	78·12	·7692	26·24	15·71	10·53
353	88·03	·8803	29·65	15·70	13·95	403	76·10	·7568	25·28	14·57	10·71
363	85·10	·8636	28·25	14·29	13·96	413	74·09	·7440	24·33	13·45	10·88

The ionisation produced by certain substances when heated on a Nernst filament. By FRANK HORTON, Sc.D., St John's College.

[*Read* 9 March 1914.]

The ionisation produced by heated solids has been investigated by many observers in recent years, but at the present time none of the theories which have been put forward to explain the origin of the emission of either positive or negative electricity is universally accepted. The substance upon which most experiments have been made is platinum, but other metals, and many compound substances, have also been used. As a rule, in testing compound substances they have been supported upon a strip or wire of platinum, and the view has been put forward that the observed effects are due to the metal support rather than to the layer of substance under test. Thus certain oxides—those of the alkaline earth metals—give a large emission of negative electricity when heated upon platinum in an exhausted tube, and it has been suggested that this effect is merely an increased electron emission from the platinum itself; the increase being caused by the lime lessening the energy required to enable an electron to escape through the metal surface. The emission of positive electricity from various salts has also usually been studied when the salt under test is heated upon a platinum strip. Dr W. Wilson failed to detect any positive ionisation from aluminium phosphate heated on a Nernst filament, although the positive emission from this salt when heated upon platinum has been investigated by several experimenters. Dr Wilson has therefore concluded that "the leak observed when the salt is heated on platinum is either mainly a leak from the platinum itself, or the latter plays an important rôle in its production*."

The author has recently been studying the thermal ionisation produced by Nernst filaments and, having the apparatus at hand, it was thought desirable to test these views as to the origin of the ionisation from glowing solids by ascertaining (*a*) whether lime heated upon a Nernst filament gives a negative emission comparable with that obtained when it is heated upon platinum, and (*b*) whether the positive emission from a glowing Nernst filament is increased by placing sodium phosphate upon it. It has already

* W. Wilson, *Phil. Mag.* 6, XXI. p. 634, 1911.

been shown that sodium phosphate heated upon platinum gives a large positive ionisation which is more permanent than that given by aluminium phosphate*.

The discharge tube used in these experiments is similar to that described in an earlier paper*, the only difference being that the two parallel platinum plates which form the anode are rather further apart in the present apparatus. These plates are situated vertically, 1·5 cms. apart, at the centre of the small bulb which forms the discharge tube. The Nernst filament when in position is parallel to the plates and mid-way between them. It can be heated by an alternating electric current led in through stout platinum wires, and the filament and its leads can easily be removed from, or replaced in, the bulb. The alternating current was obtained from the secondary of a transformer, the primary coil of which was connected to the alternating town supply (100 volts), and the current through the filament could be varied by changing the resistance in both primary and secondary circuits. One of the fine iron wire resistances supplied with Nernst lamps was always kept in series with the filament. As this resistance has a large positive temperature coefficient, it tends to keep the temperature of the filament more steady than would otherwise be the case.

In order to start the filament glowing it was taken from the discharge tube and heated by holding it above, and near to, a glowing "heater" of the kind supplied with an ordinary Nernst lamp. It was then placed in the apparatus which could be rapidly exhausted, when required, by means of a water-pump, mercury-pump, and charcoal tube cooled in liquid air. In this way the gas pressure in the apparatus could be reduced to ·0001 mm. within twenty minutes from the time that the glowing filament was placed in position.

The temperature of the filament was determined by means of a Féry optical pyrometer, which was very kindly lent to the Cavendish Laboratory by Professor T. Mather, of the City and Guilds College, London. This instrument was standardised by using a platinum tube of about the same diameter as a Nernst filament, with a thermo-couple of fine wires of platinum and platinum-rhodium welded to it. In order to have the surface of the tube exactly similar to that of the filament, a filament was finely powdered and mixed with water, and the platinum tube was then covered with a thin layer of Nernst filament material by evaporating this mixture upon it. This platinum tube was fitted up in the place of the filament in the discharge bulb, and observations of the thermo-electromotive force and

* F. Horton, *Proc. Roy. Soc.* A. LXXXVIII. p. 117, 1913.

readings of the optical pyrometer were taken at several temperatures between 900° C. and 1600° C. From these observations the temperature of a Nernst filament corresponding to any reading of the pyrometer between these limits can be ascertained with fair accuracy. In most of the experiments described in this paper the actual temperature of the filament is of little consequence, for the pyrometer was usually employed simply to adjust the temperature to a constant value.

The negative emission from Lime.

A Nernst filament was heated in a good vacuum (the residual gas having a pressure of ·0001 mm.) until the negative current obtained from it under a potential difference of 214 volts had become fairly constant, a condition which is obtained in a much shorter time than when platinum is being experimented on. A series of observations of the thermionic emission from the filament at different temperatures was then taken, and the results are shown in the curve of fig. 1. In this ordinates represent

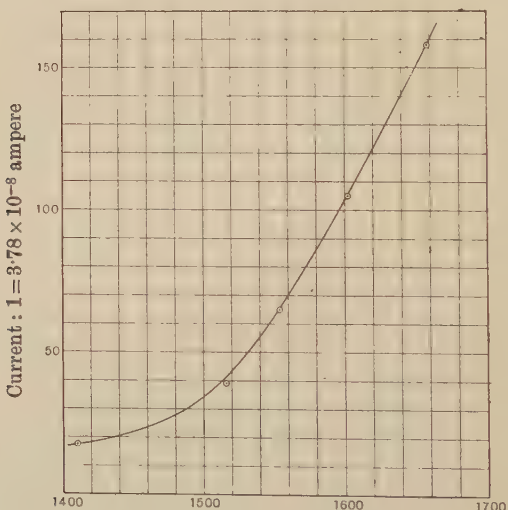

Fig. 1.

current, one division = $3{\cdot}78 \times 10^{-8}$ ampere, and abscissae represent temperatures on the centigrade scale. The curve given shows that at temperatures above about 1500° C. the thermionic current increases directly with the temperature and not in the

exponential manner usually obtained with substances heated at lower temperatures. Most of the current-temperature curves obtained with Nernst filaments at high temperatures showed this approximately linear relation, but the curve selected for fig. 1 is that in which the experimentally determined points lie most nearly on a straight line. A negative ionisation of about the magnitude represented by this curve was always obtained when a Nernst filament was heated in a good vacuum. The large potential difference of 214 volts was used to swamp the alternating potential difference of the heating current, which, between the ends of the filament, was about 60 volts when the filament was at 1500° C.

In the first experiments with lime-covered filaments, the lime was obtained by evaporating a strong solution of calcium nitrate on the filament and then igniting, but this process was found to spoil the filament, which became very difficult to start glowing, and when glowing would sometimes suddenly go out during the observations ; a filament so treated also broke through after being used for a short time. Pure lime (Kahlbaum, prepared from marble) was therefore powdered very finely in an agate mortar, and some of this powder was stirred up with distilled water. The lime in suspension in water was then placed upon the filament, a drop at a time, and the water was evaporated away by warming over the heater. The drops were all placed near the centre of the filament, and care was taken not to allow any of the liquid to go on to the platinum leads, although this precaution was probably unnecessary, for the leads never became visibly hot during the experiments. The evaporation of the lime water was continued until a uniform layer of lime was obtained over about 7 mm. of the middle of the filament. When obtained in this manner the layer of lime did not peel off on heating, and the filament could be started glowing just as easily as a new one. It should be mentioned that the filaments used were all of about the same dimensions, the length of the glowing portion being about 9·5 mm. and the diameter ·78 mm.

The glowing lime-covered filament was placed in the apparatus and the air-pressure was reduced. It was at once obvious that under these conditions the lime gives an enormous negative emission, for as the pressure was reduced to a few millimetres a brilliant glow appeared in the discharge tube, without the application of any electromotive force except that of the heating circuit. One of the platinum wire leads to the filament was kept earthed, and from the appearance of the discharge it was seen that the bare end of this wire, near the filament, was acting as an anode. By lowering the temperature of the cathode the luminous discharge could be stopped, but at the lower temperature it could

be started again by earthing the parallel plate anodes of the discharge tube by touching with the finger.

In a previous paper* the author has drawn attention to the different appearances of the luminosity obtained with a hot lime-covered platinum cathode. Under certain conditions of temperature, gas-pressure, and potential difference, the luminosity surrounds the anode only, and this luminosity always appears gradually when the conditions are slowly changed (*e.g.* the temperature slowly raised), and there is no sudden increase in the current passing through the tube. If the temperature is still further raised, or the potential difference increased, a point is reached when the luminosity suddenly leaves the anode and surrounds the cathode, and then, in some cases, luminous pencils of cathode rays are seen. These effects were also obtained with the lime-covered Nernst filament. With the temperature at 1500° C., and the gas-pressure ·0001 mm., a current of 28 milli-amperes was obtained. In this case 210 volts were put on from the high potential battery, but the magnitude of the discharge current was limited by a wire resistance of about 5500 ohms included in the circuit. The passage of the discharge greatly increased the temperature of the filament, which became much hotter near its lower end than at any other point. The gas-pressure in the apparatus was so low that no definite pencil of cathode rays could be seen, but there can be little doubt that the discharge was taking place mainly from the hottest point of the filament. The whole bulb was filled with a bluish glow (Hg vapour and CO), which could be best seen by looking in such a direction that the light from the filament itself was screened by one of the platinum plates forming the anode. The applied P.D. was lowered to 40 volts and the filament became cooler and equally luminous all over. The glow could still be seen in the bulb, but as the current was now very much reduced, this luminosity was doubtless an "anode glow." With 40 volts applied through a resistance of 5500 ohms to the terminals of the discharge tube, the following measurements of the thermionic current at different temperatures were taken:

Temperature (° C.)	1510	1626	1698	1780
Current (milli-amps.)	2·3	3·3	4·3	5·3

These currents do not increase so quickly with the temperature as is usually the case with the negative emission from glowing

* *Phil. Trans. Roy. Soc.* A. CCVII. p. 149, 1907.

solids, but the values given cannot be taken as measuring the corpuscular emission from the lime-covered filament, for the following reason. A resistance of 5500 ohms was included in the circuit to prevent damage to the high potential battery, and while this resistance remains constant during the experiments the resistance offered by the discharge tube diminishes greatly with increasing temperature. Thus the fall of potential across the tube decreases as the temperature of the filament is raised, and as the current is not saturated, its values at the different temperatures are not proportional to the number of corpuscles emitted by the filament. The potential difference required for saturating a thermionic current increases as the temperature rises, and at the same time the energy required to produce ionisation by collisions in the gas decreases. At the temperatures obtained with a glowing Nernst filament ionisation by collisions occurs under quite small electric forces—much smaller than that due to the heating circuit in these experiments—and even at the low pressure used it was not possible to obtain a saturation current.

A comparison of the results in the above table with those recorded in the curve of fig. 1 shows that the negative ionisation produced by a glowing Nernst filament is enormously increased by coating the filament with lime. It is difficult to say whether the emission is as large as that from lime-covered platinum, but the experiments at all events show that the order of magnitude is the same in both cases. To make a careful comparison of the two it would be necessary to work at much lower temperatures so as to avoid the complications introduced when the discharge becomes luminous. It was found to be extremely difficult to maintain the lime-covered filament at a constant temperature much less than 1500° C., and after several attempts the idea of trying to make an exact comparison was abandoned. The results described above are, however, sufficient to show clearly that the large negative emission obtained from lime heated on platinum cannot be attributed to the metal with which it is in contact.

The positive emission from Sodium Phosphate.

A new filament was used in these experiments, and the positive emission from this was first investigated. It was found that the emission in a good vacuum decreased on continued heating of the filament, but the decrease was not nearly so great as that obtained when platinum is first heated in a vacuum. The author has shown that the positive emission from platinum is largely due to absorbed gas, and in the case of a Nernst filament it may also be due to gas evolved during the heating, in which

case the falling off with time in the case of a freshly heated filament would not be expected to be so large as in the case of metals, for no doubt a great deal of the absorbed gas is driven off while the filament is being fitted into position and during the evacuation of the apparatus. When a filament had once been heated until the positive emission had been reduced to a steady value it was found that the emission quickly became steady on re-heating and re-testing, and the value under given conditions remained fairly constant. With the filament at about 1450° C., and with a potential difference of 205 volts (in addition to that due to the heating circuit), a positive thermionic current of $1{\cdot}36 \times 10^{-7}$ ampere was obtained at atmospheric pressure, and on gradually pumping down, the current increased to a maximum value, generally at about 30 mm. pressure, after which it decreased to a minimum at about 2 mm. pressure, and then rapidly increased again as the pressure was still further reduced. Sometimes, however, this final increase in the current did not occur, and the current decreased to a minimum value at the lowest pressure. The form of the current pressure curve for the positive emission from a Nernst filament is thus exactly similar to that given by platinum*.

The filament was next covered, except for about 1 mm. at each end, with a layer of pure sodium phosphate by evaporating a water solution of that salt upon it, a few drops at a time. It was then fitted into the discharge tube and the positive emission was re-tested. In the earlier experiments it was found that the phosphate volatilised away from the filament and condensed on the walls of the discharge tube. The heating current was therefore regulated so as to keep the filament glowing at as low a temperature as possible. Under these conditions it was rather difficult to start the filament glowing, and it often "went out" before the observations were begun; but ultimately I succeeded in obtaining a series of values of the positive emission at 1422° C. over a fairly wide range of gas-pressures. Before this series was obtained the filament was heated for about two hours in a pressure of ·002 mm. At the end of this time the emission had been reduced to a steady value. The gas-pressure in the apparatus was gradually raised to 56 mm., observations of the positive emission being taken at several pressures. After each alteration of pressure a few minutes were allowed for the current to become constant. The pressure was then gradually reduced again, and the observations of the current were repeated. Throughout the series the temperature of the filament was maintained at 1422° C. by adjusting the resistance of the heating circuit. The results

* F. Horton, *Proc. Roy. Soc.* A. LXXXVIII. p. 117, 1913.

obtained with decreasing pressures are plotted as a curve in fig. 2. In the following table some results at various pressures,

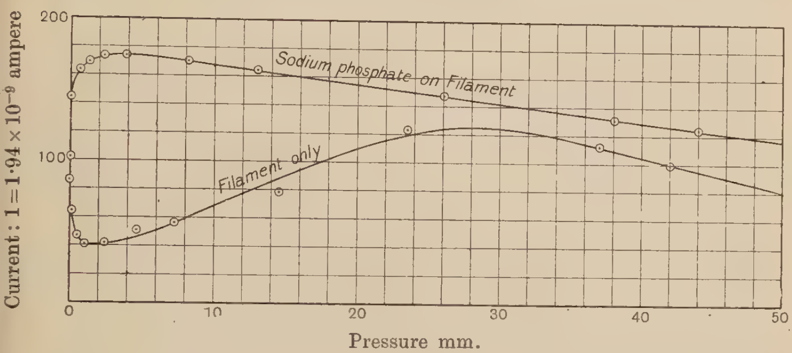

Fig. 2.

both increasing and decreasing, are given. The close agreement between the values in the two series shows that the emission had really been reduced to a steady state.

Pressure (mm.)	·0024	13·25	15·0	45·0	56·5
Thermionic current: Increasing pressures	92	160	163	123	114
Decreasing pressures	95	162	160	120	114

The curve in fig. 2 is similar to that which represents the connection between positive ionisation and pressure for sodium phosphate heated upon a platinum strip. In both cases the emission at first increases with decreasing pressure to a maximum value at a pressure of a few mm., after which it decreases rapidly as the pressure is still further reduced. In some earlier experiments* the maximum value of the positive emission from sodium phosphate heated upon platinum at 1190° C., under an applied potential difference of 200 volts, was $2{\cdot}25 \times 10^{-6}$ ampere per sq. cm., and occurred at about 10 mm. pressure. From the curve in fig. 2 the maximum current is obtained at about 3 mm. pressure, and its value is $1{\cdot}97 \times 10^{-6}$ ampere per sq. cm., the latter

* *loc. cit.*

figure being obtained on the supposition that the part of the filament which had been covered with phosphate was still covered when these observations were made. Since the current per sq. cm. calculated on this supposition is rather less than that given by platinum coated with sodium phosphate at a much lower temperature, it seems probable that in places the salt had volatilised away from the filament, so that the area covered when the observations were taken was really much less than that assumed in the calculation. The fact that the maximum emission is obtained at a lower pressure in the case of the Nernst filament is in agreement with the general characteristic of the positive emission from glowing solids, namely, that the pressure of maximum emission is lower the higher the temperature of the anode.

After these experiments with sodium phosphate, the filament was taken down and repeatedly washed in distilled water to remove any remaining sodium salt. It was then replaced in the apparatus and the positive emission at 1422° C. was again tested. The current-pressure curve obtained is also shown in fig. 2. This curve is similar to that given by a new filament after heating to a steady state, but the values of the currents shown in the curve are rather larger than those generally obtained. The difference in the form of this curve and that obtained with sodium phosphate on the filament is very marked. A similar difference is shown by the curves for platinum covered with sodium phosphate and for platinum alone. Evidently the increase of the emission to a maximum value at a few mm. pressure is due to the presence of the sodium salt on the anode; and the large emission obtained from sodium phosphate heated upon platinum is due to the salt itself and not to the platinum.

Dr W. Wilson, in his experiments*, could detect no positive emission from a Nernst filament covered with aluminium phosphate, but he obtained an emission while the heater connected with the filament was kept glowing, and he concluded that the latter effect was due to the fine platinum wire of the heater. In an earlier paper the author has shown that the activity of ordinary aluminium phosphate is due to sodium impurity which would immediately sublime away at the high temperature of the fully glowing filament, but would probably give a steady emission at the much lower temperature of the heater; so that the current obtained in Wilson's experiments while the heater was in operation may have been due to the material of which it is made, or to some of the phosphate which had fallen upon it.

The form of the current-pressure curve given by salts containing sodium is very interesting. In my paper on the discharge of

* W. Wilson, *Phil. Mag.* 6, xxi. p. 634, 1911.

positive electricity from sodium phosphate heated in different gases*, it is pointed out that this curve is similar to those obtained when the negative emission is under test, which suggests that ionisation by collisions comes in at certain pressures. In the paper referred to this view is ruled out because, at that time, it was generally thought that much larger electric forces than those employed were necessary to obtain ionisation by collisions with positive ions. Some experiments recently made by Dr Pavloff in the Cavendish Laboratory have, however, shown that ionisation by collisions with positive ions occurs even with potential differences of about 10 volts. It is thus possible that the shape of the curve obtained with sodium phosphate is due to ionisation of the gaseous molecules present, by collisions with the positive ions from the glowing anode.

The experiments made with uncoated filaments showed a pressure of maximum emission at about 30 or 40 mm. pressure, but near this maximum the thermionic current was very unsteady, varying capriciously over a range of as much as 50 per cent. in some cases. It was at first thought that this unsteadiness was due to the cooling effect of convection currents, but the emission at higher pressures was very much steadier, so it does not seem probable that convection is the cause. The points plotted in this part of the curve in fig. 2 represent the means of many determinations. In the case of the particular filament to which this curve refers, the pressure of maximum emission was at about 30 mm. The effect of placing sodium phosphate upon the filament is to reduce this pressure of maximum emission very considerably, so that at pressures of a few millimetres there is a very much greater ionisation from the phosphate covered anode than from the filament alone. The ionisation at all pressures is increased by the presence of the sodium salt, but the increase is most marked at pressures of a few millimetres.

The form of the current-pressure curve for a clean filament is similar to that given by a platinum strip which has been heated for a long time. In the latter case there are strong reasons for believing that the ionisation is due to gas evolved from the heated platinum, and it would appear probable that the ionisation from the Nernst filament is due to a similar cause. The question arises: can the form of the current-pressure curve be due to ionisation by collisions? The high pressure at which the maximum value of the current is obtained would seem to be against this view, but it must be remembered (*a*) that the electric force near the filament is very great; (*b*) that at the temperature employed, the mean free path in the surrounding gas or vapour

* *Proc. Camb. Phil. Soc.* XVI. p. 89, 1911.

is much larger than at ordinary temperatures, and the pressure of maximum current increases with both these quantities. I think, therefore, that the shape of the curves given by pure platinum and by a clean Nernst filament can be explained by ionisation by collisions coming in at certain pressures.

The effect of sodium phosphate is probably twofold: (1) it increases the formation of positive ions at the surface of the anode, and (2) it changes the nature of the gaseous material through which the discharge takes place; for the space round the anode now contains molecules of volatilised salt, or the products of dissociation of that salt. The pressure of maximum current is now much lower than before, and if ionisation by collisions is the cause of the existence of this maximum, it follows that the vapour now surrounding the anode is not so easily ionisable by collisions as was the gas previously in contact with it. On this view the increase of energy required for ionisation must be considerable to account for the large decrease in the pressure of maximum emission. If we may assume that an increase of temperature of the anode increases the percentage of difficultly ionisable molecules surrounding it (by increasing the volatilisation or dissociation of the salt) to a greater extent than is necessary to counterbalance the effect of the increased mean free path, we should expect that the higher the temperature of the anode the lower would be the pressure at which the maximum emission occurs. This is what is actually found to happen when series of observations of the positive emission are taken at different temperatures. In the case of the negative emission from glowing solids, the pressure of maximum current increases with the temperature, a fact which is explained by the larger mean free path in the surrounding gas at higher temperatures. If the above explanation of the effects obtained with the positive ionisation is correct, it follows that the difficulty in ionising the salt vapour (or its dissociation products) is experienced by the positive ions but not by electrons.

So far nothing has been said about the increase in the positive emission from the clean filament as the pressure is reduced below 1 mm. An exactly similar increase was noticed when pure platinum was being experimented upon, and it has been attributed to the mercury vapour which comes over from the pump. It is intended to test this view by taking precautions to prevent mercury vapour from entering the discharge tube.

The author wishes to acknowledge his indebtedness to the Government Grant Committee of the Royal Society for the means of purchasing some of the apparatus used in these experiments.

Fluctuations of sampling in Mendelian Ratios. By G. UDNY YULE, M.A., St John's College, University Lecturer in Statistics.

[*Read* 9 March 1914.]

In any series of Mendelian experiments, the observed proportions always exhibit greater or smaller deviations from expectation. In the material as a whole, if it be considerable, the deviation may be small: but if it be broken up into small sub-groups—*e.g.* the offspring of individual matings—the fluctuations may be very large indeed. For example, in the series of matings of Japanese waltzing mice × albino hybrids *inter se*, cited in Table A below, the expectation of albinos is 25 per cent., and the average proportion 137/555 = 24·68 per cent. is extremely close to this: but in the individual litters the percentages cover the whole range from 0 to 100 per cent. The question whether such fluctuations have any significance or whether they are merely the result of "pure chance," corresponding to the fluctuations that might be expected in the number of black balls drawn from a bag containing both black and white, is one that must no doubt have occurred to most workers in the subject. Mr Bateson and Miss Saunders, writing in the First Report of the Evolution Committee of the Royal Society (1902), were careful to emphasise that the numerical results are irregular and that the laws represent only an average result (pp. 10 and 127—8). In the Second Report (1905) a long series of results with peas (*Pisum*) is tabulated and it is stated that "the possibility that departures from the expected F_2 ratios might not be purely fortuitous was a special subject of enquiry" (p. 55). The conclusion given is that "the ratios found in individual plants, 62 for shape and 85 for colour, are not enough for full discussion, and a study of them does not, so far, suggest the presence of any disturbing factor" (p. 77). But this brief statement hardly seems to make the most of the material given in the Report.

It is difficult in fact to understand why more use has not been made of well-known results in the theory of sampling in order to compare the fluctuations observed in Mendelian experiments with those to be expected if the individual families, fruits—or whatever the sub-groups may be—are simply random samples of the whole possible material. If p be the chance of obtaining one of the two alternative characters (*e.g.* the dominant), q the chance of obtaining the other (*e.g.* the recessive), the relative frequencies of

samples of n individuals 0, 1, 2, etc. of whom shew the first character, are given by the terms of the binomial series $(q-p)^n$ or

$$q^n,\ nq^{n-1}p,\ \frac{n(n-1)}{1.2}q^{n-2}p^2,\ \frac{n(n-1)(n-2)}{1.2.3}q^{n-3}p^3, \ldots.$$

The mean of the series is np: its standard deviation is $(npq)^{\frac{1}{2}}$. If, for example, data are available for a number of litters of moderate size, litters of the same size n may be grouped together and for each such group the numbers of litters with 0, 1, 2, etc. recessives may be compared with the frequencies to be expected as given by the binomial series.

Where the sub-groups are of considerable size but not very numerous, as in counts of peas on different plants or of maize grains on different cobs, this complete comparison becomes impossible and we have to fall back on a simple comparison of the standard deviation of the observed proportions with expectation. So far as I know, not even this has been done in many instances. The work of R. H. Lock on maize, referred to further below, is a notable exception. For a number of samples, all of the same size n, the standard deviation of the proportion is $(pq/n)^{\frac{1}{2}}$: if the sizes of the samples vary, for n should be substituted the harmonic mean H of the numbers in the samples where $\frac{1}{H}=\frac{1}{N}\Sigma\left(\frac{1}{n}\right)$* if N be the number of samples. With the use of Barlow's Tables (Spon), for giving the reciprocals, $1/H$ is readily evaluated: the values of $1/n$ are written down straight from the tables, their arithmetic mean $r=1/H$ is formed, and the expected standard deviation is $(pqr)^{\frac{1}{2}}$.

The following illustrations of these theorems were for the most part obtained some years since and have frequently been used as examples in lectures, but with one exception they have not hitherto been published. I have put together these notes in the hope that they may stimulate those who are carrying out actual experiments to make more extensive and detailed tests on the same general lines. The work seems to me well worth doing.

I know of no data sufficiently extensive to give a thoroughly satisfactory test against the binomial distributions. From A. D. Darbishire's data respecting his crosses of Japanese waltzing mice with albino mice (*Biometrika*, III., 1904, p. 1) tables can be compiled in the required form, but there are only 121 litters for the case in which the expectation is 25 per cent., and 132 for the expectation 50 per cent. These numbers are too small for any close agreement with theory to be expected in the case of the

* Cf. Yule, *Introduction to the Theory of Statistics*, XIII. § 11.

few litters of each individual size, but a fair comparison results when the figures for litters of all sizes are totalled together. Table A shews what is meant. The figures in ordinary type give

TABLE A. *Hybrids of Japanese waltzing mice × albinos paired together: expectation of albino* 25 °/₀. *Observed* (*roman*) *and calculated* (*italic*) *numbers of litters of each size with* 0, 1, 2,... *etc. albinos. Data from Darbishire, Biometrika,* III.

Size of litter	Number of albinos in the litter							Total litters
	0	1	2	3	4	5	6 or more	
1	7 *5·25*	— *1·75*						7
2	3 *6·19*	7 *4·12*	1 *0·69*					11
3	7 *6·75*	5 *6·75*	4 *2·25*	— *0·25*				16
4	5 *5·39*	9 *7·18*	3 *3·59*	— *0·80*	— *0·04*			17
5	9 *6·17*	9 *10·28*	3 *6·86*	3 *2·28*	1 *0·38*	1 *0·03*		26
6	8 *5·52*	9 *11·03*	8 *9·20*	6 *4·09*	— *1·02*	— *0·14*	— *0·01*	31
7	1 *1·47*	4 *3·43*	3 *3·43*	2 *1·90*	1 *0·63*	— *0·13*	— *0·01*	11
8	— *0·10*	— *0·27*	1 *0·31*	— *0·21*	— *0·09*	— *0·02*	— —	1
9	— *0·08*	— *0·23*	1 *0·30*	— *0·23*	— *0·12*	— *0·04*	— *0·01*	1
Total obs.: calc.:	40 *36·92*	43 *45·04*	24 *26·63*	11 *9·76*	2 *2·28*	1 *0·36*	— *0·03*	121 *121·02*

the observed numbers of litters of each size with a specified number of albinos; the italic figures are those calculated from the respective binomial series. The expectations being 3/4 and 1/4, the expected proportions of litters of 1 with no albinos and 1 albino are 3/4 and 1/4: the expected proportions of litters of 2 with 0, 1, and 2 albinos 9/16, 6/16 and 1/16—and so on. The

frequencies in the individual compartments of the table are too small to shew close agreement between observation and theory, but if the observed and calculated frequencies are totalled for all sizes of litter, as in the bottom row of the table, the agreement shewn is very close indeed—closer than would be found as a rule. If Professor Pearson's test be applied (*Phil. Mag.*, July 1900, and Elderton's tables, *Biometrika*, I.), $\chi^2 = 0{\cdot}780$, $P = 0{\cdot}94$, or a worse fit between observation and theory is to be expected 94 times out of 100. Reducing the cases for which the expectation is 50 per cent. in the same way, I find the figures for all sizes of litters given in Table B. Here the agreement is not quite so good, as the observed data for litters of five or more are irregular, but it is quite fair. I find $\chi^2 = 8{\cdot}003$, $P = 0{\cdot}238$, or a worse fit might be expected nearly one time in four*. The same data may be otherwise dealt with by working out the percentage of albinos in every litter and comparing the standard deviation of such percentages with $(pq/H)^{\frac{1}{2}}$. I find:

Data of Table A.

$H = 3{\cdot}53$; s.d. theo. $23{\cdot}04$; actual $23{\cdot}09 \pm 1{\cdot}00$.

Data of Table B.

$H = 4{\cdot}73$; s.d. theo. $22{\cdot}98$; actual $21{\cdot}63 \pm 0{\cdot}90$.

The agreement here is very close.

TABLE B. *Waltzing mice × albino hybrids crossed with albinos. Observed numbers of litters with* 0, 1, 2,... *albinos and numbers calculated (as in Table A) from expectation* 50 %. *Data from Darbishire, Biometrika*, III.

Number of albinos in litter	Number of litters		Number of albinos in litter	Number of litters	
	Observed	Calculated		Observed	Calculated
0	6	6·14	6	1	4·48
1	21	19·98	7	0	1·28
2	30	31·71	8	1	0·26
3	37	32·90	9 or more	—	0·03
4	18	23·41			
5	18	11·83			
			Total	132	132·02

Turning now to the data for peas (*Pisum*) tabulated by individual plants in the Second Report of the Evolution Committee

* In calculating P the frequencies of 4 albinos and upwards in the first case, and of 6 albinos and upwards in the second, were grouped.

referred to above, it is evidently no longer possible to give the full comparison with the binomial series as the range in number of seeds per plant is too great and the plants too few, and a comparison of the actual standard deviation of the percentages for individual plants with the standard deviation $(pq/H)^{\frac{1}{2}}$ is all that can be effected. Table C shews the results. The first four lines

TABLE C. (*Data from Second Report of the Evolution Committee of the Roy. Soc. (W. Bateson and Miss Killby).) Comparison of actual and calculated standard deviations of percentages of recessive seeds in F_2 on different plants in peas. Expectation* 25 °/ₒ.

Table in original	Characters: dominant first	Number of plants	Mean percentage of recessive seeds	Standard deviation and probable error (per cent.)	$\sqrt{pq/H}$ (per cent.)
I	Yellow and green	84	25·4	6·55 ± ·34	7·31
III	,, ,,	86	24·3	4·86 ± ·25	5·30
II	Round and wrinkled	64	23·8	6·05 ± ·36	6·48
III	,, ,,	86	24·3	5·59 ± ·29	5·30
I, III	Yellow and green	170	24·8	5·79 ± ·21	6·39
II, III	Round and wrinkled	150	24·1	5·80 ± ·23	5·84
Omitting from Tables I and II plants with less than 50 seeds, and from III plants with less than 100.					
I	Yellow and green	52	24·9	4·40 ± ·29	4·40
III	,, ,,	43	24·3	4·23 ± ·31	3·71
II	Round and wrinkled	42	25·1	4·24 ± ·31	4·38
III	,, ,,	43	23·8	3·57 ± ·26	3·71

of the table give the standard deviations of the percentages of yellows for the plants included in Tables I and III of the original, and of the percentages of rounds for the plants tabulated in Tables II and III. In three cases out of the four the observed standard deviation is less than that calculated, though the differences are within the possible limits of fluctuations of sampling (taking the probable error of a standard deviation as given approximately by the formula for the case of normality $0{\cdot}6745\sigma/\sqrt{2n}$). Grouping together the data respecting yellows and greens from Tables I and III, and similarly the data respecting rounds and wrinkleds from Tables II and III, the results shewn in the next two lines are obtained: the observed standard deviation is less

than that calculated in each case, and for yellows and greens the difference is nearly three times the probable error. Speaking generally, however, the agreement is fairly close, and the data hardly suggest any fluctuations of physical significance. The range of the number of seeds per plant is, however very high—from a mere half-dozen seeds up to two or three hundred—and it was suggested to me by Mr Darbishire that the poor plants might be exceptional, and that it might be worth while to try the effect of their exclusion. Excluding plants with less than 50 seeds in the case of Tables I and II of the original, and plants with less than 100 seeds in the case of Table III, the standard deviations obtained are given in the last four lines of Table C. Here the agreement is distinctly closer than before. Finally use may be made of the fact that Table III of the Report deals with the two characters yellowness and roundness in combination. The expectation of yellow-rounds is 9/16 or 56·25 per cent. and the mean percentage given by all the plants of Table III of the Report is close to this. Including all plants the actual standard deviation (Table D) is lower than that calculated by 2·8 times the probable error: omitting plants with less than 100 seeds the agreement is extremely close. In the case of the combination of characters as in the case of the single character there is no evidence of any significant fluctuation.

TABLE D. (*Data from Table III, loc. cit. above.*) *Standard deviations of percentage of the pair of dominant characters, yellow-round seeds, in F_2. Expectation* 9/16 *or* 56·25 %.

	Number of plants	Percentage of yellow-rounds	Standard deviation and probable error	$\sqrt{pq/H}$
All plants	86	56·51	5·36 ± ·28	6·13
Excluding plants with less than 100 seeds...	43	56·77	4·27 ± ·32	4·25

I turn now to Mr Lock's paper on his experiments on maize (*Annals of the Botanic Gardens, Peradeniya*, III. 1906). Mr Lock himself worked out for one of his most extensive tables (Table 33) for a $DR \times RR$ cross the probable error and its theoretical value for simple sampling, calculating the actual value by a graphic method: he also pooled together all the cases of expectation 50 per cent. and made the same comparison. In both cases the actual somewhat exceeded the expected value of the probable error though the difference was not great. The results of my own work on Mr Lock's data are shewn in Table E. For three

tables in which the expectation is 25 per cent. the agreement is good, very good considering the small number of cobs available. When these three tables are pooled together (without respect to the fact that they deal with different characters) the actual s.d. is 2·05 per cent. against an expected value 1·99.

TABLE E. (*Data from Lock, Annals of the Botanic Gardens, Peradeniya,* III. 1906.) *Comparison of actual standard deviations and s.d.'s of sampling in experiments with maize.*

Table in original	Characters	No. of plants	Mean percentage of recessive grains	Standard deviation and probable error (per cent.)	$\sqrt{pq/H}$ (per cent.)
I	Starchy and sugary	18	24·8	2·04 ± ·23	2·20
II	,, ,,	22	23·4	1·96 ± ·20	2·23
XXXIV	Yellow and white	30	25·5	1·62 ± ·14	1·63
I, II, XXXIV	—	70	24·7	2·C5 ± ·12	1·99
III, IV, V, VI, VII	Starchy and sugary	74	50·9	3·17 ± ·18	2·60
VIII, IX, X, XI, XII	Yellow and white	49	50·2	3·16 ± ·22	2·72
XXXIII	,, ,,	95	50·0	2·48 ± ·12	2·16

In the case of the $DR \times RR$ crosses, however, the agreement is by no means so good, the actual exceeding the expected standard deviation by two to three times the probable error in each case. The divergence in the second group is largely due to the five plants of Table IX which give percentages of whites ranging from 43·3 to 56·6 and 56·9 on 644, 648 and 515 grains respectively: with standard errors of 2 to 2·2 per cent., such deviations are not probable as the results of random sampling. But, as pointed out by Mr Lock, the data as a whole shew a rather greater variation than one would expect, and the result cannot be attributed merely to one or two particular tables. The difficulty is to imagine any cause for such excessive variation (such as fertilisation by pollen from DD's or DR's instead of RR's, incorrect sorting of doubtful yellows or unsuspected heterogeneity of the whites) which would not also affect the means obtained, and the means agree very well with expectation. The contrast between the DR's selfed and the $DR \times RR$ crosses is curious. The former certainly do not suggest any significant fluctuation: the latter do, on the whole, suggest some source of disturbance or possibly error. I wish more data, for other plants or for

animals, were available for this cross. It seems to me of especial interest as it is only on the heterozygote that there are differences between the gametes.

Some years ago Miss E. R. Saunders was good enough to put at my disposal extracts from her records of experiments on stocks to test the question whether the proportions shewed any significant fluctuation when they were based on the seeds from individual fruits, and not merely on seeds from each plant as a whole. To avoid possibly serious disturbance of the proportions recorded, I excluded in the case of the plant characters all pods in which less than 70 per cent. of the seeds survived. The following are the results:

Characters	Expected ratio	No. of fruits	Actual s.d. of the percentages	$\sqrt{pq/H}$ (per cent.)
Hoariness and smoothness	1 : 1	20	6·39 ± ·68	8·62
,, ,, ,,	3 : 1	102	7·11 ± ·34	6·80
,, ,, ,,	9 : 7	236	9·01 ± ·28	8·92
Singles and doubles	3 : 1	105	8·64 ± ·40	8·09
Seed colours: Green, not-green	27 : 37	28	9·46 ± ·85	7·80
,, ,, ,,	9 : 7	53	8·13 ± ·53	7·28
,, ,, ,,	3 : 1	40	5·84 ± ·44	6·70

In five cases out of the seven the observed are greater than the predicted standard deviations, but the excesses are within the limits of sampling. Even in spite of the restriction mentioned the observed percentages of the plant characters may be somewhat disturbed by losses, so that even if the excessive variation be regarded as significant it does not follow that in these cases it represents a deviation from pure chance in the distribution of the gametes amongst the fruits. In the case of the seed colours Miss Saunders informs me that the classification was carried out chiefly for reasons of convenience, and as it was not the main object of the experiments the time and labour that would have been entailed by the strictest accuracy was not given to the sorting of difficult cases—and there are difficult cases, such as parti-coloured seeds and seeds which have remained green because they have died before ripening. Hence there may be a small proportion of missorted seeds, tending to increase the fluctuation. If the results cannot be said definitely to disprove the existence of significant fluctuation, apart from fluctuations due to varying death-rates in different groups of plants, neither do they give any certain evidence of its existence.

A Case of Repulsion in Wheat. By F. L. ENGLEDOW, B.A., St John's College. (Communicated by Professor Biffen.)

[*Read* 9 March 1914.]

The plants which supplied the evidence of repulsion comprised the second generation (F_2) of the cross

Smooth Black × Essex Rough Chaff

and the two characters concerned were:

(1) Roughness of Chaff,

(2) Black Colour of Chaff.

The term chaff, as used here, refers to the glumes only of the ear. It is, however, practically certain that an examination of the outer paleae would justify the inclusion of these parts with the glumes as far as the experimental characters are concerned.

The following details concerning the parents suffice for the purposes of this paper.

"Smooth Black" is a variety of wheat obtained by Professor Biffen as one constituent of the second generation of the cross

Rivet × Fife.

Its genetic constitution has not yet been determined but, as two years of growing have shown, it breeds true.

The glumes are glabrous and of a deep and burnished black colour, the black pigment occurring in the sub-epidermal tissues.

Types which possess glumes of this nature will be referred to as "Smooth Black."

"Essex Rough Chaff" is, agriculturally, a very familiar variety. In common with most other types of wheat it has glumes entirely devoid of the black colour which characterises the other parent. There is a further distinction between its glumes and those of Smooth Black, viz. the presence of numerous hairs. This combination of characters in the glume is designated by the term "Rough White."

In all, the second generation which resulted from the crossing of these two varieties contained 213 plants. Sorting by eye—which was fully confirmed by an examination with the dissecting microscope—furnished the following classification:

Rough Black	Rough White	Smooth Black	Smooth White
120	43	47	3

If repulsion occur on a 1 : 3 :: 3 : 1 basis between "Roughness" and "Blackness," the theoretical expectation for this case is:

109·8 49·9 49·9 3·3

The results of the single character classifications are as follows:

Rough : Smooth :: 163 : 50
Black : White :: 167 : 46

the expectation being in each case 160 : 53. The probable error of the number of dominants is 4·3 and hence the agreement with expectation is satisfactory.

In another note* there appears a method of determining the best coupling or repulsion series for a set of observed data. Employing that method for this case, the best series is found to be:

1 : 2·56 :: 2·56 : 1.

In the paper already referred to, the results are examined by Pearson's Method† for the probability of the fit of an observed to an expected series. It is apparent from the examination that the probability of the existence of repulsion on the 1 : 3 :: 3 : 1 basis between "Roughness" and "Blackness" in wheat is quite as great as that of the existence of coupling and repulsion which has been described in the cases of other plants.

A point of some interest lies in the fact that blackness appears not to be a simple character—not simple in the sense of not being present always to the same extent. Some of the second generation plants are so black that they are indistinguishable from the black parental type, while others show merely a small black patch on the glume. The interval between these extremes is fairly uniformly filled by plants of intermediate blackness. It must however, be remarked that blackness when present is readily detected. White glumed plants which have been discoloured by disease sometimes resemble plants with small black patches but the patches caused by disease are not similarly placed on all the glumes of the ear and they may be almost completely removed by scraping gently with a sharp knife.

The presence of blackness to so varying a degree can with difficulty be conceived to be produced and controlled by one factor only. One factor or combination of factors common to all the black plants and accompanied in some cases by an intensor or partially inhibitor factor is the explanation which at once suggests itself.

* Yule and Engledow, *Proc. Camb. Phil. Soc.*, Vol. XVII., Part 5, "The Determination of the Best Value of a Coupling Ratio from a given set of data."
† Pearson, *Phil. Mag.* vol. L. 1900.

That still other factors must be assumed to be concerned in this colour effect is clear from an examination of the parentage of "Smooth Black." The black appeared in the second generation of a cross in which neither of the parents showed the black colour. This, of course, would lead us to assume that in each parent is a factor which alone cannot produce blackness but united with the other has this effect. No simple explanation can, however, be furnished on these lines and it seems probable that before the question can be settled more knowledge will have to be obtained of the nature of true "black" and "grey." Transverse sections of the glumes may be of some assistance in this respect. Meantime, the repulsion above described seems certain and from the nature of the case there appears to be a strong probability that Rivet "grey" and true "black" are very closely related. It is hoped to elucidate this matter by further crosses next summer.

A double confirmation of the existence of repulsion between blackness and roughness seems possible. The doubly heterozygous members of the second generation ought to exhibit repulsion; and coupling between the two characters should result from the crossing of "Rough Black" and "Smooth White." These two types can, of course, be obtained from the second generation which exhibited the repulsion.

The determination of the best value of the coupling-ratio from a given set of data. By F. L. ENGLEDOW, B.A., St John's College, and G. UDNY YULE, M.A., St John's College, University Lecturer in Statistics.

[*Read* 9 March 1914.]

Many workers in Mendelism who have come across cases in which coupling or repulsion occurred must have felt the necessity for some general method by which to determine from their data the best value to assign to the coupling-ratio, apart from any theory as to the ratios that are possible. Mr G. N. Collins (*Am. Nat.*, vol. XLVI., 1912) is, so far as we are aware, the only writer who has suggested any such method. He worked out the value of a coefficient of association for the whole series of possible ratios, 1 : 1 : 1 : 1, 2 : 1 : 1 : 2, etc., and then used the observed value of the same coefficient to decide which ratio gave the best agreement with the facts. While this method is very simple and convenient, it does not seem to lead to the most advantageous value for the ratio.

The test to be used for the closeness of agreement between the theoretical and observed frequencies seems clearly to be that developed by Professor Pearson (*Phil. Mag.*, vol. L., 1900). If $F_1F_2F_3F_4$ etc. are a set of theoretical or expected frequencies and $F_1'F_2'F_3'F_4'$ etc. are those observed, and if

$$\chi^2 = \Sigma \frac{(F' - F)^2}{F},$$

the probability P that in random sampling deviation-systems of equal or greater improbability will arise is a function of χ^2 which decreases continuously as χ^2 increases. The values of this function for any number of frequencies from 3 to 30 have been tabulated by Mr Palin Elderton (*Biometrika*, vol. I.). In order to measure the closeness of agreement between an observed set of the four frequencies for any pair of characters, and the expectation based on any assumed ratio, it is only necessary to work out the value of χ^2 and turn up in Mr Elderton's table the column headed $n' = 4$, where the probability that an equally bad or worse set of deviations might arise in sampling will be found. If P is high, the agreement is good; if low, it is bad. That value of the ratio, then, which gives the most satisfactory agreement with the data is the value which makes the probability P a maximum or χ^2 a minimum. The value of P is not accurate if any frequencies are small, as a

normal distribution of errors is assumed in the calculation of the tables, but even from the empirical point of view the method suggested seems much better than any now in use.

Suppose the two factors to be A and B, and let the gametes be produced by the heterozygote in the following proportions:

AB	Ab	aB	ab
p	$(0{\cdot}5-p)$	$(0{\cdot}5-p)$	p

then, assuming random mating, zygotic forms will be produced in the proportions:

AB	Ab	aB	ab
$(p^2+0{\cdot}5)$	$(0{\cdot}25-p^2)$	$(0{\cdot}25-p^2)$	p^2.

Let the observed *proportions* of zygotes be $f_1 f_2 f_3 f_4$, where $f_1+f_2+f_3+f_4=1$. Then we have to make a minimum the quantity

$$\frac{(p^2+0{\cdot}5-f_1)^2}{p^2+0{\cdot}5}+\frac{(0{\cdot}25-p^2-f_2)^2}{0{\cdot}25-p^2}+\frac{(0{\cdot}25-p^2-f_3)^2}{0{\cdot}25-p^2}+\frac{(p^2-f_4)^2}{p^2}.$$

Differentiating with respect to p and equating to zero, we find

$$\begin{aligned}(f_2^2+f_3^2-f_1^2-f_4^2)\,p^8+(0{\cdot}5\,f_1^2+f_2^2+f_3^2-0{\cdot}5\,f_4^2)\,p^6&\\ +(0{\cdot}25\,f_2^2+0{\cdot}25\,f_3^2+0{\cdot}1875\,f_4^2-0{\cdot}0625\,f_1^2)\,p^4&\\ +0{\cdot}0625\,f_4^2\,p^2-0{\cdot}015625\,f_4^2=0 \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots(1).&\end{aligned}$$

This is an equation of the fourth degree for p^2. A first approximation to the root required may be obtained by Collins' method or from the formula

$$p^2=0{\cdot}25\,(f_1+f_4-f_2-f_3) \quad \ldots\ldots\ldots\ldots\ldots\ldots(2)$$

(which gives the value of p that makes the sum of the squares of differences least), or by comparison with various calculated series, and the solution is then readily obtained by Newton's method.

To take the data of the preceding note as an illustration, the values of the proportions f are 0·5634, 0·2207, 0·2019, and 0·0141; writing x for p^2, this gives the equation

$$\begin{aligned}-0{\cdot}228147x^4+0{\cdot}248083x^3+0{\cdot}002566x^2&\\ +0{\cdot}00001244x-0{\cdot}0000031094=0.&\end{aligned}$$

The data suggested a gametic ratio 1 : 3 : 3 : 1, which gives $p=0{\cdot}125$, $p^2=0{\cdot}015625$. Trial shewed that 0·0156 was not a very close approximation to a root; 0·02 proved nearer to a solution, and Newton's method gave by two approximations $p^2=0{\cdot}019715\ldots$. Hence $p=0{\cdot}1404$ and this gives a ratio 1 : 2·56. The observed frequencies were then compared with the frequencies

to be expected from this ratio, and from the ratio 1 : 3 : 3 : 1. The results obtained were

Ratio 1 : 3	$\chi^2 = 2.0974$	$P = 0.554$
„ 1 : 2·561	$\chi^2 = 1.9918$	$P = 0.574$

It will be observed that while the calculated ratio does give the better agreement, the difference is slight. In both cases results equally or more divergent from expectation would occur nearly as often as not owing to mere fluctuations of sampling. The result is an illustration of the now recognised fact that a considerable alteration in the coupling-ratio may mean but a small alteration in the closeness of fit.

Two other cases have been tried and gave the following results. Collins (*loc. cit.*, p. 579) gives the following data for the characters coloured aleurone and horny endosperm in maize.

Coloured-horny	1774
Coloured-waxy	263
White-horny	279
White-waxy	420

We find $p = 0.3891$ or a ratio 3·509 : 1. For this value of the ratio χ^2 is 0·60435 or $P = 0.947$, the calculated frequencies 1782, 270, 270, 414 being in very close agreement with those observed. For the 3 : 1 ratio, χ^2 is 9·106 or $P = 0.028$, and the divergence is therefore one that would only be likely to occur once in some 36 trials owing to the fluctuations of random sampling.

Finally we took the data given by Bateson, Saunders and Punnett in the Fourth Report of the Evolution Committee (p. 16) for coupling between dark axils and fertility in sweet peas. Here we find $p = 0.4745$, which is equivalent to a ratio 18·608 : 1, as compared with the ratio 15 : 1 suggested in the Report and a value "about 20 : 1" by Collins. The relative merits of the ratios are apparent from the following:

Ratio 18·608 : 1	$\chi^2 = 3.7539$	$P = 0.294$
„ 15 : 1	$\chi^2 = 5.9226$	$P = 0.116$
„ 20 : 1	$\chi^2 = 3.8975$	$P = 0.275$

The ratio 15 : 1 is clearly much the poorest of these three: a worse fit is only likely to occur, owing to fluctuations of sampling, some 12 times in 100. A worse fit than that given by 20 : 1 may occur some 27 times in 100, and a worse fit than that given by our calculated ratio some 29 times in 100. The figures again shew, however, how great differences may be made in the coupling-ratio assumed without creating an impossible discordance between assumptions and fact. The mere agreement of the data, within the possible limits of fluctuations of sampling, with the frequencies

deduced from some assumed ratio—as in the case of the above data for peas and the ratio 15 : 1—is very slight evidence in favour of the truth of the assumption, especially where the coupling-ratio is high, at least with such moderate numbers of observations as are at present available. Some light might, however, be thrown on the theory of reduplication by carrying out an examination of all the available cases, determining p or the coupling-ratio for each by equation (1). Such an examination we hope to carry out.

As p is not expressed explicitly as a function of the proportionate frequencies f by equation (1), we do not see our way to give its probable error by this method of determination. The value given by (2), however, is in some cases close to the value given by (1), viz. if no one of the frequencies is very small (cf. the data below), and its standard error can be determined without difficulty on the usual, though hardly quite justifiable, assumption that deviations in the frequencies are small compared with their mean values. As the standard errors by the two methods of determination are likely to be of the same order of magnitude, it seems worth while stating the result as at least a rough guide to the possible magnitude of fluctuations. Differentiating both sides of equation (2), squaring and summing, we have, utilising known results for the sums of squares and product sums (cf. *e.g.* Yule, *Jl. Stat. Soc.* 1912, p. 601),

$$\epsilon_p{}^2 = \frac{1}{4N} \frac{(f_1+f_4)(f_2+f_3)}{(f_1+f_4)-(f_2+f_3)} \quad \ldots\ldots\ldots\ldots\ldots(3),$$

where ϵ_p is the standard error of p (to be multiplied by 0·6745 to obtain the probable error) and N is the number of observations. If there is coupling ($p > 0·25$), the coupling-ratio $r = p/(0·5 - p)$. Differentiating, squaring and summing again, we have

$$\epsilon_r{}^2 = \epsilon_p{}^2 \frac{1}{4(0·5-p)^4} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(4).$$

Case	Number of observations	Value of p from		r from		Standard error of the values from (2)	
		(1)	(2)	(1)	(2)	p	r
Wheat	213	0·1404	0·1968	2·56	1·54	0·0430	0·56
Maize	2736	0·3891	0·3885	3·51	3·48	0·0049	0·20
Peas	885	0·4745	0·4744	18·6	18·5	0·00385	2·94

If there is repulsion ($p < 0·25$), the repulsion-ratio is $(0·5-p)/p$ and p^4 must be read for $(0·5-p)^4$ in the denominator of the above

expression. The table shews for comparison the values of p and r given by equation (1) and equation (2) respectively, and the standard errors of p and of r as obtained by the latter method.

In the first case the two equations give very divergent results, the unsuitability of equation (2) for general use being shewn by its failure to give a good approximation to the best value of the ratio. In this case, no doubt, we must also regard the standard error of r (0·56) as of very uncertain validity. The magnitude of the standard error of r in the last case—nearly 3 units—again emphasises the caution that must be used before attaching importance to the precise values of these high coupling-ratios.

*A Contribution to the Theory of Relative Position**. By Norbert Wiener, Ph.D. (Communicated by Mr G. H. Hardy.)

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The theory of relations is one of the most interesting departments of the new mathematical logic. The relations which have been most thoroughly studied are the *series*: that is, relations which are contained in diversity, transitive, and connected or, in Mr Russell's symbolism, those relations R of which the following proposition is true:

$$R \Subset J \,.\, R^2 \Subset R \,.\, R \cup \breve{R} \cup I \upharpoonright C\text{‘}R = C\text{‘}R \uparrow C\text{‘}R.$$

Cantor, Dedekind, Frege, Schröder, Burali-Forti, Huntington, Whitehead, and Russell, are among those who have helped to give us an almost exhaustive account of the more fundamental properties of series. There is a class of relations closely allied to series, however, which has received very scant attention from the mathematical logicians. Examples of the sort of relation to which I am referring are the relation between two events in time when one completely precedes the other, or the relation between two intervals on a line when one lies to the left of the other, and does not overlap it, or, in general, the relation between two stretches α and β, of terms of a series R, when any term lying in α bears the relation R to any term lying in β. Relations of this sort, which I shall call relations of complete sequence, differ in general from series in not being *connected*: that is, for example, it is not necessary that of two distinct events, each of which wholly precedes or follows some other event, one should *wholly* precede the other, for the times of their occurrence may overlap. But in all the instances we have given, the relation of complete sequence is closely bound up with some serial relation: the relation of succession between the events of time is intimately related to the series of its instants, the relation between two intervals on a line one of which lies completely to the other's left is intimately related to the series of the points on the line, and so on. These considerations lead us to the general questions, (1) what are the formal properties which characterise relations of the sort we have

* The subject of this paper was suggested to me by Mr Bertrand Russell, and the paper itself is the result of an attempt to simplify and generalize certain notions used by him in his treatment of the relation between the series of events and the series of instants.

called relations of complete sequence? and (2) what is the nature of the connection between relations of complete sequence and series?

One very general property which belongs to relations of the sort we have called relations of complete sequence is that they never hold between a given term and itself. This property—that of being contained in diversity—they share with series proper. Writing cs for the class of relations of complete sequence, we can represent this fact in the symbolism of the *Principia Mathematica* of Whitehead and Russell by the formula

$$\mathrm{cs} \subset \mathrm{Rl}{}^{\backprime}J.$$

Another property they share with series is that of transitivity. If, for example, the event x wholly precedes the event y, while the event y wholly precedes the event z, the event x wholly precedes the event z. But they possess another property more powerful logically, which may be called a generalized form of transitivity. If the event x wholly precedes the event y, and the event y neither wholly precedes nor wholly follows the event z, while the event z wholly precedes the event w, then the event x will wholly precede the event w. All the other relations which we have mentioned as examples of relations of complete precedence will be found to possess the same property, which, moreover, will be satisfied by all those relations which we would naturally call relations of complete precedence. We may, then, so define "relations of complete precedence" as to regard this as a property common to all such relations. In symbols, we shall then have

$$\vdash . \mathrm{cs} \subset \hat{R}\{R \mid (\dot{-}R \dot{-} \breve{R}) \mid R \mathrel{\ooalign{$\subset$\cr\hidewidth$\cdot$\hidewidth}} R\}.$$

The relation $(\dot{-}R \dot{-} \breve{R})$, with its field limited to that of R, is what we ordinarily know as simultaneity. In most theories of time and of relations of complete precedence, it has been thought necessary to treat precedence and simultaneity as coördinate primitive ideas. Nevertheless, those who hold such theories have to assume such propositions as the following, in order to make simultaneity and precedence possess the appropriate formal properties*:

$$\vdash . S \dot{\cap} P = \dot{\Lambda},$$

$$\vdash . S \mathrel{\dot{\cup}} P \mathrel{\dot{\cup}} \breve{P} = C{}^{\backprime}S \uparrow C{}^{\backprime}S,$$

$$\vdash . S \mathrel{\dot{\subset}} \breve{S},$$

$$\vdash . C{}^{\backprime}S = C{}^{\backprime}P.$$

* In the following list of propositions, S stands for 'is simultaneous with,' and P for 'precedes.'

From these it is an easy matter to deduce that

$$\vdash . S = (\dot{-} P \dot{-} \breve{P}) \upharpoonright C\text{‘}P,$$

while on the hypothesis that $P \subset J$, the converse deduction can readily be made. Therefore, we may define simultaneity as that relation which holds between x and y when both either follow or precede something and neither precedes the other. The second property of relations of complete sequence may, then, be interpreted to state that if R is such a relation, then if xRy, y-is-simultaneous-with-respect-to-R to z*, and zRw, then xRw.

We shall find that most of the properties of relations of the sort of complete temporal succession between events follow from the two conditions which we have mentioned above—indeed, many of the most important ones follow from the second alone—so that we shall *define* a relation of complete succession as one which satisfies those two conditions: in other words, we shall make the following definition:

$$*0{\cdot}01\dagger. \quad \mathrm{cs} = \mathrm{Rl}\text{‘}J \cap \hat{R}\,\{R \mid (\dot{-} R \dot{-} \breve{R}) \mid R \subset R\} \quad \text{Df.}$$

Moreover, as we shall have frequent cause to refer to the relation $(\dot{-} P \dot{-} \breve{P}) \upharpoonright C\text{‘}P$, and as this expression is rather unwieldy, we shall abbreviate it as follows:

$$*0{\cdot}02. \quad P_{se} = (\dot{-} P \dot{-} \breve{P}) \upharpoonright C\text{‘}P \quad \text{Df.}$$

Now the question arises, how are the members of cs related to series? How, for example, is the relation between an event and another that completely succeeds it related to the relation between an *instant* and another that follows it? Two methods of procedure are open to us; we may define an event as a class of instants, and derive succession between events from that between instants, or we may define an instant as the class of all the events that occur at it. Both these methods seem to have certain inherent disadvantages: if we choose the first method, then we cannot consider the possibility of several events occurring with the same times of beginning and ending, whereas if we choose the second alternative, we cannot consider the possibility of all the events of one moment happening also at another and vice versa. However, we shall choose the latter method of procedure, since cs is a more general notion than ser. This can be proved as follows:

$$\begin{aligned} \vdash . R \mid (\dot{-} R \dot{-} \breve{R}) \mid R &= R \mid [(\dot{-} R \dot{-} \breve{R}) \upharpoonright C\text{‘}R] \mid R \\ &= R \mid \hat{x}\hat{y}\,(x \dot{-} Ry \,.\, y \dot{-} Rx \,.\, x, y \in C\text{‘}R)\, R \qquad (1) \end{aligned}$$

* In this paper, 'x-is-simultaneous-with-respect-to-R to y' will be interpreted as meaning $x\,[C\,(\dot{-} R \dot{-} \breve{R}) \upharpoonright C\text{‘}R]\,y$.

† I follow the method of the *Principia Mathematica* of Russell and Whitehead.

$$\vdash : R \in \text{connex} \,.\, \supset .\, R \,|\, (\dot{-} R \dot{-} \breve{R}) \,|\, R = R \,|\, \hat{x}\hat{y}\,(x = y) \,|\, R$$
$$= R \;\; I \,|\, R$$
$$= R \,|\, R \qquad (2)$$

$$\vdash : R \in \text{ser} \,.\, \supset .\, R \,|\, (\dot{-} R \dot{-} \breve{R}) \,|\, R \subseteq R \,.\, R \in \text{Rl}`J \,.$$
$$\supset .\, R \in \text{cs} \qquad (3)$$

$\vdash . (3) . \supset \vdash . \text{ser} \subset \text{cs}.$

Moreover, it has been shown by Mr Russell that it is advantageous for purposes of methodological simplicity to regard the instants of time as constructions from its events. This is an additional reason for starting from the members of cs and forming certain members of ser as functions of them. Let us, then, agree that an instant, for example, is to be regarded as a class of events, and a point on a line as a class of the segments of the line, for the purposes of this paper. The question then arises, when is a class of events an instant, and when is a class of segments a point? It is obvious on inspection that not every class of events is an instant: all the events which make up a given instant must be simultaneous with one another, and all the events which are simultaneous with every member of the instant must belong to that instant. Moreover, Λ must not be an instant. It can also be seen readily that any class satisfying these conditions will be an instant. That is, if P is the relation of an event to an event which completely follows it, it is a simple matter to show that the class of all instants is

$$\hat{\alpha}\,\{\alpha = p`\overrightarrow{P_{se}}``\alpha\}\,*.$$

One instant precedes another when and only when some event belonging to the one entirely precedes some event belonging to the other. That is, calling the relation of precedence between instants inst$`P$, we can easily show that we have

$$\vdash . \text{inst}`P = (\breve{\epsilon} \,;\, P) \upharpoonright \hat{\alpha}\,\{\alpha = p`\overrightarrow{P_{se}}``\alpha\}.$$

Let me now make the following definitions for any value of P:

$*0{\cdot}03. \quad \tau_P = \hat{\alpha}\,\{\alpha = p`\overrightarrow{P_{se}}``\alpha\} \qquad$ Df.

$*0{\cdot}04. \quad \text{inst} = \hat{Q}\hat{P}\,\{Q = (\breve{\epsilon} \,;\, P) \upharpoonright \tau_P\} \qquad$ Df.

I wish to show that

$$\vdash . \text{inst}``\hat{R}\,\{R \,|\, R_{se} \,|\, R \subseteq R\} \subset \text{ser},$$

* This definition is due to Mr Russell.

and hence that

$$\vdash . \text{inst}``\text{cs} \subset \text{ser}.$$

This shows us how we can construct a serial relation from any relation of the same sort as complete succession; or, indeed, from any relation agreeing with it in only one respect.

$*0·1.$ $\vdash . \text{inst}``\hat{R}\{R \mid R_{se} \mid R \mathrel{\text{G}} R\} \subset \text{ser}.$

Proof.

It is easy to show that

$\vdash : \alpha\, \text{inst}`P\beta . \equiv .$

$$\alpha = p`\overrightarrow{P_{se}}``\alpha . \beta = p`\overrightarrow{P_{se}}``\beta . (\exists x, y) . x \in \alpha . y \in \beta . xPy \quad (1)$$

from the definitions of inst and τ_P. From this we can deduce

$$\vdash : \alpha\, \text{inst}`P\beta . \supset . \beta = p`\overrightarrow{P_{se}}``\beta . (\exists x, y) . x \in \alpha . y \in \beta . \sim xP_{se}y,$$

since, by the definition of P_{se}, xPy and $xP_{se}y$ are incompatible. This reduces to

$$\vdash : \alpha\, \text{inst}`P\beta . \supset . \beta = p`\overrightarrow{P_{se}}``\beta . (\exists x) . x \in \alpha . \sim(x \in p`\overrightarrow{P_{se}}``\beta),$$

from which we can deduce

$\vdash : \alpha\, \text{inst}`P\beta . \supset_{\alpha,\beta} . \alpha J\beta$

or $\vdash . \text{inst}`P \in \text{Rl}`J$ (2)

Also, we find from (1) that

$\vdash : \alpha\, \text{inst}`P\beta . \beta\, \text{inst}`P\gamma . \supset .$

$$\alpha = p`\overrightarrow{P_{se}}``\alpha . \beta = p`\overrightarrow{P_{se}}``\beta . \gamma = p`\overrightarrow{P_{se}}``\gamma .$$
$$(\exists x, y, u, v) . x \in \alpha . y, u \in \beta . v \in \gamma . xPy . uPv.$$

This implies

$\vdash : \alpha\, \text{inst}`P\beta . \beta\, \text{inst}`P\gamma . \supset .$

$$\alpha = p`\overrightarrow{P_{se}}``\alpha . \gamma = p`\overrightarrow{P_{se}}``\gamma . (\exists x, v) . x \in \alpha . v \in \gamma . xP \mid P_{se} \mid Pv.$$

This, together with (1), gives us

$\vdash . \text{inst}``\hat{R}\{R \mid R_{se} \mid R \mathrel{\text{G}} R\} \subset \text{trans}$ (3)

By the definitions of inst and τ_P, we find that

$$\vdash : \alpha, \beta \in C`\text{inst}`P . \supset . \alpha = p`\overrightarrow{P_{se}}``\alpha . \beta = p`\overrightarrow{P_{se}}``\beta.$$

By an easy deduction, we can arrive, from this proposition and the definition of P_{se}, at the proposition

$$\vdash :: \alpha, \beta \in C`\text{inst}`P . \supset :. \alpha = p`\overrightarrow{P_{se}}``\alpha . \beta = p`\overrightarrow{P_{se}}``\beta :.$$
$$x \in \alpha . y \in \beta : \supset_{x,y} : xPy . \vee . yPx . \vee . xP_{se}y,$$

whence we get

$$\vdash :: \alpha, \beta \in C\text{‘‘}\mathrm{inst}\text{‘}P \,.\, \supset :. \, \alpha = p\text{‘}\overrightarrow{P_{\text{se}}}\text{‘‘}\alpha \,.\, \beta = p\text{‘}\overrightarrow{P_{\text{se}}}\text{‘‘}\beta :.$$

$$x \in \alpha \,.\, y \in \beta \,.\, \supset_{x,y} .\, \sim xPy \,.\, \sim yPx : \supset : u \in \alpha \,.\, \supset_u .\, u \in p\text{‘}\overrightarrow{P_{\text{se}}}\text{‘‘}\beta,$$

or $$\vdash :: \alpha, \beta \in C\text{‘‘}\mathrm{inst}\text{‘}P \,.\, \supset :. \, \alpha = p\text{‘}\overrightarrow{P_{\text{se}}}\text{‘‘}\alpha \,.\, \beta = p\text{‘}\overrightarrow{P_{\text{se}}}\text{‘‘}\beta :.$$

$$x \in \alpha \,.\, y \in \beta \,.\, \supset_{x,y} .\, \sim xPy \,.\, \sim yPx : \supset .\, \alpha \subset \beta.$$

By an exactly similar argument,

$$\vdash :: \alpha, \beta \in C\text{‘‘}\mathrm{inst}\text{‘}P \,.\, \supset :. \, \alpha = p\text{‘}\overrightarrow{P_{\text{se}}}\text{‘‘}\alpha \,.\, \beta = p\text{‘}\overrightarrow{P_{\text{se}}}\text{‘‘}\beta :.$$

$$x \in \alpha \,.\, y \in \beta \,.\, \supset_{x,y} .\, \sim xPy \,.\, \sim yPx : \supset .\, \beta \subset \alpha.$$

Combining these, we get

$$\vdash :: \alpha, \beta \in C\text{‘‘}\mathrm{inst}\text{‘}P \,.\, \supset :. \, \alpha = p\text{‘}\overrightarrow{P_{\text{se}}}\text{‘‘}\alpha \,.\, \beta = p\text{‘}\overrightarrow{P_{\text{se}}}\text{‘‘}\beta :.$$

$$x \in \alpha \,.\, y \in \beta \,.\, \supset_{x,y} .\, \sim xPy \,.\, \sim yPx : \supset .\, \alpha = \beta.$$

This we may write as

$$\vdash :: \alpha, \beta \in C\text{‘‘}\mathrm{inst}\text{‘}P \,.\, \supset :. \, \alpha = p\text{‘}\overrightarrow{P_{\text{se}}}\text{‘‘}\alpha \,.\, \beta = p\text{‘}\overrightarrow{P_{\text{se}}}\text{‘‘}\beta :.$$

$$(\exists x, y) \,.\, x \in \alpha \,.\, y \in \beta \,.\, xPy : \vee : (\exists x, y) \,.\, x \in \alpha \,.\, y \in \beta \,.\, yPx : \vee : \alpha = \beta.$$

By (1), this becomes

$$\vdash :. \alpha, \beta \in C\text{‘‘}\mathrm{inst}\text{‘}P \,.\, \supset : \alpha \, \mathrm{inst}\text{‘}P \beta \,.\, \vee \,.\, \beta \, \mathrm{inst}\text{‘}P \alpha \,.\, \vee \,.\, \alpha = \beta,$$

or $\vdash .\, \mathrm{inst}\text{‘}P \in \mathrm{connex}$ (4)

Combining (2), (3), and (4), we get the desired conclusion: namely,

$$\vdash .\, \mathrm{inst}\text{‘‘}\hat{R}\,\{R \mid R_{\text{se}} \mid R \mathrel{\text{⋐}} R\} \subset \mathrm{ser}.$$

From this we can easily conclude that

$$\vdash .\, \mathrm{inst}\text{‘‘}\mathrm{cs} \subset \mathrm{ser}.$$

It will be noticed that two of the three serial properties of inst‘P—its being contained in diversity and its connexity—are independent of the properties of P itself. It is especially noticeable that no use is made of $P \mathrel{\text{⋐}} J$ in proving inst‘$P \mathrel{\text{⋐}} J$, nor, indeed, in deducing any of the serial properties of inst‘P. inst is a valuable tool for what Mr Russell calls "fattening out" a relation: i.e. deriving from a non-serial relation a relation with many of the properties of series*.

It is interesting to consider under what conditions inst‘P will be compact. If we define csd as follows:

$$*0{\cdot}2. \quad \mathrm{csd} = \mathrm{cs} \cap \hat{R}\,\{R \mathrel{\text{⋐}} R \mid R_{\text{se}} \text{‘} R \,.\, \breve{R} \mid R_{\text{se}} \mathrel{\text{⋐}} \breve{R} \text{‘} \min_R \text{‘} \overrightarrow{R_{\text{se}}}\} \quad \mathrm{Df},$$

* Since writing this article, I have discovered an operation which will turn *any* relation into a series (though not necessarily an existent one) and will leave unchanged the relation-number of any series to which it is applied. It is the operation which transforms P into inst‘[(inst‘P)$_{pv}$].

we shall find that $R \epsilon \mathrm{csd}$ is a sufficient condition for the density of $\mathrm{inst}{}^{\backprime}R$. This condition says that (1) R is a relation of complete sequence, (2) if x precedes y by the relation R, there are two members of the field of R neither of which bears the relation R to the other, while x precedes the one by R, while the other precedes y by R, (3) if x follows by R some R-contemporary of y, it follows some *initial* R-contemporary of y. This latter condition, which was first formulated by Mr Russell, ensures that if $x \epsilon C{}^{\backprime}R$ and $R \epsilon \mathrm{csd}$, $\overrightarrow{\mathrm{min}}_R{}^{\backprime}\overrightarrow{R}_{\mathrm{se}}{}^{\backprime}x \epsilon \tau_R$. This I now wish to prove.

$*0{\cdot}21.\quad \vdash : P \epsilon \mathrm{csd} \,.\, x \epsilon C{}^{\backprime}P \,.\, \supset .\, \overrightarrow{\mathrm{min}}_P{}^{\backprime}\overrightarrow{P}_{\mathrm{se}}{}^{\backprime}x \epsilon \tau_P.$

Proof.

It follows from the definition of $p{}^{\backprime}\kappa$ and $\overrightarrow{\mathrm{min}}_P{}^{\backprime}\alpha$ that

$$\vdash .\, p{}^{\backprime}\overrightarrow{P}_{\mathrm{se}}{}^{\backprime\backprime}\overrightarrow{\mathrm{min}}_P{}^{\backprime}\overrightarrow{P}_{\mathrm{se}}{}^{\backprime}x = \hat{y}\,\{\alpha \epsilon \overrightarrow{P}_{\mathrm{se}}{}^{\backprime\backprime}[\overrightarrow{P}_{\mathrm{se}}{}^{\backprime}x \cap C{}^{\backprime}P - \breve{P}{}^{\backprime\backprime}\overrightarrow{P}_{\mathrm{se}}{}^{\backprime}x] \,.\, \supset_\alpha .\, y \epsilon \alpha\}.$$

Since it follows from the definition of P_{se} that $\vdash .\, C{}^{\backprime}P_{\mathrm{se}} \subset C{}^{\backprime}P$, this reduces to

$$\vdash .\, p{}^{\backprime}\overrightarrow{P}_{\mathrm{se}}{}^{\backprime\backprime}\overrightarrow{\mathrm{min}}_P{}^{\backprime}\overrightarrow{P}_{\mathrm{se}}{}^{\backprime}x = \hat{y}\,\{\alpha \epsilon \overrightarrow{P}_{\mathrm{se}}{}^{\backprime\backprime}[\overrightarrow{P}_{\mathrm{se}}{}^{\backprime}x - \breve{P}{}^{\backprime\backprime}\overrightarrow{P}_{\mathrm{se}}{}^{\backprime}x] \,.\, \supset_\alpha .\, y \epsilon \alpha\}.$$

This becomes by a little manipulation

$$\vdash .\, p{}^{\backprime}\overrightarrow{P}_{\mathrm{se}}{}^{\backprime\backprime}\overrightarrow{\mathrm{min}}_P{}^{\backprime}\overrightarrow{P}_{\mathrm{se}}{}^{\backprime}x = \hat{y}\,\{zP_{\mathrm{se}}\,x \,.\, z \mathbin{\dot{-}} \breve{P} \mid P_{\mathrm{se}}\,x \,.\, \supset_z .\, yP_{\mathrm{se}}\,x\} \qquad (1)$$

On the other hand, it follows from the definition of $\overrightarrow{\mathrm{min}}_P{}^{\backprime}\alpha$ that

$$\vdash .\, \overrightarrow{\mathrm{min}}_P{}^{\backprime}\overrightarrow{P}_{\mathrm{se}}{}^{\backprime}x = \hat{y}\,\{yP_{\mathrm{se}}\,x \,.\, y \mathbin{\dot{-}} \breve{P} \mid P_{\mathrm{se}}\,x\}.$$

Since by definition any R which belongs to csd satisfies the condition, $\breve{R} \mid R_{\mathrm{se}} \in \breve{R} \mid \mathrm{min}_R \mid \overrightarrow{R}_{\mathrm{se}}$, we get

$$\vdash : P \epsilon \mathrm{csd} \,.\, \supset .\, \overrightarrow{\mathrm{min}}_P{}^{\backprime}\overrightarrow{P}_{\mathrm{se}}{}^{\backprime}x = \hat{y}\,\{yP_{\mathrm{se}}\,x \,.\, y \mathbin{\dot{-}} \breve{P} \mid \mathrm{min}_P \mid \overrightarrow{P}_{\mathrm{se}}\,x\}.$$

From this we may deduce

$$\vdash : P \epsilon \mathrm{csd} \,.\, \supset .\, \overrightarrow{\mathrm{min}}_P{}^{\backprime}\overrightarrow{P}_{\mathrm{se}}{}^{\backprime}x = \hat{y}\,\{yP_{\mathrm{se}}\,x :.$$

$$zP_{\mathrm{se}}\,x \,.\, z \mathbin{\dot{-}} \breve{P} \mid P_{\mathrm{se}}\,x : \supset_z : yPz \,.\, \vee .\, y \mathbin{\dot{-}} Pz \,.\, z \mathbin{\dot{-}} Py \,.\, y, z \epsilon C{}^{\backprime}P\}.$$

But when yPz is the correct alternative in the conclusion of the second proposition in the brackets, together with $yP_{\mathrm{se}}\,x$, this gives us $z\breve{P} \mid P_{\mathrm{se}}\,x$, which contradicts the hypothesis. Hence, by the definition of P_{se}, we have

$$\vdash : P \epsilon \mathrm{csd} \,.\, \supset .$$

$$\overrightarrow{\mathrm{min}}_P{}^{\backprime}\overrightarrow{P}_{\mathrm{se}}{}^{\backprime}x = \hat{y}\,\{yP_{\mathrm{se}}\,x : zP_{\mathrm{se}}\,x \,.\, z \mathbin{\dot{-}} \breve{P} \mid P_{\mathrm{se}}\,x \,.\, \supset_z .\, yP_{\mathrm{se}}\,z\} \qquad (2)$$

Now, it is part of the hypothesis $P \in \mathrm{csd}$ that $P \mathrel{\dot\subset} J$. From this it is easy to deduce that $I \upharpoonright C\lq P \mathrel{\dot\subset} P_{se}$, or that $x \in C\lq P \,.\supset_x .\, xP_{se}x$. Moreover, it follows from the definition of P_{se} that yPx and $y P_{se} x$ are incompatible hypotheses, and hence that $x \mathbin{\dot-} \breve{P}\;P_{se}\, x$. This fact, combined with (1), gives us

$$\vdash : P \in \mathrm{csd} \,.\, x \in C\lq P \,.\supset .\, p\lq\overrightarrow{P_{se}}\lq\lq \mathrm{min}_P\lq\overrightarrow{P_{se}}\lq x$$

$$= \hat{y}\,\{yP_{se}x : zP_{se}x \,.\, z \mathbin{\dot-} \breve{P}\;P_{se}\,x \,.\supset_z .\, yP_{se}z\} \qquad (3)$$

From (2), (3), and the definition of τ_P, we have

$$\vdash : P \in \mathrm{csd} \,.\, x \in C\lq P \,.\supset .$$

$$\mathrm{min}_P\lq\overrightarrow{P_{se}}\lq x = p\lq\overrightarrow{P_{se}}\lq\lq\mathrm{min}_P\lq\overrightarrow{P_{se}}\lq x \,.\supset .\, \mathrm{min}_P\lq\overrightarrow{P_{se}}\lq x \in \tau_P.$$

This is the desired proposition.

It will be observed that the only portions of the hypothesis of $P \in \mathrm{csd}$ of which we actually make use in this theorem are $P \mathrel{\dot\subset} J$ and $\breve{P}\,|\,P_{se} \mathrel{\dot\subset} \breve{P}\;\mathrm{min}_P\,|\,\overrightarrow{P_{se}}$. The theorem ensures us that $\vdash . C\lq P \subset s\lq\tau_P$: that is, in the case of time, that each event shall be at some instant—the instant at which it begins. For, since $P \mathrel{\dot\subset} J$, $I \upharpoonright C\lq P \mathrel{\dot\subset} P_{se}$. This ensures that $x \in \overrightarrow{P_{se}}\lq x$. Moreover, as we have just seen, $x \mathbin{\dot-} \breve{P}\,|\,P_{se}\,x$, or $x \in -\breve{P}\lq\lq\overrightarrow{P}\lq x$. Therefore, if $x \in C\lq P$, $x \in \overrightarrow{P_{se}}\lq x \cap C\lq P - \breve{P}\lq\lq\overrightarrow{P_{se}}\lq x$, or $x \in \mathrm{min}_P\lq\overrightarrow{P_{se}}\lq x$. As we have proved in $*0{\cdot}21$ that $\mathrm{min}_P\lq\overrightarrow{P_{se}}\lq x \in \tau_P$, we get the formula

$$\vdash . C\lq P \subset s\lq\tau_P.$$

I now wish to prove that $\mathrm{inst}\lq\lq\mathrm{csd} \subset \mathrm{comp}$.

$*0{\cdot}22.$ $\vdash . \mathrm{inst}\lq\lq\mathrm{csd} \subset \mathrm{comp}.$

Proof.

As we saw in $*0{\cdot}1$, (1),

$$\vdash : \alpha\,\mathrm{inst}\lq P\beta \,.\equiv .\, \alpha = p\lq\overrightarrow{P_{se}}\lq\lq\alpha \,.\, \beta = p\lq\overrightarrow{P_{se}}\lq\lq\beta \,.$$

$$(\exists x, y) \,.\, x \in \alpha \,.\, y \in \beta \,.\, xPy.$$

Since $R \in \mathrm{csd}$, by definition, implies $R \mathrel{\dot\subset} R\,|\,R_{se}\;R$, this gives us

$$\vdash :. P \in \mathrm{csd} : \supset : \alpha\,\mathrm{inst}\lq P\beta \,.\supset .$$

$$\alpha = p\lq\overrightarrow{P_{se}}\lq\lq\alpha \,.\, \beta = p\lq\overrightarrow{P_{se}}\lq\lq\beta \,.\, (\exists x, y) \,.\, x \in \alpha \,.\, y \in \beta \,.\, xP\,|\,P_{se}\,|\,Py.$$

Since $R \in \text{csd}$ also implies $\breve{R} \,|\, R \Subset \breve{R} \,|\, \text{min}_R \,|\, \overrightarrow{R}_{\text{se}}$, this becomes

$$\vdash :. P \in \text{csd} : \supset : \alpha \, \text{inst}`P\beta \, . \supset . \, \alpha = p`\overrightarrow{P}_{\text{se}}``\alpha \, . \, \beta = p`\overrightarrow{P}_{\text{se}}``\beta \, .$$

$$(\exists x, y) \, . \, x \in \alpha \, . \, y \in \beta \, . \, xP \,|\, \breve{[\text{min}_P \,|\, \overrightarrow{P}_{\text{se}}]} \,|\, Py.$$

$xP \,|\, \breve{[\text{min}_P \,|\, \overrightarrow{P}_{\text{se}}]} \,|\, Py$ says that there are a u and a v such that xPu, vPy, and $v \, \text{min}_P \,|\, \overrightarrow{P}_{\text{se}} \, u$. This latter proposition is equivalent to $v \in \overrightarrow{\text{min}_P}`\overrightarrow{P}_{\text{se}}`u$. We have just seen, moreover, that $u \in \overrightarrow{\text{min}_P}`\overrightarrow{P}_{\text{se}} \, u$, and that $\overrightarrow{\text{min}_P}`\overrightarrow{P}_{\text{se}} \, u \in \tau_P$. This gives us

$$\vdash :: P \in \text{csd} :. \supset :. \alpha \, \text{inst}`P\beta : \supset :$$

$$(\exists u, v) : \alpha = p`\overrightarrow{P}_{\text{se}}``\alpha \, . \, \beta = p`\overrightarrow{P}_{\text{se}}``\beta \, . \, \overrightarrow{\text{min}_P}`\overrightarrow{P}_{\text{se}}`u \in p`\overrightarrow{P}_{\text{se}}``\overrightarrow{\text{min}_P}`\overrightarrow{P}_{\text{se}}`u :$$

$$(\exists x, y) \, . \, x \in \alpha \, . \, y \in \beta \, . \, u, v \in \overrightarrow{\text{min}_P}`\overrightarrow{P}_{\text{se}}`u \, . \, xPu \, . \, yPv.$$

From this and $*0{\cdot}1$, (1) it is an easy matter to deduce that

$$\vdash :. P \in \text{csd} : \supset : \alpha \, \text{inst}`P\beta \, . \supset . \, (\exists \gamma) \, . \, \alpha \, \text{inst}`P\gamma \, . \, \gamma \, \text{inst}`P\beta.$$

This gives us immediately

$$\vdash . \, \text{inst}``\text{csd} \subset \text{comp}.$$

It will be noticed that it is not true that $\text{csd} \subset \text{comp}$. For example, if P is the relation of complete succession between one-inch stretches on a line, P will be a member of csd, and an inch stretch beginning half an inch after the end of another will bear the relation P to it, yet there will be no inch stretch to which the first bears the relation P and which bears the relation P to the second. $P \Subset P \,|\, P_{\text{se}} \,|\, P$ is a weaker hypothesis than $P \Subset P^2$, which implies it if $P \Subset J$.

On an Application of the Molecular Field in Diamagnetic Substances. By A. E. Oxley, B.A., Coutts Trotter Student, Trinity College.

[*Read* 23 February 1914.]

It is well known how Nernst and Lindemann* have extended the formula, given by Einstein, for the variation of the specific heat of substances with temperature. Although the new formula expresses the experimental results with moderate degree of accuracy, yet, in the neighbourhood of the fusion point, there is an abnormally large departure, the experimental value of the specific heat being always greater than the calculated value. The empirical expansion term, $\alpha\vartheta^{\frac{3}{2}}$ (α, coefficient of expansion; ϑ, absolute temperature), used by Lindemann does not satisfactorily account for the discrepancy†.

Later Debye‡ has modified the Planck-Einstein theory and given a relation between the thermal properties and the elastic constants of a substance.

In a crystalline substance we must regard the molecules as subjected to large local forces which hold the molecules in position in the crystalline structure, and if at the higher temperatures the molecules begin to vibrate under the control of these forces, then we should expect the experimental value of the specific heat to be greater than that calculated on Debye's theory§.

It has been shown‖ that we may interpret the forces which hold the molecules in position in a diamagnetic crystalline structure, magnetically, and if we do so the magnetic energy associated with one gramme of the substance may be written¶

$$\frac{a_c' I^2}{2\rho} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (1),$$

* *Sitz. d. preuss. Akad. d. Wiss.*, p. 347, 1911.

† See the memoirs by Nernst and Einstein, *La Théorie du Rayonnement et les Quanta*, Paris, 1912; particularly p. 272.

‡ *Ann. der Phys.*, IV. vol. 39, p. 789, 1912.

§ See Jeans, *Phil. Mag.*, VI. vol. 17, p. 771, 1909. I am indebted to Mr Ezer Griffiths for pointing out the work of Jeans in this connection.

‖ A. E. Oxley, *Phil. Trans. Roy. Soc.*, 1914 (Unpublished).

¶ On the electron theory of magnetism developed by Langevin, a diamagnetic molecule has no initial magnetic moment and the resultant force due to it will be negligibly small except at points whose distances from the molecule are comparable with molecular dimensions.

If i be the local magnetic moment between any two molecules and h the local intensity of the magnetic field, then the magnetic energy associated with one c.c. of the substance is

$$\tfrac{1}{2}\Sigma|i||h|,$$

where a_c' is the constant of the molecular field of the diamagnetic substance, I is the aggregate of the local intensities of magnetization per unit volume, and ρ is the density of the substance. This is analogous to the case of ferro-magnetism, given by Weiss*, where the magnetic energy term is

$$\frac{NI^2}{2D} \text{ ergs per gram.}$$

N is the constant of the ferro-magnetic field, I the saturation intensity of magnetization and D the density of the substance.

If, as the diamagnetic crystalline substance is heated, the molecules perform rotational vibrations under the influence of the intense local forces, more heat must be supplied for a given rise of temperature than would be necessary if the molecules did not rotate. The corresponding increase of specific heat is given by the term

$$\frac{a_c'}{2J\rho} \cdot \frac{I\partial I}{\partial \vartheta} \quad \text{.........................(2),}$$

where J is the mechanical equivalent of the calorie.

The molecular field, by which we may interpret the forces within the crystalline structure, is $a_c'I$, and is of the order of magnitude of the ferro-magnetic field of Weiss (10^7 gauss). Hence, as in the case of ferro-magnetic substances, we may expect that the above term will form an appreciable fraction of the specific heat. Moreover, the expression (2) passes through a maximum in the neighbourhood of the fusion point.

In so far as we can test the above experimentally, there seems to be evidence in favour of the additional specific heat represented by (2). For *sodium*† and *mercury*‡ there is a decided maximum of the specific heat in the neighbourhood of the fusion point such as (2) demands. A large number of substances have been investigated by Nernst and Lindemann§, and they found that in general the specific heat is abnormally high as the fusion point is approached.

This work will be continued in a future paper.

if all the contained elementary systems are independent. Let n be the number of molecules per c.c., then we may write $\frac{1}{2}\Sigma|i||h| = \frac{1}{2}nih$. h corresponds to the molecular field in ferro-magnetism. If we write $ni = I$ and $h = a_c'.I$, we find that the energy associated with one c.c. of the substance is $\frac{a_c' I^2}{2}$.

* *Journ. de Phys.*, Sér. IV. vol. 7, p. 249, 1908.

† Ezer Griffiths, *Proc. Roy. Soc.*, vol. 89 A, p. 561, 1914.

‡ The values for mercury were taken from the Tables of Physical Constants published by the Société Française de Physique, 1913, p. 305.

§ *La Théorie du Rayonnement et les Quanta*, Paris, 1912.

PROCEEDINGS

OF THE

Cambridge Philosophical Society.

Thompsonia, a little known Crustacean Parasite. (Preliminary Note.) By F. A. Potts, M.A., Trinity Hall.

[*Read* 4 May 1914.]

Thompsonia is a genus of Cirripedia belonging to the parasitic family the Rhizocephala, and was first noted by Kossmann* in 1874 as a parasite upon the small crab *Melia tesselata* from the Philippines. Since that time it has been described by Coutière† under the name of *Thylacoplethus* as a parasite of Alpheids in East Indian and Australian waters, and by Häfele‡ and Kruger§ from the crab *Pilumnus* and the hermit crab *Eupagurus middendorfii* off the coasts of Japan. I have little hesitation in including all these forms in the same genus, even though *Thompsonia* thus enjoys a far greater diversity of hosts than any other Rhizocephalan, for as a rule each genus is strictly confined to the same subdivision of the Decapoda. In 1913 I accompanied the expedition of the Carnegie Institution of Washington to Torres Straits at the kind invitation of Dr A. G. Mayer, and during a short stay at Murray Island I collected nearly twenty individuals of a species of *Synalpheus*, commensal with the crinoid *Comatula parvicirra*, infected by a parasite which is clearly identical with Coutière's genus *Thylacoplethus*. I also obtained a specimen of the swimming

* *Arb. Zoot. Inst.*, Würzburg, Bd. I. p. 97, 1874.
† *C. R. Acad. Sci. Paris*, T. CXXXIV. pp. 913 and 1452, 1902.
‡ *Abh. Akad. München*, 2 Suppl. Bd. 7 Abh, 1911.
§ *loc. cit.* 2 Suppl. Bd. 8 Abh. 1912.

crab *Thalamita prymna*, strongly parasitised by a form resembling *Thompsonia.* An examination of the material from these two sources and a careful comparison with the published accounts of Coutière and Häfele leads me to conclude that Coutière's genus should not be retained.

The cardinal point which comes from this research is that previous investigators were entirely mistaken in their conception of the organisation and life history of the parasite. The reason for this is to be found in the fact that they were unable to examine living or well fixed material which is necessary for successfully tracing the course of the root system.

Externally the parasite consists of a number of small *external sacs*, sometimes as many as 200 on a single host, which spring

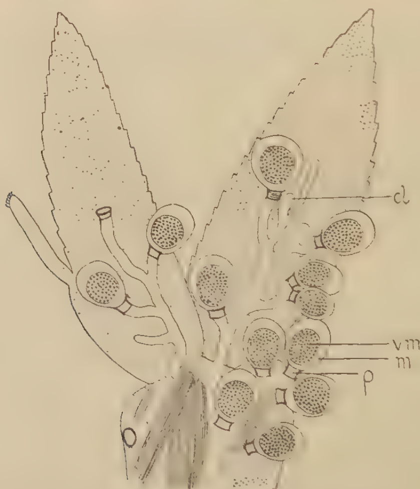


FIG. 1. Abdominal appendage of *Synalpheus* infected by *Thompsonia* × 120. To show the branching root system connecting the very young external sacs. The specimen was fixed with corrosive sublimate and stained with borax carmine. The roots therefore have taken up very little stain compared with the tissues of the host and are shown unshaded. *v.m.* visceral mass, *m.* mantle, *p.* peduncle, *cl.* blind branch of root system which after further development will become an external sac.

from the limbs, both thoracic and abdominal, occurring so thickly as to seem to embarrass motion. Coutière and Häfele regard each of these external sacs as a separate individual with a root system of its own, and derived from a Cypris larva. I claim, on the other hand, that the external sacs are all budded off from one root system continuous throughout the host, and so that each host is parasitised by a single individual and not by a hundred or more gregariously inclined Rhizocephalans.

This conclusion was reached first after an examination of the abdominal appendages of an infected *Synalpheus*. These were snipped off from the living animal and examined under the low power of a microscope. Each flattened ramus of the appendage was seen to contain at least one slender root strand of a slightly greyish colour (owing to the doubly refringent yolk granules), giving off a secondary root to each external sac and generally a number of others which end in bulbous enlargements within the tissues of the host. The significance of these last will be explained later.

Dissection of the trunk of the Alpheid also revealed the existence of a network of slender grey roots in the neighbourhood of the nerve-cord. The extent of the root system in the body of the host was best demonstrated in preserved material. The infected Alpheids were fixed both in corrosive sublimate with acetic acid, and Flemming's fluid. The latter gave by far the most favourable results, as the yolk granules were blackened by the osmic acid of the fluid and it was thus possible to recognise the roots and trace their course with the greatest ease. The root system is diffuse and is best developed in the ventral body wall, and particularly round the nerve cord, but in the thorax it passes laterally and dorsally, and in all places gives off branches into the musculature. The alimentary canal is not affected, so the root system is not really so widely distributed as that of *Sacculina*, though much more so than that of *Peltogaster*. From this *central* root system within the trunk there pass out into the walking legs, the abdominal swimmerets and the tail fans, lateral roots which together with their branches may be distinguished as the *peripheral* root system. The branches of the central system show a syncytial epithelium surrounding a central lacunar space, and containing large yolk granules: their function is *nutritive*. The peripheral part of the system has a reproductive function on the other hand. The lacunar space is filled with small nuclei in scanty cytoplasm, destined to give rise to the reproductive cells which develop in the external sacs. The vitelline granules are small and broken up, suggesting that katabolic metabolism prevails here over anabolic. In addition to transmitting nourishment to the existing external sacs these peripheral roots are charged with the function of replacing them at the proper time by a new crop.

The external sacs are probably homologous with the visceral mass of the typical Rhizocephalan, but differ from this in the simplicity of their structure, a simplicity which is partly due to their large number and small size. Of the organs contained in the visceral mass of the higher Rhizocephala, the nerve ganglion, reproductive ducts, muscular tissue, and probably too the testis,

have been lost. The mantle is a thin layer of tissue, also without muscles, existing to secrete the outer envelope of chitin. The mantle cavity has virtually disappeared, the inner surface of the mantle being adherent to the visceral mass. Until the larvae are advanced in development no aperture is found in the mantle, and the envelope of chitin is entire. The visceral mass is occupied entirely by the ovary. In this the ova grow, mature, and undergo the whole of their development. Reproduction appears to be parthenogenetic, for I have been unable to find any trace of a testis or of free spermatozoa. The description of a testis in *T. japonica* by Häfele seems to me of very doubtful value: no spermatozoa were observed by him. The Nauplius stage is omitted from the life history, and the larvae reach the Cypris stage while still within the egg membranes. When development has reached this point the ovary has completely disintegrated, and eventually the larvae burst their membranes and lie freely within the mantle. At the same time a definite aperture is formed in the mantle at the apex of the sac and a moult of the external cuticle allows the larvae to escape. On this point my observations are in agreement with those of Coutière, and Häfele's statement as to the absence of an aperture is probably due to the fact that the individuals he examined were not sufficiently mature to show this point.

As the visceral mass thus disintegrates with the birth of a brood of larvae, the question arose whether the stage of reproductive maturity is terminated when a single brood has been produced. This was answered conclusively by observations made upon infected *Synalpheus* which had been kept in captivity for some days. One or two individuals moulted, and it thus became evident that the effect on the host in this respect is more comparable to that of *Peltogaster* on the hermit crab than of *Sacculina* on the true crabs, where the moulting function is suppressed. In the case which was followed most carefully the external sacs were of large size and approaching maturity when moulting took place and they were carried away with the cast skin. Three days afterwards the appendages were examined to see whether any new external parasitic structures were visible. A number of pink sacs, much smaller than any observed hitherto, but with the characteristic structure of the external sac, were found on the abdominal appendages. Clearly, then, successive crops of sacs containing the reproductive organs are produced by the root system. It was mentioned above that the finer divisions of the roots observed in the appendages ended sometimes in external sacs and sometimes in club-shaped enlargements which did not penetrate the cuticle. It is these structures which are destined to produce a new crop of external sacs. Already in many cases the typical organisation can be recognised. The peripheral

layer, which becomes the mantle, is thicker, and its external cuticle is more developed, while the distinct central part, crowded with small nuclei, represents the visceral mass. A comparison of the cast skin with the appendage after moulting shows that the new external sacs appear in different positions to their predecessors. Evidently then the rootlets which communicated with the old external sacs do not regenerate new ones at once.

It seems very likely that there has been an adjustment of the development period of the parasite to the time elapsing between

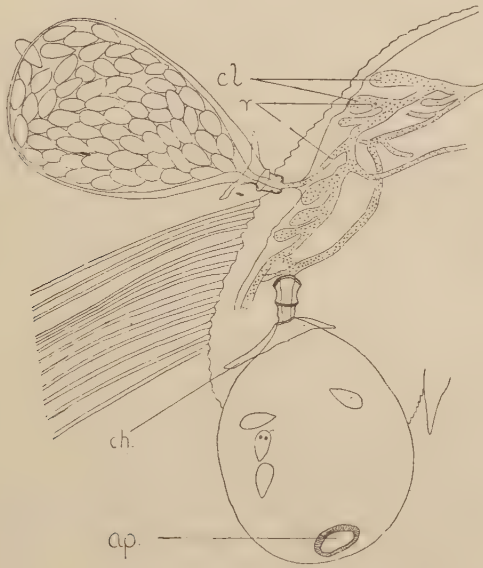


FIG. 2. *Thompsonia* parasitic on *Synalpheus* showing mature external sacs springing from a tail-fan of the host. One is full of mature Cypris larvae: in the other the larvae have mostly escaped through the apical aperture *ap*. The root system, *r*, is shown up by the blackened yolk globules it contains. *ch.* the remnant of the chitinous envelope of the external sac. *cl.* enlargements of roots which will later develop into external sacs. Fixed in Flemming's fluid.

moults of the host. When the Alpheid under my observation cast its skin the external sacs contained advanced larvae, and I imagine that the disturbance connected with capture and change of conditions brought on the moult slightly before it would have normally occurred. If there is no such correspondence between the two periods, surely the moult of the host will suspend the development of the parasite and interfere with the mechanism for securing the liberation of the larvae.

The existence of crowded nuclei with the thinnest investment

of cytoplasm in the lacunar space of the peripheral roots has been noticed before. When a club-shaped swelling is formed at the end of a rootlet these cellules appear to migrate into its interior and form the ovary of the future external sac. An interesting parallel may be drawn between *Thompsonia* and those Hydromedusae in which the gonophores are degenerate in structure and the reproductive cells are formed in the coenosarc, migrating thence into the gonophores.

A curious thing is noticed on further examination of the peripheral roots. That is the occurrence, among these very small cellules, of much larger bodies, which appear to be segmenting eggs corresponding in size and structure with those found within the external sacs. I have not satisfied myself that the stage of development of these bodies corresponds to that of the ovarian eggs. But what can be their fate? Unless they migrate into the external sacs they must disintegrate and disappear before development proceeds much further. What are the conditions which stimulate the reproductive cells to development whatever their situation in the organism? It must however be noticed that they are far fewer than those developing within the external sacs.

The new light which these observations shed upon *Thompsonia* show it to be, not a primitive Rhizocephalan as Coutière and Häfele maintain it to be, but on the contrary the most specialised member of the group. The simplicity of the structure of the external sacs is a secondary condition. *Sacculina* and *Peltogaster* still show in their external sacs a certain morphological resemblance to the non-parasitic Cirripede. But the external sacs of *Thompsonia* are little more than ovaries placed externally to allow of the escape of the larvae. The adoption of parthenogenesis (I feel little doubt that this is the method of reproduction here, as in *Sylon* and *Mycetomorpha*) has made it possible to dispense with testes and gonadial ducts. The fact that these external sacs can be produced with great economy of material allows the appearance of successive crops. But assuming, as I thus do, that *Thompsonia* with its highly specialised root system and peculiar method of reproduction is descended from an ancestor, resembling *Sacculina* and *Peltogaster* in many respects, can we in any way trace the course of evolution? I think some clue is afforded by the consideration of *Peltogaster socialis*.* This parasite of hermit crabs is rarely if ever alone; often there are a hundred or more external sacs. But it differs from *Thompsonia* in two most important particulars. Firstly, the external sac is well developed, being similar in almost all respects to that of other species of *Peltogaster*; and secondly, the root systems of adjacent external sacs, as well as the sacs

* Smith, *Fauna u. Flora Golfe v. Neapel. Rhizocephala*, 1906, p. 57. Compare also *Peltogasterella socialis*, Krüger, if this form is really distinct.

themselves, are said to be distinct even in the internal stage. As in *Thompsonia* so here, the external sacs on each host are always in the same stage of development, so that the only alternative to a theory of origin by internal budding from a single Cypris larva, advanced by Geoffrey Smith, is that of simultaneous fixation of a crowd of gregariously inclined larvae. But the phenomena are so similar in the two forms that I venture to think that the same broad explanation must cover both. The proof of a budding process in *Thompsonia*, which I offer here, is in my view important, if indirect, support for Geoffrey Smith's theory for *Peltogaster socialis*. It would be absurd to suppose that this latter species is on the same line of descent as *Thompsonia*, but it shows that the latter genus may well have had an ancestor with many external sacs of normal Rhizocephalan type.

A full account of this form, and also of *Hapalocarcinus*, described in the next paper, will appear later in the publications of the Department of Marine Biology of the Carnegie Institute of Washington, and I am indebted to Dr A. G. Mayer for permission to publish these preliminary notes.

The gall-forming Crab, Hapalocarcinus. (Preliminary Note.) By F. A. POTTS, M.A., Trinity Hall.

[*Read* 4 May 1914.]

Hapalocarcinus is a genus of Brachyrhynch crabs very small in size and profoundly modified owing to the fact that they pass the greater part of their lives confined in small cavities in coral colonies. At an early age the crab settles between two branchlets, usually terminal, and so influence their further growth that they broaden and, later, unite to form the so-called gall, a lenticular or spherical structure, about the size of a hazel nut. Within this is the living chamber of the crab which communicates with the outside water by a series of apertures. Information about the biology of this form is however particularly scattered and incomplete, and while the structure of the adult female is fairly well known and the systematic position has been adequately discussed by Dr W. T. Calman*, the male has remained undiscovered up till the present. I am greatly obliged to Dr Calman who directed my attention to this interesting creature and the need for a connected account of it.

I found *Hapalocarcinus* during the month of October, 1913, existing in great numbers on the reefs of Murray Island at the north end of the Great Barrier Reef of Australia. It here forms galls on two species of branching Madreporarian corals *Pocillopora caespitosa* and *Seriatopora hystrix*, both belonging to the same family, the Pocilloporidae and characterised by dichotomous branching. Both are widely distributed and dominant forms but the former is the favourite host of *Hapalocarcinus* and in the still waters which cover the inner reef there is hardly a colony which does not bear at least one of the galls, while some show nine or ten in various stages of development. The ease with which the growth of the colonies can be modified by external agencies accounts for the attraction which *Pocillopora* possesses for the gall-forming crab. No coral shows more variation under different environmental conditions. *Seriatopora* shows less power of response to the influence of currents. Galls are by no means so common as in *Pocillopora* for they represent a much greater interference with the type of branching. The remarks which follow are then confined to *Pocillopora*, the abundance of the material and the uniformity of development giving great facility to study.

It the first place it may be stated that though the galls themselves vary greatly in development, there is a general relation

* W. T. Calman, *Trans. Linn. Soc. Zoology*, 2nd Ser. Vol. VIII. Pt I. p. 43.

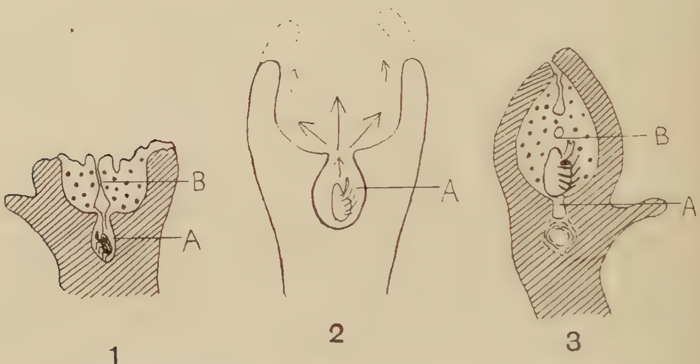
between the state of development and the size of the crab it contains. In all cases the gall appears to be formed and inhabited by a solitary female individual. Those in the younger "open" galls, where the two constituent branches have not yet approached each other and fused, are immature: those in the older "closed" galls are mature and often the abdominal appendages are laden with developing eggs.

The young female crab probably commences its sedentary life by settling down in the notch at the apex of a recently divided branchlet. She is at this period a small flat creature, little more than a millimetre in carapace length. Her position at the growing point allows her to control the future development of the branch.

The initial modification of growth is probably due to the mere mechanical effect of the continued presence of the animal. The two branches, instead of remaining cylindrical, broaden out. They then approximate above and laterally, thus partially enclosing a chamber which is large enough to contain the crab with comfort. In the diagrams which illustrate this paper the chamber is referred to as *A*. In the second stage of construction of the gall a much larger upper chamber (*B*) is formed by the continued growth of the two branches. During nearly the whole time the crab remains an inhabitant of *A*. It is not until the two walls of the upper chamber have almost met, that she finds her earlier quarters too restricted and transfers herself to the upper chamber at the time when it is ready for occupation. Just before fertilisation the ovary begins to grow rapidly, causing so great a development of the abdomen that more spacious accommodation is quickly needed. This is provided by the new chamber (*B*) of the gall.

It follows then, that since the crab only enters into occupation on completion of the work, she must influence the growth of the branches in a manner more indirect than, though quite as effective as, at first. It is rather difficult to explain the precise influences which give the gall its characteristic and constant shape, but it can be stated that if the respiratory current of the crab is not the sole external factor which is responsible it is at any rate the most important. By means of powdered carmine spilt in a small quantity of sea water containing a crab I was able to assure myself that, as a general rule, the water needed for respiration is sucked into the branchial chamber behind and expired upwards and outwards, as usually happens in the Brachyura. The form of the initial cavity *A*, a narrow crevice, hardly allows the animal to move except in one plane. Thus the upward and outward current has sufficient fixity of direction and constancy of power to secure a definite result. This current either flows straight upwards or impinges on the lip of the chamber *A*. It the latter case it would be deflected obliquely toward the opposite lip, tending to

repress growth on the inner edge though not on the outer. As the outer edge was built up these oblique currents would become circular eddies and the growth of the wall of chamber B would follow their outside line. The two opposed walls meet naturally at the intersection of their curves but that part of the current flowing directly upwards, in its escape prevents concrescence. But after the crab has migrated into chamber B, partial fusion does occur so that what was at first a wide slit is converted into a series of small circular holes occurring laterally as well as above, and all of equal size and, presumably, importance. There is nothing in my material to support the curious observation of Semper* in



Transverse sections of Galls to show stages of formation.

1. An 'open' gall, chamber B in course of formation. 2. Diagram of an open gall to show the way in which the respiratory current may influence the growth of the coral. The arrows represent the expired current. 3. A 'closed' gall, chamber B completely formed. Chamber A has been almost filled up.

The crab inhabiting the gall lies in a plane at right angles to the paper and so is represented in side view. The black spots inside B represent the thecae of polypes.

which he describes the concrescence of the edges proceeding "till at length only two fissures,...are left, which plainly show by their position opposite to each other that it is through them that the current for respiration passes: one fissure serves for the influx, the other for the exit, of the water." I think that though the crab cannot turn round, it moves about freely in a vertical plane and that all the apertures are used in turn.

A certain number of secondary changes occur after the formation of the gall. The polypes inside the gall do not seem to be greatly affected by their life within a closed dark space and the thickness of the coenenchyme is added to very distinctly on the inside as well as the outside, so much so as to encroach

* Karl Semper, *Animal Life*, English Translation, 1881.

seriously upon the space in chamber *B* and partially fill up chamber *A*. The formation of the gall does not permanently suppress the further development of the branch of which it forms part. A perfect forest of twigs cover the surface of closed galls of some standing and occasionally examples are found in which the gall is the foundation of a complex branching system. In these cases, the galls can evidently lay claim to a respectable antiquity.

The youngest females, which are found in galls in which chamber *A* alone is formed, have a carapace length of 1·5 mm. and their sex is hardly recognisable. They have a narrow abdomen with no trace of swimmerets, there are no reproductive apertures and the genital gland has not developed. But there is equally no sign of male characters and a perfect gradation exists from these apparently sexless forms up to the adult females. In the next stage, though the abdomen is very little broader, rudiments of the swimmerets appear and also the female apertures on the sternum opposite the third thoracic legs. The succeeding intermediates show a gradual increase in the size of the abdomen which becomes accelerated when the great growth of the ovary begins.

For some time I was unable to find the male of the species though I examined as many as a hundred galls. But at length on opening one I found it occupied by a female with her recently moulted skin and a much smaller individual of about 1 mm. carapace length. This was identified as a male on account of the well developed and typical copulatory styles and a pair of enormous testes full of mature spermatozoa which showed up as opaque white structures in the cephalothorax. From this discovery I conclude that the male is normally very much smaller than the adult female and not even so large as the young immature females which are found in the least developed galls. Also that he is free-living and visits the females within the galls, copulation taking place at a period when the gall is still open and the ovary is beginning to grow. As in the other Brachyura so here it only occurs just after the female has moulted. Soon after a stock of sperm has thus been secured the gall closes up so far that the visits of other males are barred. But the female begins to lay eggs and lays apparently brood after brood which develop within the ample shelter of the abdomen until they reach the Zoaea stage. Then the larvae are liberated to the exterior through the tiny circular outlets of the gall.

This curious life history presents points of resemblance to and difference from that of the related genus, *Cryptochirus** so far as

* J. R. Henderson, *Ann. Mag. Nat. Hist.*, Ser. 7, vol. XVII. pp. 211—219. These remarks only apply to *C. dimorphus*. Such observations as I have been able to make upon *C. coralliodytes* Heller will be given in the full paper.

the latter is known. This crab inhabits deep pits within Astraeid colonies, each pit in its origin a single theca. The pit contains, as a rule, a male as well as a female, and though neither so rare nor so roving as in *Hapalocarcinus* he is also far smaller than the female (one quarter her size). The Hapalocarcinidae (a family formed by Calman to include these two genera) thus furnishes by far the most marked cases of sexual dimorphism to be found in the Decapoda.

Certain points of structure may perhaps be mentioned now as shedding light on the biology of *Hapalocarcinus.* Calman has described the wide buccal area and pointed out that there is a certain amount of reduction in the third maxilliped. This is very much accentuated in the maxillae and mandibles which Calman was not able to examine owing to paucity of material. Each member of the two pairs of maxillae is reduced to a single elongated plate. The mandible is a well developed triangular piece of chitin but without the denticulate biting border of the typical Decapod mandible. Especially noticeable too is the absence of a mandibular palp. It is plain that none of these appendages are used for mastication.

A similar modification is to be noticed in the stomach. In all the higher Crustacea the chitinous wall of the cardiac chamber is thickened locally to form a system of plates bearing teeth, the so called gastric mill, which continue the task of breaking up the food into very small particles. The pyloric chamber is occupied by a very efficient filtering mechanism composed of interlacing setae which only allows food in an easily absorbed condition to pass into the midgut. In *Hapalocarcinus* this arrangement is very much simplified. Many of the plates have disappeared and while the more important constituents of the gastric mill, the urocardiac and zygocardiac ossicles, are still present, they are much weaker and the teeth they bear instead of being stout and blunt are long and slender, passing into setae. Their function is not mastication but they apparently aid the setae of the pyloric valve in sieving the food current. In accordance with this forward shifting of the sieving mechanism the structure of the pyloric chamber is simpler and the pyloric ampullae which are so prominent a feature in other Decapoda are entirely unrepresented. This modification, both of the mouth appendages and of the stomach, is far greater than any which occurs in other Brachyura and we must look to a very distinct cause for the explanation. This cause is, without doubt, the change in the feeding habits of the animal caused by its voluntary imprisonment in an almost totally closed space. The holes which allow entrance to the "closed" gall are exceedingly small in the skeleton and must be smaller still if we allow for the coating of living coenosarc. It has

been pointed out before how erroneous was the early description of *Hapalocarcinus* as a parasite on corals. There is no doubt that the crab must live on the plankton which is drawn in with the respiratory current. But the larger constituents of the plankton fauna are too large to enter the apertures of the gall and I suggest that the food of *Hapalocarcinus* consists almost entirely of the so-called "nanno-plankton" which embraces all those animals and plants less than 3-4μ in measurement. This suggestion is borne out by my examination, so far as it goes, of the stomachs of a dozen or so gall-crabs. These appear empty on a cursory investigation but on one or two occasions there were very small representatives of the phytoplankton. It is suggested that these organisms are collected from the respiratory current in the first place by the action of the close-set combs of setae springing from the interior of the palps of the maxillipeds which cover the whole of the buccal area. The close likeness which exists between the oral appendages of *Hapalocarcinus* and those of the Branchiopoda in the lower Crustacea is explained by a similarity of diet. In this latter group too, the mandibular palp is absent, the mandible is weak, and the maxillae are perfectly simple lobes, one pair sometimes being lost altogether. Mr J. T. Saunders, of Christ's College, informs me that these animals undoubtedly feed upon nanno-plankton.

A Note on Leaf-Fall as a Cause of Soil Deterioration. By W. LAWRENCE BALLS, M.A., St John's College.

[*Read* 4 May 1914.]

In 1907 a portion of the cotton-breeding plot of the Khedivial Agricultural Society at Giza, Egypt, was sown with Caravonica, Senaar, and Nyam-nyam Tree cottons*, the second of these in particular having abundant leafage. The portion of the plot under these trees was about 15 metres square. The trees stood till 1912.

In 1912 this land was taken over by a horticultural colleague, who in the summer of 1913 pointed out to me that one portion refused to grow anything, the immediate crop being orange seedlings. On referring to old plans it was found that this infertile spot exactly coincided with the tree-cotton portion. The land had been heavily manured, and the cause of its infertility appeared to be the presence of some toxic substance. Analysis showed 0·5 °/₀ of NaCl as against 0·1 °/₀ elsewhere, in the top 10 centimetres of soil.

Similar phenomena were shown during 1913 by soil in the wire-gauze cages which had been erected for the protection of pure strains from natural crossing in 1912. The growth in these cages during 1912 was extremely rank, though the plants were prolific, and the amount of leaf shed appeared to have been approximately ten times that which the land would have received in normal field crop.

In both cases the particular sites were situated above patches of clay, so that no natural under-drainage existed, and any salts, or other soluble bodies, once brought up to or placed upon the surface, could not be washed down and away by irrigation.

No valid explanation could be found which would apply to both cases, excepting that the leaves had deposited something in the upper soil, of which NaCl was one component. The amount of NaCl was, however, scarcely sufficient to produce the observed effects, and possibly toxic bodies had resulted from decomposition of the leaves. Some support was lent to this conclusion by the unsatisfactory nature of cotton leaf-mould for potting purposes.

If substantiated, this conclusion forms a strong additional argument against the use of tree-cottons in intensive cultivation. Established trees flourish in such soil, because their deep roots are well below the toxic surface layer, but new sowing cannot be effected.

These episodes led to the enquiry into salinity of the cell-sap.

* See Sir G. Watt, *The Wild and Cultivated Cottons of the World.*

Specific Salinity in the Cell Sap of Pure Strains. By W. LAWRENCE BALLS, M.A., St John's College.

[*Read* 4 May 1914.]

The following determinations were made on the leaf-lamina of various species and pure strains of cotton growing at Giza, Egypt, on "sweet" land, which contained not more than 0·1 °/₀ of salt in the surface layers even after long deprivation of water, except where otherwise stated.

The percentages are expressed as NaCl in terms of dry-weight (air-dry) of leaf tissue; dry weight is approximately 25 °/₀ of fresh weight.

The method employed was to incinerate slowly in a porcelain crucible under a layer of calcium carbonate, dissolving, filtering, adding slight excess of silver nitrate, and back-titrating with potassium thiocyanide. In making the later analyses I have to acknowledge the assistance of M. Jacques Garboa.

The results are strictly preliminary, my departure from Egypt having interrupted the work.

Ordinary field crop. From plots yielding about 500 lbs. of lint to the acre, sown with Domains Afifi variety, leaf-salt percentages were 1·68, 1·65, 1·47, 1·2, 1·1.

Determinations were made as to the number of leaves borne on, and shed by, various individual plants; these were then compared with the mean plant of the flowering-curves for these plots, and from these data it appears that the amount of salt brought up from the deep soil and deposited upon the surface during a single season's growth amounts to about 15 kilogrammes per acre.

Tree cottons, and plants growing in cages (see previous communication) will shed about 100 kilogrammes per annum, which is sufficient to produce bad effects if not removed by drainage.

Pure strain No. 77. This strain was cultivated over thirty acres of land. Determinations made in various normal places gave percentages: 0·63, 0·80, 0·76, 0·76, 0·74, 0·69, 0·65, 0·60, 0·46, with a mean of 0·68.

The last three figures were obtained in a part of the land where there was exceptionally free natural under-drainage.

In a clay patch, where the subsoil water was isolated, and exceptionally salt*, the leaf-salt of the plants was found on two samples to be 0·85 and 0·88.

* W. L. B., "A study of some water-tables at Giza," Well No. 8. *Cairo Scientific Journal*, 1914.

First-year plants in the salted cages gave 0·90 and 1·09, while second year plants (rattoons) in the cages gave 1·53 and 1·86, and earlier analyses from the same plants gave 1·03, 1·36, 1·47, 1·72, 1·93.

There are thus variations in the content of a pure strain, directly with, but not proportionate to, the variations in salinity of the soil.

Pure strain No. 310. It is interesting to note that this strain is agriculturally useless near Cairo, but flourishes in the salter lands of the cooler northern Delta, where No. 77 is useless.

Several plants were growing so close to plants of No. 77 that their roots were interlacing. The analyses from three such pairs, two samples from each, are given below:

							Mean	Ratio
No. 310	0·96	1·17	0·90	1·04	1·30	1·52	1·15	100
No. 77	0·76	0·76	0·80	0·62	0·90	1·09	0·82	71

Similar indications were found with other strains, that a definite specific difference in salinity existed between them.

These numerical differences might be due to differences in the leaf-structure, but there is no evidence in favour of such a view. The probable error of dry-weight determinations from equal leaf-areas of 15 sq. cm. on all kinds of Egyptian cotton is only $\pm 4\ ^\circ/_\circ$.

It is interesting to note that the highest figure recorded for American Upland cottons grown in the U.S.A. that I have been able to find, is only 0·1 $^\circ/_\circ$, while a few determinations at Giza on Russell and King varieties ranged from 0·37 to 0·81.

Conclusions. Egyptian cotton growing in typical field crop has a salt content which indicates a concentration of 0·3 $^\circ/_\circ$ NaCl in the cell-sap.

This concentration varies with the salinity of the soil, though not proportionately.

It also varies with the particular pure strain or variety of Egyptian cotton employed.

Plants of two Egyptian strains growing with interlacing root-systems may show differences of as much as 10 : 7 in the salinity of their cell-sap.

The latter fact may have some utility in future breeding of strains for the salty lands of the Northern Delta of Egypt, where large schemes of reclamation are in progress.

Egyptian cotton (*Gossypium peruvianum*, &c.) may be classed as a facultative halophyte.

Pre-Determination of Fluctuation. (Preliminary Note.) By W. LAWRENCE BALLS, M.A., St John's College.

[*Read* 4 May 1914.]

A. A simple example of this phenomenon is provided by fluctuations in the length and wall-thickness of the unicellular epidermal hairs which coat the seed of the cotton plant, forming the commercial "lint."

Cytology shows that the elongation of the lint-hair is completed within the first 25 days from the opening of the flower, after which the thickening of the cell-wall begins. It might be expected that length would be mainly determined by environmental conditions acting round about the 16th day of maturation of the capsule, and thickness by conditions round about the 40th day, the capsule opening on the 50th day.

Statistical examination of lint ripened from flowers opening on sixty successive days, with determinations of thickness made in the form of Breaking-strains of single fibres, gives fluctuation curves showing rapid changes of considerable magnitude.

The correlation between length and strength in one set of samples was $-0{\cdot}42$. If the curves are shifted 21 days, so as to bring them into relative positions which the cytological evidence has indicated ($40 - 16 = 24$ *circa*), they are seen to be very closely similar, and the value of "r" turns over to $+0{\cdot}60$.

Thus the length of lint in capsules which were going to open on Sept. 21st could be foretold by determining the strength of those capsules which were already open on Sept. 1st.

Subsequent events cannot affect the pre-determined length, though the subsequent thickening may be of any degree.

B. Flowering is a character which fluctuates enormously, and is not usually regarded as a "character" at all. There are nevertheless definite differences between species and varieties in this respect, largely dependent on the branching habit.

From a large series of statistical records of the flowering day by day in plots of Egyptian cotton, assisted lately by comparison with similar records taken by other workers using the writer's methods*, it has become clear that daily fluctuations in the number of flowers opening on the Mean Plant are independent of the site (over several square miles)†, the water-supply, age of the plant, space allowed per plant, and the variety grown.

* *Egyptian Government Survey Department Papers*, 24 and 31.

† Since communicating this note I have found the fluctuations to be simultaneous in sites separated by 170 kilometres; the plants of all Egypt all attempt to behave alike.

This remarkable parallelism lasts till mid-July at least, and may be resumed after an interval of confusion.

The cause can only be climatic, not edaphic, since differential soil treatment does not disturb it. Yet at the time when it is manifest the plant has passed under edaphic control*.

Statistical investigations by Mr H. E. Hurst and by the writer, comparing these fluctuations of flowering with those of wind, temperature, humidity, evaporation, cloudiness, &c., both at the time of flowering and during the few preceding days, gave no result.

After some three years, as further data accumulated, the writer traced the cause to pre-existing fluctuations in the growth-curve of the central axis, nearly a month before.

There is a similarity between the daily curves of growth and flowering, while the smoothed curves from at least two separate years—one of them very abnormal—are practically identical.

The growth-curve of the central axis is the same as those of the flowering branches until mid-June. The curve thus records the rate at which the scaffolding is laid down, upon which the flower buds are differentiated. The bud takes rather less than a month to open from its first differentiation, so that fluctuations in the rate of construction of the scaffolding are repeated in the rate of opening of the flowers.

Under the conditions of ordinary cultivation of cotton in Egypt, subsequent environmental influences exercise very little deforming effect.

The same applies to the fruiting-curve, which is closely similar to the flowering-curve, though of less amplitude, so that the arrivals of Egyptian cotton at Liverpool in October are almost entirely determined by the night-temperatures, &c. in Egypt in May.

Conclusions.

1. That we are largely in error when we seek to ascertain the causes of Fluctuation in some feature of an organism by integrating the effects of those environmental factors which were operating at, or nearly before, the time when the feature in question was manifest.

2. That such factors merely exercise a subsidiary deforming influence on a scaffolding which was pre-determined—or pre-destined—often at a very much earlier stage in the life of the organism.

3. A conception of Discontinuity is thus introduced into the study of Fluctuation, which has formerly been regarded as typical of Continuity.

* W. L. B. "*The Cotton Plant in Egypt.*" London, 1912, chap. III.

The Ammonia Content of the Waters of Small Ponds. By
J. T. SAUNDERS, M.A., Christ's College.

[*Read* 4 May 1914.]

NOTE. The ammonia referred to in this paper is the "free" ammonia and the amount is expressed in grammes of NH_3 per million cubic centimetres of water.

Free ammonia is always present in the waters of small ponds, but the amount varies very greatly. In ponds where there is sewage contamination such as horseponds, the amount is very high, as much as 16·2. Here the variation is chiefly affected by the amount of sewage that is introduced. Where there is no sewage contamination the amount is much lower, the maximum quantity for such a pond is about ·500—·600.

Broadly speaking it is in the winter that the greatest quantities of ammonia are found in uncontaminated ponds, and the amount falls off during the spring to rise again in the autumn to its winter maximum. This is the annual cycle of changes. Changes however take place from day to day, and even different layers of water show slightly different quantities of ammonia.

One of the chief factors controlling the ammonia content in these small ponds is the amount of the rainfall. Copious rain after a period of sunshine effects a very considerable reduction in the ammonia present, a reduction that is out of all proportion to the extra amount of water that has entered the pond.

During the present year February was a dry month; there was only a total of ·85 inch* of rain on 13 days. But March was extremely wet, the wettest but one on record for 50 years. On 24 February the ammonia content of a small pond on Sheeps' Green, Cambridge, was ·425. It continued to remain at about this level until the beginning of March. On 4 March the amount contained was ·400, but on 10 March the amount had fallen to ·213. This fall I attribute to the heavy rains, for on the two previous days ·58 inch of rain had fallen and there had been ·75 inches since 1 March. The ammonia content continued at a low level during the wet weather of March. On 17 March the ammonia content had sunk to ·115, and on 18 March it was further lowered to ·060. This second fall again may be attributed to heavy rain, for on 16 March ·25 inch of rain had fallen and

* The rainfall was measured by Mr Pain of Sidney Street, Cambridge, who very kindly allowed me to use his records.

on the four days previous to this, ·63 inch. After this the rain did not fall so heavily but it still continued to be wet weather until 12 April, when there commenced a period of 10 days without rain. On 14 April the ammonia content was ·082—still very low. On 21 April the amount had risen to ·175. I think that there can be no doubt that the rainfall is influencing the ammonia content, since a rapid drop in the ammonia follows closely on heavy rains. There is also an increase during a dry period but the increase takes very much longer to manifest itself than does the decrease. The annual cycle in which the ammonia content rises to its maximum in the winter and falls to a minimum in the summer cannot be explained as being due to the rainfall, for it is during the winter maximum that the greatest rainfall occurs. All that the rainfall does is to cause fluctuations.

Variations in the Ammonia Content in different layers of water.

In small ponds, even though they may be only a few feet deep, there are differences in the ammonia content between water collected from the surface and water collected from just above the bottom. On 17 March the water in a pond contained ·138 grs. p.m. NH_3 at the top and ·088 at the bottom. On 18 March, in the same pond, there were ·075 at the top and ·05 at the bottom, and again on 21 April there were ·15 at the top and ·175 at the bottom.

Variation in Ammonia Contents of different ponds.

Ponds that are liable to sewage contamination contain large and very varying quantities of ammonia, while others which are uncontaminated contain a smaller quantity and the range of variation from pond to pond is very much less. Thus in different uncontaminated ponds on different dates the quantities of ammonia were ·180, ·210, ·310, and ·400, whereas in different contaminated ponds the quantities were 1·65, 3·020 and 16·200.

Variations in the Ammonia Content produced artificially.

In the Laboratory I have four aquaria, two of which are aerated, one by a rapid and the other by a slow stream of air, and two others which have no aeration, one of which is left standing and the other has a plunger in it working up and down so as to keep the water in motion. The two which are aerated always have less ammonia in them than do those which have no aeration. It seems to make very little difference whether the aeration be rapid or slow, nor does the plunger which stirs the water make

much difference between the two which are not aerated. Thus on 6 March 1914, after the aquaria had been kept for 15 months, we find the following quantities of ammonia:

Rapid Aeration	·125
Slow Aeration	·138
Standing with Plunger	·25
„ no „	·238.

Thus it is clear that aeration helps to diminish the quantities of ammonia that may be present.

A reduction in the ammonia content can also be produced in the laboratory by dilution, but the experiments on this point are not yet complete.

Effect of Variations of Ammonia Content on the Fauna.

It is well known that the smaller green algae derive their supplies of nitrogen from ammonia and consequently these algae will be affected by variations in the ammonia content. Some of these small green algae live a planktonic life in the water, and where they occur in sufficient numbers they provide nourishment for swarms of crustacea and other small zooplankton. In fact the zooplankton only reaches large proportions when there are plenty of these minute algae. Daphne and Simocephalus feed on these algae, the "nannoplankton" as it is called, and their presence may be taken to indicate the presence of minute algae as well. Sometimes the algae occur in such vast numbers as to be visible to the naked eye, as in the "green water" of stagnant puddles, but as often as not it is difficult to detect their presence except by special means.

During the latter part of February this nannoplankton existed in such numbers in a pond on Sheeps' Green as to be visible to the naked eye, but directly the rains of March began it disappeared. At the time I was inclined to attribute the disappearance to disturbance of the water and increased turbidity, but now since I have been able to find a large quantity of nannoplankton in turbid horseponds, I do not think that turbidity is accountable for the extinction. I am inclined to think that it was due to the reduction in the ammonia content, which I have shewn above to have been caused by the heavy rainfall, and this is supported by further evidence.

All ponds that have a low ammonia content, less than ·4, never contain the swarms of Cladocera which are dependent on the nannoplankton for food. Thus in two ponds in the same field the ammonia contents were 1·65 and ·2 (the high ammonia content is due to this pond being used as a watering-place for cattle), and

the pond with the high ammonia content contained swarms of Daphne while the other had none. In my aquaria the two which are aerated have no small crustacea living in them, while those which are unaerated and have a higher ammonia content contain plenty. A high ammonia content, coinciding with the presence of quantities of nannoplankton, was also found in a jar, which was one of two kindly given to me by Mr Elton, of Christ's College. Both jars were started with collections from a small pond, but in one the water had turned green through the presence of minute algae while in the other it had remained clear. The green water contained 1·520 and the clear water ·300. In these cases we find that the presence of much nannoplankton coincides with a high ammonia content and its absence with one below ·400. From this we must conclude that the ammonia content of the water is one of the factors controlling the appearance in large quantities of these minute algae.

The rainfall, then, through its influence on the ammonia content, will react on the swarms of crustacea, for the swarms are dependent on the minute algae and these in their turn on the amount of ammonia. Thus, in all probability, one of the factors—there must be others—which cause the appearance and disappearance of large swarms of Cladocera and Copepods in small ponds is the presence or absence of much rain.

Optically active substances of simple molecular constitution. By Professor POPE and JOHN READ, M.A.

[*Read* 18 May 1914.]

Notwithstanding numerous attempts, it has not hitherto been possible to prepare an optically active substance containing fewer than three carbon atoms in the molecule and the assumption has therefore been made that a considerable degree of complexity is necessary to enable the molecule to exist in stable enantiomorphous forms.

After unsuccessful attempts to resolve chlorosulphoacetic acid and chlorobromomethanesulphonic acid the preparation and investigation of chloroiodomethanesulphonic acid were undertaken with a similar object in view, and eventually the resolution of this substance was effected with *d*- and *l*-hydroxyhydrindamine, strychnine and brucine. The purest optically active ammonium salt of this acid yet obtained, having $[M]_{5461} + 43{\cdot}7°$ in dilute aqueous solution, was prepared by repeated fractional precipitation with brucine, followed by decomposition of the brucine salt with ammonia; but the separation of the substance in a state of optical purity presents great difficulty. The above-mentioned salt crystallises from alcohol in colourless scales melting at 227—228°. The corresponding salt of the externally compensated acid crystallises similarly and melts at 221—222°. Barium *dl*-chloroiodomethanesulphonate, $(CHClI.SO_3)_2Ba + 2H_2O$, crystallises from water, in which it is exceedingly soluble, in large, lustrous, colourless plates. *dl*-Chloroiodomethanesulphonic acid is a colourless, hygroscopic liquid which evolves heat when mixed with water; it crystallises slowly when kept in vacuo over sulphuric acid.

It is remarkable that the optically active ammonium salt, which, containing only one carbon atom in the molecule corresponding to less than five per cent. of carbon, is the simplest optically active substance known, retains its activity with great persistence and cannot be caused to racemise by any of the ordinary agents employed for that purpose.

Some further experiments on eutectic growth. By F. E. E. LAMPLOUGH, M.A., Trinity College, and J. T. SCOTT, B.A.

[*Read* 18 May 1914.]

The method of "quenching" an alloy during the solidification of the eutectic has been used to ascertain the character of the eutectic during its growth. Resulting from the investigation it has been possible to classify eutectics into two classes, (1) those of spherical radiating growth, (2) those exhibiting definite crystal contours. The former are always produced when both primaries are of rounded contour, the latter if one primary is of crystal shape. The cause of "halos" surrounding primary crystals has been demonstrated.

Note on the detection of Malonic Acid. By Dr H. J. H. FENTON, Christ's College.

[*Read* 18 May 1914.]

An account was given, in a previous communication to the Society, of a reaction of malonic ester with bromomethylfurfural which serves for the identification of hexoses. When these two compounds are mixed together in alcoholic solution and the mixture is made just alkaline with alcoholic potash, an intense blue fluorescence results, which is persistent at very great dilution.

Conversely, this reaction may be employed as a distinguishing test for free malonic acid, or its salts. The substance is mixed with anhydrous methyl or ethyl alcohol containing hydrogen chloride, or sulphuric acid warmed and allowed to stand for a few minutes; the mixture is then neutralised and tested with bromomethylfurfural in the manner described. The production of this blue fluorescent substance, under the conditions specified, appears to be entirely characteristic of malonic ester and ethyl malonic ester. Other esters containing labile hydrogen atoms, such as acetoacetic and dicarboxyglutaconic esters, do not behave similarly.

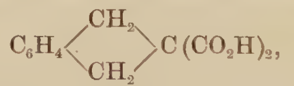
On the Resolution of 5-*Nitrohydrindene*-2-*carboxylic Acid.* By W. H. Mills, M.A., Jesus College, H. V. Parker, B.A., and R. W. Prowse, B.A.

[*Read* 18 May 1914.]

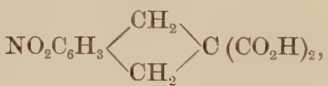
With the object of obtaining an optically active derivative of benzene in which to account for the optical activity it would be necessary to take into consideration the relative distribution in space of the groups attached to the benzene nucleus 5-nitrohydrindene-2-carboxylic acid (III) has been prepared and has been shown to be resolvable into two optically active components.

The racemic compound was prepared by nitrating hydrindene-2-2-dicarboxylic acid (I) and eliminating one carboxyl group from the product by heating in an indifferent solvent.

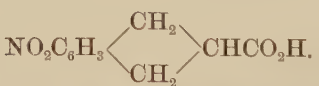
I.



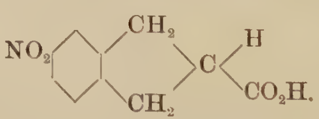
II.



III.



The position of the nitro group was established by oxidising the nitrodicarboxylic acid (II) with potassium permanganate. The product was 4-nitrophthalic acid from which it follows that in nitrohydrindene carboxylic acid the nitro group must occupy the 5-position.



The acid was resolved by means of quinine, the recrystallised quinine salt giving on decomposition the *l*-acid. The *d*-acid after extraction from the quinine salt mother liquors was separated from the accompanying racemic acid by crystallisation from benzene in which the latter is relatively sparingly soluble. The racemic acid melts at 122°, the active acids at 116°. The specific rotation of the active acids for the mercury green light is $[\alpha]_{5461}^{20^\circ} + 36{\cdot}4$ and $-36{\cdot}5^\circ$.

PROCEEDINGS AT THE MEETINGS HELD DURING THE SESSION 1913—1914.

ANNUAL GENERAL MEETING.

October 27th, 1913.

In the Cavendish Laboratory.

PROFESSOR HOBSON, IN THE CHAIR.

The following were elected Officers for the ensuing year:

President:

Dr Shipley, Master of Christ's College.

Vice-Presidents:

Prof. Pope.
Dr Barnes.
Prof. Seward.

Treasurer:

Prof. Hobson.

Secretaries:

Mr A. Wood.
Mr F. A. Potts.
Mr G. H. Hardy.

Other Members of the Council:

Prof. Sir J. J. Thomson.
Mr J. E. Purvis.
Mr R. P. Gregory.
Dr Cobbett.
Mr J. Mercer.
Dr Marshall.
Mr G. R. Mines.
Mr F. J. M. Stratton.
Prof. Woodhead.
Mr C. Forster Cooper.
Dr Duckworth.

The following Communications were made:

1. On the dependence of the relative ionisation in various gases by β rays on their velocity, and its bearing on the ionisation produced by γ rays. By R. D. KLEEMAN, B.A., Emmanuel College.

2. Note on a Dynamical system illustrating Fluorescence. By N. P. McCLELAND, M.A., Pembroke College.

November 17*th*, 1913.

In the Comparative Anatomy Lecture Room.

DR SHIPLEY, PRESIDENT, IN THE CHAIR.

The following was elected an Associate of the Society:

H. D. L. Minton, Jesus College.

The following Communications were made:

1. A possible connexion between abnormal sex-limited transmission and sterility. By Dr DONCASTER, King's College.

2. The Flight of the House-fly. By E. HINDLE, B.A. (Communicated by Mr C. Warburton.)

3. Sex proportions of *Forficula auricularia* in the Scilly Islands. By H. H. BRINDLEY, M.A., St John's College.

November 24*th*, 1913.

In the Cavendish Laboratory.

PROFESSOR NEWALL, IN THE CHAIR.

The following were elected Fellows of the Society:

S. Lees, M.A., St John's College.
D. G. Lillie, M.A., St John's College.

The following Communications were made:

1. Distribution of Stars in relation to Spectral Type. By Professor A. S. EDDINGTON, Trinity College.

2. (1) The comparison of nearly equal electrical resistances.

(2) An experiment on the harmonic motion of a rigid body. By Dr G. F. C. SEARLE, Peterhouse.

3. A Double-Four Mechanism. By G. T. Bennett, M.A., Emmanuel College.

4. On the Presence of certain Lines of Magnesium in Stellar Spectra. By F. E. Baxandall. (Communicated by Professor Newall.)

February 9th, 1914.

In the Botany School.

Professor Seward, Vice-President, in the Chair.

The following Communications were made:

1. Exhibition of specimens illustrating the damage caused by certain wood-boring beetles. By Hugh Scott, M.A., Trinity College.

2. The History of the occurrence of Azolla in the British Isles and in Europe generally. By A. S. Marsh, B.A., Gonville and Caius College. (Communicated by Professor Seward.)

3. Amitosis in the Parenchyma of water plants. By R. C. McLean. (Communicated by Professor Seward.)

4. On Root Development in *Stratiotes aloides* L. with special reference to the occurrence of Amitosis in an embryonic tissue. By Mrs Arber. (Communicated by Dr E. A. Newell Arber.)

February 17th, 1914.

In the Cavendish Laboratory.

Professor Sir J. J. Thomson, in the Chair.

The following Communication was made:

Recent work at the Heidelberg Observatory on the Nebulae and their Spectra. By Dr Max Wolf.

February 23rd, 1914.

In the Cavendish Laboratory.

PROFESSOR SIR J. J. THOMSON, IN THE CHAIR.

The following Communications were made:

1. (1) Determination of the effective aperture of the stop of a photographic lens.

(2) Experiments with a prism of small angle.

By Dr G. F. C. SEARLE, Peterhouse.

2. On the Molecular Field in Diamagnetic Substances. (Preliminary Note.) By A. E. OXLEY, B.A., Trinity College.

3. The Superior and Inferior Indices of Permutations. By Major P. A. MACMAHON.

4. A Simplification of the logic of Relations. By N. WIENER. (Communicated by Mr G. H. Hardy.)

5. The Domains of steady motion for a liquid Ellipsoid, and the oscillations of the Jacobian figure. By R. HARGREAVES, M.A., St John's College.

6. The oxygen content of the river Cam before and after receiving the Cambridge sewage effluent. By J. E. PURVIS, M.A., Corpus Christi College, and E. H. BLACK.

March 9th, 1914.

In the School of Agriculture.

DR SHIPLEY, PRESIDENT, IN THE CHAIR.

The following were elected Fellows of the Society:

P. C. Varrier Jones, M.A., St John's College.
H. W. Leatham, B.A., Trinity College.

The following Communications were made:

1. A Statistical Study of Feeding Trials with Oxen and Sheep. By Professor WOOD and G. UDNY YULE, M.A., St John's College.

2. Fluctuations of Sampling in Mendelian Ratios. By G. UDNY YULE, M.A., St John's College.

3. Inheritance in Brassicæ. By M. S. PEASE, B.A., Trinity College. (Communicated by Professor Biffen.)

4. A Determination of the Best Value of the Coupling Ratio from a given set of data. By G. Udny Yule, M.A., St John's College and F. L. Engledow, B.A., St John's College.

5. A Case of Repulsion in Wheat. By F. L. Engledow, B.A., St John's College. (Communicated by Professor Biffen.)

6. Soil and Crop Relations in the Biggleswade Market Garden Area. By T. Rigg. (Communicated by Professor Wood.)

7. Digestibility of Pentosans. By H. A. D. Neville, B.A., Emmanuel College. (Communicated by Professor Wood.)

8. A case of Correlation in Wheat. By W. H. Parker, B.A., Trinity College. (Communicated by Professor Biffen.)

9. The Factorization of large Numbers. By H. C. Pocklington, M.A., St John's College.

10. The Ionisation produced by certain substances when heated on a Nernst Filament. By Dr F. Horton, St John's College.

May 4th, 1914.

In the Comparative Anatomy Lecture Room.

Dr Shipley, President, in the Chair.

The following were elected Fellows of the Society:

W. Dawson, M.A., Gonville and Caius College.
K. J. J. Mackenzie, M.A., Christ's College.
A. S. Marsh, B.A., Gonville and Caius College.
R. V. Southwell, B.A., Trinity College.
T. L. Wren, B.A., St John's College.

The following Communications were made:

1. (1) A note on Leaf-fall as a factor in Soil-deterioration, and Specific salinity in the cell-sap of Pure Strains.

(2) Pre-determination of Fluctuating characters.

By W. L. Balls, M.A., St John's College.

2. The ammonia content of the waters of small Ponds. By J. T. Saunders, M.A., Christ's College.

3. (1) *Thompsonia*, a little known Crustacean parasite.

(2) The gall-forming Crab, *Hapalocarcinus*.

By F. A. Potts, M.A., Trinity Hall.

May 18th, 1914.

In the Chemical Laboratory.

DR SHIPLEY, PRESIDENT, IN THE CHAIR.

The following were elected Honorary Members of the Society:

Dr H. E. Armstrong.
Professor J. Bordet.
Madame Curie.
Professor F. Czapek.
Professor T. W. Edgeworth David.
Colonel W. C. Gorgas.
Professor P. H. von Groth.
Professor Jacques Hadamard.
Dr George E. Hale.
Dr François A. A. Lacroix.
Dr Charles Lapworth.
Professor H. Lebesgue.
Dr Jacques Loeb.
Professor Arthur Looss.
Professor H. A. Lorentz.
Professor M. Planck.
Lt.-Col. Leonard Rogers.
Professor Gustav Schwalbe.
Dr Karl Schwarzschild.
Dr D. H. Scott.
Professor E. B. Wilson.
A. F. Yarrow.
Professor P. Zeeman.

The following Communications were made:

1. Optically active substances of simple molecular constitution. By Professor POPE and JOHN READ, M.A.

2. Note on the detection of Malonic Acid. By Dr FENTON, Christ's College.

3. Some further experiments on eutectic growth. By F. E. E. LAMPLOUGH, M.A., Trinity College and J. T. SCOTT, B.A., Queens' College.

4. On the Resolution of 5-Nitrohydrindene-2-carboxylic Acid. By W. H. MILLS, M.A., Jesus College, H. V. PARKER, B.A. and R. W. PROWSE, B.A.

5. (1) On the nature of the Internal Work done during the Evaporation of a Liquid.
(2) The Work done in the Formation of a surface Transition Layer of a Liquid Mixture of Substances.

By R. D. KLEEMAN, B.A., Emmanuel College.

6. A contribution to the Theory of Relative Position. By N. WIENER. (Communicated by Mr G. H. Hardy.)

INDEX TO THE PROCEEDINGS

with references to the Transactions.

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